UNIVERSITY OF TORONTO
3 1761 01087634 0

20

THE TUTORIAL PHYSICS

VOLUME I.

A TEXT-BOOK OF SOUND

The Tutorial Physics.

I.—TEXT-BOOK OF SOUND. By E. Catchpool, B.Sc Lond. *Fifth Edition.*

II.—HIGHER TEXT-BOOK OF HEAT. By R. Wallace Stewart, D.Sc. Lond. *Second Edition.*

III.—TEXT-BOOK OF LIGHT. By R. Wallace Stewart, D.Sc. Lond. *Fifth Edition.*

IV.—ADVANCED TEXT-BOOK OF MAGNETISM AND ELECTRICITY. By R. W. Hutchinson, M.Sc., A.M.I.E.E. In Two Vols. *Second Edition.*

V.—PROPERTIES OF MATTER. By C. J. L. Wagstaff, M.A. *Third Edition.*

VI.—PRACTICAL PHYSICS. By W. R. Bower, B.Sc. A.R.C.S., and John Satterly, D.Sc., M.A. *Second Edition.*

INTERMEDIATE TEXT-BOOK OF MAGNETISM AND ELECTRICITY. By R. W. Hutchinson, M.Sc., A.M.I.E.E.

Acoust
C

THE TUTORIAL PHYSICS

VOLUME I.

A TEXT-BOOK OF SOUND

BY

EDMUND CATCHPOOL, B.Sc.

REVISED AND ENLARGED

BY

JOHN SATTERLY, D.Sc., M.A., F.R.S.C.

ASSOCIATE PROFESSOR IN PHYSICS IN THE UNIVERSITY OF TORONTO

Fifth Edition (*Twelfth Impression*)

384021
9.9.40

LONDON : W. B. CLIVE

University Tutorial Press Ltd.

HIGH ST., NEW OXFORD ST., W.C.

1925

QC
225
C35
1913
cop. 3

PRINTED IN GREAT BRITAIN BY THE UNIVERSITY TUTORIAL PRESS LD. AT THE BURLINGTON PRESS, FOXTON, NEAR CAMBRIDGE.

PREFACE TO THE FIRST EDITION.

In writing this text-book I have tried to keep in view the following aims:—

To include all the facts of which a knowledge is expected in elementary examinations.

To present a clear picture of the external physical processes which cause the sensation of sound. As the mathematical symbols for a physical process are easily made a substitute for a clear conception of the process, the mathematical symbols have been avoided as far as possible.

To keep before the reader the distinction between phrases which describe actual processes or conditions and phrases which, while they facilitate the prediction of real processes and real phenomena, do not themselves stand for any physical condition or event. Such phrases are used in every department of physics, and are only mathematical symbols in disguise. Where I have departed from, or added to, the usual forms of explanation, it has usually been with the intention of making this distinction clearer.

As the book is intended as a physical treatise, it is the physical processes which cause the sensations of sound, and not the sensations themselves, which form its subject-matter. Only those peculiarities of the sensation are considered which throw light on the external physical processes which are taking place.

E. C.

PREFACE TO THE FIFTH EDITION.

In view of the continued popularity which this book has maintained during the fourteen years which have elapsed since its publication, it is sufficiently obvious that it must possess considerable merit. At the same time it has now been considered advisable to subject the book to a careful revision in order to ensure that it should include the results of research and satisfy modern requirements.

The revision has consisted more of additions than of alterations: a few alterations and changes in order have been made, and a few pages have been replaced by matter taken from other books in the series, but practically the whole of the original text has been retained. The additions include a large number of instructive experiments, with full descriptions of apparatus and manipulation, and a good collection of questions and numerical examples.

In the original edition mathematical formulae were almost entirely excluded, with the object of concentrating the reader's attention on the physical processes, and not allowing them to become obscured by mathematical symbols. In the course of the revision a certain number of formulae have, however, been introduced.

In its present form the book will be found to provide a suitable course of work for Intermediate and Final University Examinations for a Pass Degree.

J. S.

CONTENTS.

CHAPTER I.

PAGE

VIBRATORY MOTION 1

Examples I. 34

CHAPTER II.

PROGRESSIVE UNDULATION 37

Examples II. 64

CHAPTER III.

VELOCITY OF SOUND... 67

Examples III. 98

CHAPTER IV.

SUPERPOSITION 101

Examples IV. 116

CHAPTER V.

FREQUENCY 117

Examples V. 139

Examination Questions I. 139

CHAPTER VI.

FORCED VIBRATIONS. RESONANCE. 142

Examples VI. 153

CHAPTER VII.

FOURIER'S THEOREM 155

Examples VII. 171

CHAPTER VIII.

PAGE

THE EAR AND HEARING 172

Examples VIII. 215

CHAPTER IX.

REFLECTION AND REFRACTION OF SOUND 217

Examples IX. 248

CHAPTER X.

STATIONARY UNDULATION 251

Examples X. 264

Examination Questions II. 264

CHAPTER XI.

VIBRATIONS OF AIR IN PIPES 267

Examples XI. 305

Examination Questions III. 307

CHAPTER XII.

TRANSVERSE UNDULATION 311

Examples XII. 365

Examination Questions IV. 368

ANSWERS 371

TEXT-BOOK OF SOUND.

CHAPTER I.

VIBRATORY MOTION.

1. Cause of Sound.—In every case in which the sensation of sound can be traced to an external cause, we find that the cause is something in a state of vibration. This vibration is often sufficiently great to be evident to the eye; not that the movements can be actually watched, for they are always too rapid for that, but they often produce the blurred indistinctness of outline which we know from experience to result from rapid movement. This is well seen in the string of a harp, or along the edge of a large bell, when these are producing loud sounds. When the vibration has become too small to be visible, it can still be detected by touching the string or bell with the tip of the finger, which is able to perceive, as separate and successive, movements which follow one another much too rapidly for the eye to distinguish. If sound is still heard when even touch detects no vibration, we find that a suspended pith ball, held so as to hang just in contact with the sounding body, is driven away every time it touches the surface, and we naturally attribute this to blows or taps given to the ball by the sounding body, and infer that there is still vibration.

Exp. 1.—Mount a large bell-jar so that it can easily be set in vibration by bowing with a violin bow (Fig. 1). By its side suspend a pith ball by a silk fibre so that the ball rests in contact with the upper rim of the jar. Gently bow; the pith ball is repeatedly driven away as though it had received a number of rapidly delivered blows. The experiment may be varied by using a tuning-fork instead of a jar.

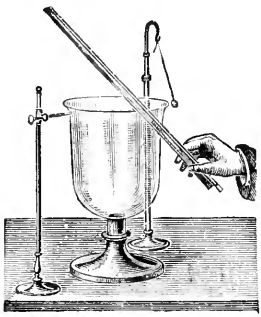


Fig. 1.

There are many other simple experiments which illustrate the same point. A tumbler or wine-glass can be held by the base and set into vibration by rubbing a wet finger along the rim. If the front be taken off a piano it will be seen that the hammer sets the wire into quick vibration and that this causes the sound. If a bicycle bell is rung and the edge of the rim lightly touched by the finger it will be found to be in rapid vibration. A drum, a tuning-fork, a stretched piece of elastic, all show the vibration clearly. It is not so obvious in a whistle or a flute, but later on it will be shown that the vibrating substances in these two instances is air.

The vibration of the body cannot, of course, be the immediate cause of the sensation of sound. It is a fact so familiar as to seem an axiom that "nothing acts except where it is"; the immediate cause of the sensation must be something in contact with the sense organ, the ear. If we stretch a thin membrane, such as goldbeater's skin, on a vertical ring of metal, so as to form a sort of tambourine, and hang a pith ball just in contact with the membrane, it will be found that the pith ball is driven away whenever the arrangement, which is called an **acoustic pendulum**, is in any place where sound can be heard; by special devices for detecting the movement of the pith ball, it is found that this is true even for the faintest audible sounds. We infer that sound is only heard in places where a stretched membrane would be in vibration.

The vibration of the membrane depends on some special condition of the air surrounding it, for if the "acoustic

pendulum" is under the exhausted receiver of an air-pump, it does not vibrate even when loud sounds are audible everywhere round the receiver. And this condition of the air not only requires for its production a vibrating body: the vibrating body must be in contact with the air. Thus, if one of the deep-toned clocks, which strike on a coiled wire, is suspended by pieces of cotton inside the exhausted air-pump receiver, no sound is heard when the hammer strikes the wire, although the wire visibly vibrates. If the clock, instead of being suspended, stands on the air-pump plate, it will be heard to strike, but in this case it can be shown that the plate itself is vibrating, and the plate is in contact with the air.

Exp. 2.—*Show that sound cannot pass through empty space.* Under the receiver of an air-pump place an alarum clock set to ring in about five minutes. The clock must be supported on a thick pad of soft felt or suspended by silk threads so that the vibrations may not be transmitted to the metal plate of the pump and so to the outer air. Now exhaust the air in the receiver to the highest degree possible (for this purpose a very good air-pump is requisite, for unless the pressure can be reduced to a quarter of an inch of mercury at the very most, it is quite useless to attempt the experiment). By the time the exhaustion is effected, the alarum will begin to go off and the hammer may be seen to strike the bell rapidly. If, however, a sufficiently high vacuum has been obtained no sound will be heard. Before the alarum ceases readmit the air; the sound will become at first slightly audible and then louder and louder.

Instead of an alarum clock an electric bell may well be used in this experiment.

With the simple apparatus shown in Fig. 2, we can exhaust the air without the help of a pump, and at the same time so vary the experiment as to bring out a number of striking results.

Exp. 3. *Show that the intensity of the sound depends on the density of the medium.* Fit an indiarubber stopper to a strong glass flask (Fig. 2). Bore two holes through the stopper, and through one pass a tightly fitting glass rod. Attach a small bell by a piece of india-rubber tubing to the rod so that when in position the bell can be rung by shaking the flask. Pour now an inch of water into the flask and boil it briskly over a Bunsen flame. The steam rushes out through the other hole in the cork. After five minutes boiling remove the burner and push another glass rod into the other hole in

the stopper. All the air has been expelled, and when the flask cools the space around the bell contains only water vapour at a very small density.

Fig. 2.

Now shake the flask: only a feeble sound is heard. That *any* sound is heard is due to the fact that the water vapour transmits the vibrations from the bell to sides of the flask; the sound is but a feeble one on account of the lesser density of the water vapour as compared with air. We are led to suspect that the sounds will be louder or fainter just as the gas or vapour is heavier or lighter—that the intensity of the sound varies with the density. If the flask be filled with vapours or gases of various densities, we can see if the results bear out this inference, and it will be found that when the flask is filled with *coal gas*, which is lighter than air, a fainter sound will be produced. On using *hydrogen* (the lightest known gas) the sound heard is still more feeble. *Carbon dioxide* gas (carbonic "acid") is heavier than air, and when the bell is rung in a flask filled with this gas, the sound produced is slightly louder than in air.

In these experiments, it is to be observed that the density of the surrounding air does not necessarily vary. The loudness of the sound, then, depends on the density of the gas or vapour in which it *originates*. Of course we are leaving out of account in the meantime other things which influence the intensity of the sound, such as the violence of the blow which produces the sound, and the material of which the sonorous body is composed.

On ascending into mountainous regions it is found that sounds become more faint, and in high altitudes, where the atmosphere is very rare, all sounds are much enfeebled.

Exp. 4.—*To show that sound can be transmitted by a solid body.* A. Place a watch sufficiently far away from the ear that its ticks cannot be heard through the air. Now take a long plank (a bench will do), place the watch on one end and apply the ear to the other. The sound is readily heard.

B. In Fig. 3 *R* is a long rod fixed to a resonance box *B*.* A tuning-fork *F*, to which the box resounds, is struck; when held in the air only a feeble sound is heard. When the shank of the fork

* See Art. 72.

touches the end of the rod the sound is louder than before. The vibration has been conducted down the rod and communicated to the air by the resonance box *B*.

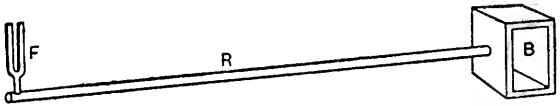


Fig. 3.

C. *Make a toy telephone.* Take two cardboard boxes, bore a small hole in the bottom of each, thread the ends of a long cord through these holes and tie knots at the ends of the cord inside the boxes. Keeping the cord stretched, let one observer hold one box to his ear and another speak into the second box. The sounds will be transmitted along the cord. In some cases the transmission is so clear that conversation can be carried on.

D. Surround a musical box with several layers of felt. No noise is heard. Introduce a wooden rod connected with a wooden plate through a hole in the layers so as to touch the box. The sound will be propagated along the rod and will be heard outside.

It is also, in common experience, a noticeable fact that sound does not readily pass from one medium to another of very different density. Thus the sound made by the wheel of a train on the rails of the line is transmitted to great distances along the rails, but it is not audible in the air even quite close to the rail. If, however, the ear be applied directly to the rail, or to the end of a rod of wood touching the rail, the sound of the wheels on the rail is distinctly heard. The wave motion passes directly from the iron to the bones of the head, or from the iron to the wood, and from the wood to the bones of the head, and so reaches the inner ear. If a square of light wood were fastened to one end of the rod of wood and the other end applied to the rail the sound might become audible in the air near the square of wood.

Similarly the sound of water running in pipes underground is transmitted through the ground and can be heard by applying the ear to the surface of the ground or to the end of a sounding-rod resting on the ground. Unless the sound is very loud it cannot, however, be heard in air.

The action of the stethoscope used by medical men is another illustration of the same point. The sounds of the lungs and heart in action are transmitted through the surrounding tissues of the body, but not, to any appreciable extent, to the external air. They cannot therefore be heard by placing the ear close to the chest, but if the ear and side of the head rest on the chest wall the sounds can be heard with some distinctness. They are, however, more conveniently and more distinctly heard by means of the stethoscope. In the wooden form of the instrument the sound is conducted to the air by the wood, and also, and perhaps mainly, by the column of air extending in the tube and ear from the surface of the body to the drum of the ear. In the biaural forms of the instrument the sound is conducted to the ear by the air in the cup and tubes of the instrument.

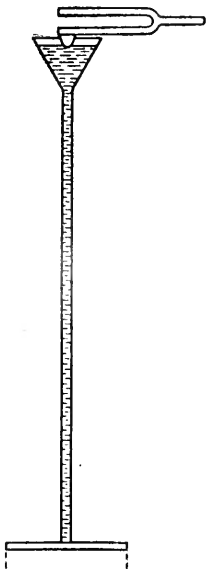


Fig. 4.

Exp. 5.—*To show that sound can be propagated through a liquid.* Fix a long tube (Fig. 4) into the top of a resonance box. Fill it with water. Take a tuning-fork which matches the resonance box and stick a cork cone to one of the prongs. Sound the fork. Now dip the cone into the water. The sound is increased, showing that the vibrations are transmitted down to the water to the resonance box.

2. Speed of Sound.—In the foregoing experiments it appeared as if the commencement of the vibration and the arrival of the sound at the ear were simultaneous. But if a person be at a considerable distance from the body producing the sound, it becomes evident that sound takes some time to travel. The sound of thunder is not heard until some time after the lightning flash is seen, and the flash from a distant cannon is observed before the

report is heard. But in each case the flash and the noise start simultaneously, being produced by the same operation.

Soldiers, it is said, on the wall of a besieged town have observed the flash of the enemy's gun some time before the shot struck the fortifications, and heard the report last of all. Therefore the speed of a cannon-ball, though much less than the speed of light. is greater than the speed of sound in air.

Though movements of the air cannot be seen or felt, like those of solid bodies, it seems obvious that a condition of the air which is produced by the vibrations of a body in contact with it, which spreads from the air near the vibrating body to that at a distance, and which enables the air to set in vibration light membranes which expose a large surface to it, must be a vibratory motion of the air itself, and this becomes a certainty when we find that the velocity with which the condition travels is exactly that with which it can be calculated that a vibratory movement would spread from one part of the air to another.

This condition of vibratory movement spreads not only through air, but through all known kinds of molecular matter, and air is not in all cases the substance whose movement produces the sensation of sound. Thus persons whose ears are so defective in structure that vibrating air does not affect them can often hear a watch held between their teeth, and in this case it is through the bones, not through air, that the vibration spreads.

The sensation of sound is never produced by vibrations of the ether of space,* nor does the vibratory condition which causes sound ever spread from one material substance to another through a region which contains ether only; this has been shown in the air-pump experiment described. It is also proved by the fact that the tremendous explosions which constantly take place on the sun are quite inaudible on earth.

Though vibratory movements which are very slow, like that of a pendulum, or extremely rapid, like those of the air in some very small whistles, do not produce any sensation of sound, yet, as such vibrations do not differ in their

* See *Text-Book of Light*, § 132.

physical nature from those which are audible, it is convenient to include all in the same department of physics. The physical study of sound then includes all kinds of vibration of molecular matter, but not the vibrations of the ether.

3. Strict Meaning of "Vibration."—Any point is said to vibrate when it goes through the same, or nearly the same, series of movements at regular or nearly regular intervals. In popular language the term is confined to motion backwards and forwards along the same straight line, but the scientific use of the term is not so limited, and a point moving in a circle or a figure of 8 is also said to be vibrating. The time occupied by the complete series of movements which constitute a vibration, from the moment when the point passes any given position, to the moment when it passes the same position, in the same direction, to go through the same series of movements again, is called the **period** of the vibration, and is usually denoted by t. The number of such complete periods in one second is called the **frequency** of the vibration, and is usually denoted by n. Evidently n is always equal to $\frac{1}{t}$.

When a point moves backwards and forwards in a straight line, its vibrations are called **rectilinear**. In the case of rectilinear vibrations, most French, and some English, writers, count the time taken in moving from one end of the line to the other as the period of the vibration; we shall follow the usual English custom, and consider it only half a period. One (complete) vibration is therefore a double swing or two oscillations.

4. Curves of Displacement.—A point may vibrate in a straight line in a great variety of ways; for instance, it may move rapidly in one direction and slowly in the other, like the piston of a Cornish pumping-engine; or quickly at one end of its path and slowly at the other, like an elastic ball dropped on the ground and rebounding again and again; or quickly in the middle of its path and slowly at the ends, like a pendulum-bob, or a ball dancing up and

down at the end of a piece of elastic; and so on. These differences constitute the *character* of a vibration.

In all these cases, if we marked on paper the actual track of the vibrating body, that track would be simply a straight line, and there would be nothing to indicate the differences in the characters of the movements. In order to show these differences graphically, the following device is often used: Suppose the body is vibrating vertically; imagine that a sheet of paper is drawn horizontally at a uniform

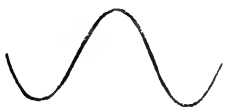


Fig. 5.

Fig. 6.

rate from right to left behind the body, and that the body leaves a trace on the paper. The form of this trace will depend on the relative velocities of the body at different times of its vibration. Thus, a ball dancing up and down at the end of a piece of elastic would leave a trace like Fig. 5, while a ball dropped on a level surface and rebounding would mark out Fig. 6. (In each of these cases successive vibrations are drawn exactly similar in extent and period; the gradual loss of velocity due to imperfect elasticity is neglected.)

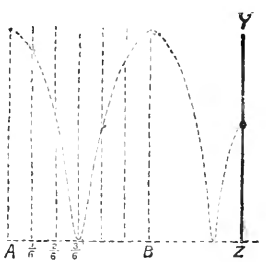


Fig. 7.

Suppose in this last instance the rebounding ball moved up and down a fixed vertical line *YZ*, Fig. 7,* *Z* being the point of the level surface from which it rebounded, and let the ball have left a trace, shown by the dotted curve, on a sheet of paper drawn uniformly from

* In all curves in this book which represent the condition of the same body at different times, a point to the *left* of another represents the condition at an earlier moment.

right to left behind it. A will be a point of the paper which was behind Z when the ball was at Y, and B will be a point on the paper which was behind Z when the ball was at Y again after an interval t, which is the period of the vibration. If we divide AB into say six equal parts, and draw on the paper verticals through the points of division, these verticals are lines which coincided with YZ at instants $\frac{t}{6}$, $\frac{2t}{6}$, etc., after the instant when A passed behind Z, and the distance up any vertical from AB to the curve is the distance that the ball was from Z at the moment when that vertical coincided with YZ.

So that, instead of supposing the curve actually traced by the moving body, we may construct it geometrically. Mark off any length AB on a horizontal line to represent the period of vibration, and divide it into, say, six equal parts. At the first point of division put up a perpendicular called an *ordinate*, equal in height to the distance of the moving body from some fixed point, when it has been moving for $\frac{1}{6}$ of its period, and so on. The curve can then be drawn through the ends of these ordinates. Or, if the movements of the point are inconveniently long or short for such a diagram, the ordinates may be made longer or shorter than the actual distances in any convenient ratio; the curve will still indicate the character, though no longer the actual extent, of the vibratory movement.

Such a curve, in which equal distances along a horizontal line represent equal intervals of time, while the distance of the curve from the line at each point is proportional to the distance of the moving point from a fixed one at the corresponding instant of time, is called a **curve of displacement**. Practical methods of determining the frequencies and displacement curves of points on vibrating bodies will be described later.

EXAMPLES.—(1) The displacement curve of a particle moving uniformly along a straight line is a straight line inclined to the axis of time and passing through the point on this axis which represents the time when the particle began to move.

(2) The displacement curve of a particle moving with uniform acceleration is a parabola whose axis is perpendicular to the axis of time.

5. Curve of Velocities.—Another way of distinguishing between rectilinear vibrations of different characters, which, though not quite so simple, is of much greater value in the study of sound, is to draw a curve whose ordinate at each point is proportional to the *velocity* instead of the *displacement* of the vibrating body at the moment represented by the position of the ordinate along the horizontal line; velocities in one direction being represented by heights above that line, velocities in the opposite direction by distances below it.

As an instance of this, we take again the case of a ball falling and rebounding vertically, whose curve of displacements was given in Fig. 7, and construct its curve of velocities. Mark off along a horizontal line any length, such as AB (Fig. 8), to represent the time of a complete vibration, and let A represent the moment when the ball is at its highest point and just about to begin falling, while B represents the moment when it has just reached the highest point again. At the moment corresponding to A the ball has no velocity, and the length of the ordinate to the curve at A is zero. After the lapse of $\frac{1}{6}$ of a complete period the ball has a downward velocity, represented by FG, and this downward velocity increases uniformly with the time, as shown by the straight portion AD. Just before half the period is completed, at the moment represented by H, the ball touches the ground, and in the short time represented by HJ, the downward velocity represented by HD is changed into the upward one represented by JE. At the instant represented by J, the ball ceases to touch the ground, and from this moment till the end of the period the upward velocity uniformly diminishes, as shown by the straight line EB.

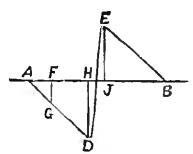


Fig. 8.

The character of a rectilinear vibration is fully defined by either its displacement curve or its velocity curve, and either curve can be easily drawn when the other is given. The advantage of the velocity curve will appear later (Art. 18).

6. The Motion of a Pendulum.—An ideal simple pendulum consists of a small heavy particle suspended by a weightless string. Of course we can never reach this ideal in practice, but a spherical metal bob attached to a length of cotton thread is a near enough approximation.

The bob of a pendulum moves under the influence of two forces, its own weight and the stretching force in the string by which it is supported. The resultant of these two forces may be shown to be almost exactly proportional to the displacement of the bob from its position of rest.

Exp. 6.—*To show that when a pendulum-bob is pulled aside the force required to keep it aside is proportional to the displacement.* Suspend a heavy bob B (Fig. 9) by means of a *long* thread from a support. Tie a thread to the suspension thread at a point P near the bob and attach it to a spring balance. Hold the spring balance horizontal and fix a horizontal scale on the level of P so that when B is at rest the reading is zero. Now pull the spring balance aside, keeping the string horizontal until the displacement of P is about 5 cms. Take the scale reading and the reading of the spring balance. Correct the latter for the horizontal position of the balance, and so find the pull in the horizontal cord. Repeat the observations at different displacements. Plot a curve, taking pulls as ordinates and deflections as abscissae. It will be a straight line, showing that the *force giving rise to the displacement is proportional to the displacement.* Hence when the pendulum is oscillating the bob moves towards the position of rest impelled by a force proportional to the displacement, and therefore the acceleration is proportional to the displacement. This also admits of an easy mathematical proof.

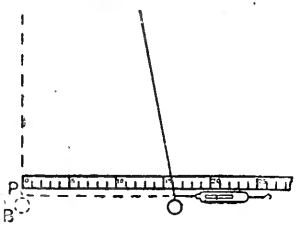


Fig. 9.

7. Circular Motion.—Suppose that a particle is constrained to move uniformly with speed v round a circle ABD of radius a (Fig. 10). If the constraint were removed the particle would move along a straight line. The constraint must therefore exert such a force on the particle that the acceleration produced is sufficient to keep the particle moving in the circle. Let A and B be two very near

positions of the particle, the particle arriving at B a short time t after leaving A. Let AP represent the velocity at A, BQ that at B. To find the change in velocity between A and B draw the triangle of velocities apq in which ap represents the velocity AP, aq the velocity BQ, and therefore pq the change of velocity between A and B. Let O be the centre of the circle ABD. Join OA, OB. Now the triangle apq is similar to the triangle OAB.

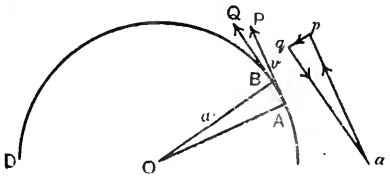


Fig. 10.

$$\therefore \frac{pq}{ap} = \frac{AB}{OA},$$

i.e.
$$\frac{\text{Change of Velocity}}{v} = \frac{vt}{a},$$

$$\therefore \text{Change of Velocity} = \frac{v^2t}{a}.$$

But the acceleration is equal to the change of velocity divided by the time, therefore

$$\text{acceleration of the moving particle} = \frac{v^2}{a}.$$

The direction of the acceleration is given by pq which is parallel to the bisector of the angle AOB. It is therefore directed towards the centre of the circle ABD.

The acceleration may also be expressed in terms of the angular velocity and the radius. Let $\omega =$ the angular velocity of the line joining the particle to the centre of the circle. Then $v = a\omega$,

$$\therefore \text{ acceleration} = \frac{v^2}{a} = \frac{a^2\omega^2}{a} = \omega^2 a.$$

The acceleration may also be expressed in terms of the periodic time. For if $T =$ periodic time,

$$\omega = 2\pi/T,$$

$$\therefore \text{ acceleration} = \left(\frac{2\pi}{T}\right)^2 a = \frac{4\pi^2 a}{T^2}.$$

The force acting on the particle

$$= \text{mass} \times \text{acceleration, and}$$

$$\therefore = \frac{mv^2}{a},\ m\omega^2 a,\ \text{or}\ m\frac{4\pi^2 a}{T^2}.$$

8. Simple Harmonic Motion.—Let B (Fig. 11) be a point moving uniformly round a circle of radius a in a period T, and A be a point moving up and down a diameter CD so that AB is always perpendicular to CD. Then A is said to execute a **simple harmonic vibration** or to move with **simple harmonic motion.***

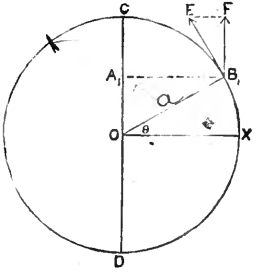


Fig. 11.

The maximum distance a vibrating particle moves from its central position is called the **amplitude.** The amplitude may also be defined as one-quarter of the total distance travelled by the vibrating point in a complete vibration. In the case taken the amplitude is equal to the radius of the circle along which B moves. From a casual glance at the figure it is evident that a body executing a simple harmonic vibration moves fastest in the middle of its path and slower as it nears the ends.

We will now consider the motion in greater detail. Let X and O be the initial positions of B and A, and B_1, A_1 the positions after a time t. The displacement of $A = OA_1 = a \sin B_1OX = a \sin \theta$ (say) where $\theta = \angle B_1OX$. The velocity of A equals the component, parallel to CD, of the speed of B. Now the speed of B is equal to

$$\frac{\text{circumference of circle } CDX}{\text{periodic time}},\ i.e.\ \text{to}\ \frac{2\pi a}{T},$$

$\therefore$ the velocity of A when in the position A_1

$$= \frac{2\pi a}{T} \cos EB_1F = \frac{2\pi a}{T} \cos \theta = \frac{2\pi a}{T} \sin (\theta + 90).$$

* Simple harmonic vibration may be more concisely defined the projection of uniform circular motion.

Now θ is the angle through which OB has rotated from OX, and $\theta + 90°$ is the angle which OB will have rotated after another quarter of a period. Therefore **the velocity of a harmonically vibrating point any instant is $\frac{2\pi}{T}$ times the displacement which it will have a quarter of a period later.**

If v be the velocity, then

$$v = \frac{2\pi a}{T} \cos \theta,$$

$$\therefore\ v^2 = \frac{4\pi^2 a^2}{T^2} \cos^2 \theta = \frac{2\pi^2 a^2}{T^2} \{1 + \cos 2\theta\}.$$

Now as θ takes all values from 0 to π, $\cos 2\theta$ goes through a complete cycle of values in which it is as often positive as negative. Hence the average value of $\cos 2\theta$ is zero, and therefore the average value of v^2 is $\frac{2\pi^2 a^2}{T^2}$. The largest value of v^2 is when $v = \frac{2\pi a}{T}$, and is therefore $\frac{4\pi^2 a^2}{T^2}$; hence the average is half the maximum. Since the kinetic energy of the particle is $\frac{1}{2}$ mass $\times v^2$, the average kinetic energy is one half of the maximum kinetic energy. This can be expressed in another way. The whole energy of the system, kinetic and potential, does not alter. When the particle is passing its equilibrium position, its potential energy is zero and its kinetic energy greatest, hence the whole energy = maximum kinetic energy, and is double the average kinetic energy. It is a general rule in all vibratory motion that the average kinetic energy = the average potential energy reckoned from the equilibrium position; each of these being half the total energy.

EXAMPLE.—Show that the energy of a particle of mass m vibrating with amplitude a and period T or frequency n is $\frac{2\pi^2 m a^2}{T^2}$ or $2\pi^2 m a^2 n^2$.

The acceleration of A is the rate of change of the velocity of A. The velocity varies harmonically, therefore

the acceleration is $\frac{2\pi}{T}$ times the velocity which A will have a quarter of a period later, *i.e.* $\frac{4\pi^2}{T^2}$ times the displacement half a period later, *i.e.* $\frac{4\pi^2}{T^2} a \sin(\theta + 180)$ or $-\frac{4\pi^2}{T^2} a \sin\theta$. This is $-\frac{4\pi^2}{T^2} \times$ the present displacement of A. The acceleration of A is therefore proportional to the displacement of A, but is in the opposite direction.

This last relation could also be got by considering the motion of B. For the acceleration of A = the component, parallel to CD, of the acceleration of B. But the acceleration of B is $\frac{4\pi^2 a}{T^2}$ directed towards the centre of the circle, therefore the acceleration of A, when at A_1, is $\frac{4\pi^2 a \sin\theta}{T^2}$.

The ratio of the acceleration to the displacement is $-\frac{4\pi^2}{T^2}$, and so is independent of a, the amplitude. If, therefore, the force on a body is such as to produce an acceleration which is always $-\mu$ times the displacement where μ is constant, the body will vibrate harmonically in a period T such that $\mu = \frac{4\pi^2}{T^2}$ or $T = 2\pi\sqrt{\frac{1}{\mu}}$, and this period is independent of the amplitude. So that when, for instance, a steel rod is fixed in a vice, bent aside, and let go, the period of its vibration does not alter as the vibration itself dies away. Vibrations whose periods do not alter are called Isochronous. We infer that harmonic vibrations are isochronous; but isochronous vibrations are not necessarily harmonic.

Exp. 7.—*To show that the motion of the bob of a simple pendulum is harmonic.* Set up two long simple pendulums of the same length from the same support. If possible suspend them from the ceiling. Pull them aside and then start them moving, one in a vertical plane and the other along a conical surface. With considerable trouble they can be started together so that the line joining the two bobs is always perpendicular to the straight line along which the former

bob moves. The former bob is therefore moving in Simple Harmonic Motion.

Exp. 8.—*To show that the time of swing of a pendulum is independent of the amplitude as long as this is small.* Set up a simple pendulum, say one metre long, by the edge of the bench. Draw a chalk mark on the bench behind the equilibrium position. Set the pendulum in vibration and standing well back take the time of 50 vibrations. Repeat the observations and from the mean find the time of one vibration. Do this for different angular amplitudes, say for 5°, 10°, 15° . . . . Your results will show that for angular amplitudes not greater than 20° the time of swing is very nearly constant, while as the angular amplitude increases beyond 20° the time of swing also increases.

9. Development of a Sine Curve.—Let A (Fig. 12) be a point performing a simple harmonic vibration along the line CD, the amplitude being OC. It is required to plot a curve showing the relation between displacement of A and the time which has elapsed since it was at O.

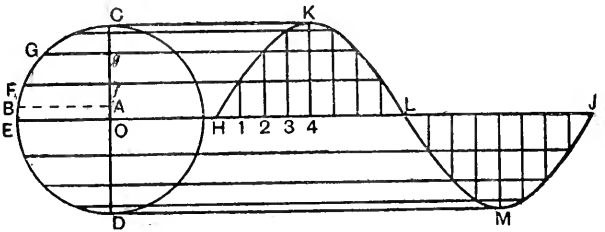


Fig. 12.

Draw a circle ECD on CD as diameter and take a moving point B on the circumference, so that BA is always perpendicular to CD. Then B moves uniformly around the circle ECD. Divide the circumference of ECD into any number of equal parts, say sixteen, so that B takes equal times in moving from E to F, F to G, and so on, *i.e.* A takes equal times in moving from O to f, f to g, etc.

To draw the displacement curve for this movement, we take a horizontal line and cut off a part HJ so that HJ represents the time occupied in one complete vibration. Divide HJ into sixteen equal spaces, to represent the

equal intervals of time occupied by A in passing from O to f, f to g, etc., back to O, and at these points draw ordinates equal (or, if preferred, proportional) to the distances of O, f, g, etc., from some fixed point, which may conveniently be the middle point O of CD. The distances from O of the points below O must be considered as of opposite sign to the distances of those above, and the corresponding ordinates drawn in the opposite direction. If we start at the moment when A is passing through O on its way upwards, the curve drawn through the ends of the ordinates will be like the one shown in Fig. 12. Such a curve is called a harmonic or sine curve, the latter name being given because, as can easily be shown, the ordinate at each point of $HKLMJ$ is proportional to the sine of the angle which OB makes with OE at the instant represented by that point.

It is very often necessary to draw a harmonic or sine curve with a semblance of accuracy. An easy way of doing this is got directly from the fact that the sines of 0°, 30°, 90° are respectively zero, one-half, and unity. Let AK (Fig. 13) represent one complete vibration

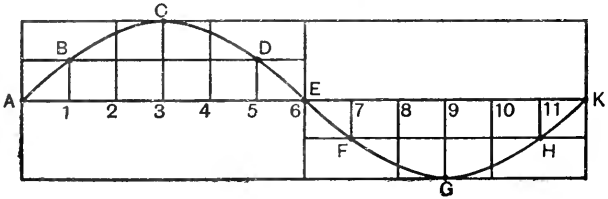


Fig. 13.

either in angle or time. Draw rectangles on each side of AK whose heights are equal to the amplitude. Bisect each rectangle by a line parallel to AK. Divide AK into twelve equal parts, and through the 1st, 3rd, 5th, 7th, 9th, and 11th points of division erect perpendiculars to AK, thus obtaining the points $ABCDEFGHK$. Draw the curve through these points freehand. To show that the point D is on the sine curve:—AK represents 360° $\therefore$ AE represents 180° and $\therefore$ $A5$ represents 150°. The sine of 150° = sine of 30° = one-half.

Any quantity is said to vary harmonically with the time when it changes so as to be at every instant proportional to

the distance of a harmonically vibrating point, such as A (Fig. 12), from some fixed point in CD; so that we may have harmonic velocities, pressures, or electric currents. In any case where a quantity varies harmonically, if we put up ordinates at equal distances along a line, and make the ordinates proportional to the value of the quantity at equal intervals of time, the curve through the ends of the ordinates is a harmonic or sine curve. Then the rate at which the quantity is altering is, by Art. 8, $\frac{2\pi}{T}$ times the value which the quantity itself will have a quarter of a period later.

We have seen that, in general, the velocity curve of a vibrating point is quite different to its displacement curve. But if a point vibrates harmonically its velocity also varies harmonically (see Art. 8), so that its velocity curve is also a sine curve. Its velocity at any moment is, however, not proportional to its displacement at that moment, but to the displacement which it will have a quarter of a period afterwards. Therefore the curve of velocities is a quarter of a period behind the curve of displacement, as shown in Fig. 14.

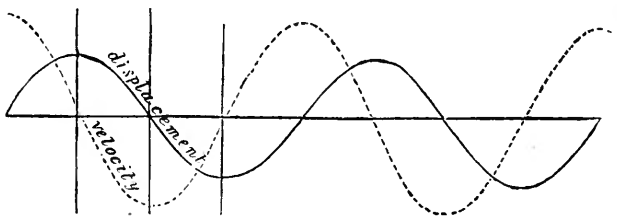


Fig. 14.

When a point vibrates harmonically, it was shown in Art. 7 that its acceleration is always towards, and proportional to its displacement from, its mean position. So that if the force on a material particle, when the particle is displaced from its position of equilibrium, is at every stage of displacement directed towards the position of

equilibrium, and proportional to the amount of displacement, the particle will vibrate harmonically. We shall see later that, even when a vibration is not harmonic in character, it may be conveniently treated as the resultant of two or more harmonic vibrations. It is to this fact that the importance of harmonic vibrations is due.

10. Phase.—The *phase* of a harmonically vibrating point usually means the fraction of a whole period of its vibration which has elapsed since it last passed its mean position in the direction which we are counting as positive. Some writers, however, count the time which has elapsed since it started from the positive end of its vibration. Upwards and to the right are usually counted the positive directions for vertical and horizontal vibrations respectively.

If two points, vibrating harmonically in the same period, are in the same phase at one instant, they are, of course, always in the same phase, and if they are not, the difference of phase between them may be conveniently defined by stating the difference between the times at which they cross their mean positions in the positive direction as a fraction of the whole period of either; it may also be stated as the corresponding angle of revolution of the uniformly revolving point, either in degrees or in radian measure. Thus, in Fig. 12, if we suppose f and g to be positions at the same instant of two points vibrating harmonically in the same period, and that they are moving in the same direction, their phase-difference may be called $\frac{1}{16}$ or $\frac{\pi}{8}$ or $22\frac{1}{2}°$, as we prefer.

When the two vibrations are of different periods, their phases change at different rates, and there is no constant phase-difference between them, and there will be instants at which the two vibrations are in the same phase. The "phase-difference" is, in this case, usually taken to mean the difference between the times at which the points pass through their mean positions in the positive direction, expressed as a fraction of the greatest common measure of their periods, but other notations are also used.

11. Composition of Harmonic Vibrations.—A point may execute two or more harmonic vibrations at the same time. This does not mean, as it appears to, that a point A can be moving, at any one instant, with respect to a given point of reference such as the earth, with more than one velocity or in more than one direction. A conventional meaning is attached to the phrase. Harmonic motion, like all motion, is, of course, relative; thus the piston of the engine of a steamer which travels up and down the cylinder in such a way that its distance from the end varies harmonically, is vibrating harmonically with respect to the cylinder, though if the ship is rolling at the same time the movements of the piston with respect to the earth may be very complex. If, then, a point A is moving harmonically with respect to a body B, while B is moving harmonically with respect to the earth, the movement of A with respect to the earth is said to be compounded, or the resultant, or the sum, of the two harmonic motions, or, more shortly, A is said to be executing both movements at once. This is not very accurate, but it is not misleading, since there is no other sense in which a point can be moving in two ways at once.

When a point is executing two rectilinear harmonic vibrations at once, its motion need not be either rectilinear or harmonic; if the periods are incommensurable, it is not even a vibration. Of this we shall have many instances.

12. Lissajous' Figures.—An important case occurs when the two harmonic motions are at right angles to one another; for instance, let A vibrate horizontally with respect to a point B, while B vibrates vertically with respect to the paper.* If the periods of the two vibrations are incommensurable, the motion of A will never exactly repeat itself; but if m horizontal vibrations occur in exactly the same time as n vertical ones, then at the end of this period the movements of A, both horizontal and vertical, will exactly repeat themselves, and the point will describe the same closed curve over and over again. The form of this curve will depend upon, and may therefore be

* We shall call lines parallel to the length of the page vertical lines, and lines drawn across the page horizontal lines.

used to determine, the amplitudes, relative periods, and phase-difference of the two vibrations.

When these are given, the curve may easily be constructed by a method which will be best understood from the following example. Suppose we require to know what curve will be traced by a point which vibrates vertically and horizontally at the same time, the periods of the vertical and horizontal vibrations being equal and their amplitudes equal to, say, 4 cms., and the two vibrations being in the same phase. Imagine that there is a sheet of glass lying on the page, with a dot on it which we will call *A*, and a horizontal line through *A*. If a point *B* moves harmonically right and left along this line, with an amplitude of 4 cms. on each side of *A*, and at the same time the plate of glass moves harmonically up and down the page with an amplitude of 4 cms., then *B* is said to be vibrating

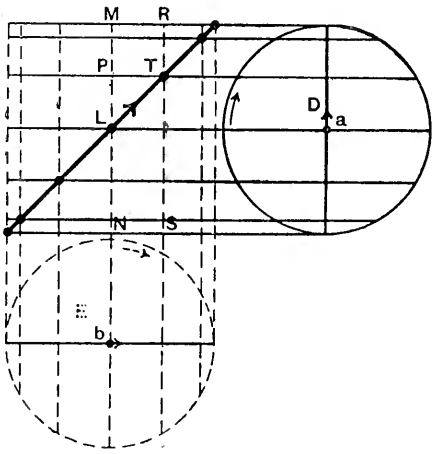


Fig. 15.

both vertically and horizontally with respect to the paper, and the line on the paper over which *B* passes is the curve required.

(1) **Let us suppose the particles are in the same phase.**—In this case suppose that L (Fig. 15) is the point on the paper which is under the dot A at the same moment when A and B are both at the middle points of their vibrations, and therefore coincide. MN is the line over which A moves up and down, and if we draw a circle D of 4 cms. radius on the paper with the centre on the same level as L, and divide the circumference of this circle into 12 equal parts, and draw horizontal lines to MN through the points of division, then LP is the distance A will move in $\frac{1}{12}$ of its period, and so on.

If we draw *on the glass* another circle E of 4 cms. radius, with its centre directly below A, and divide its circumference into 12 equal parts, and draw vertical lines such as RS on the glass through the points of division, then, as B starts moving on the glass from A to the right at the same moment that A starts from L, B will reach the line RS on the glass at the same moment that A is above the point P on the paper. B is therefore at this moment above the point T on the paper, for B is always on the same level as A; and LT is the line on the paper over which B has passed. Similarly, B crosses each of the rectangles diagonally, so that it moves over a line on the paper like the one shown in the figure. The path is a straight line and B travels as indicated by the arrow.

In Figs. 15 to 18 the complete lines are supposed drawn on the paper, and to remain stationary, while the system of dotted lines is supposed drawn on the glass and to be moving up and down with it.

(2) **Let us suppose the particles have a phase-difference of $\frac{1}{4}$.**—Suppose that L (Fig. 16) is the point on the paper which is under the dot A at the moment when A is at the middle of its vibration and B is at the end of its vibration. With the same construction as before, LT is the first part of the path, and it is evident that the complete path is a circle traversed as indicated by the arrow.

(3) Let us now suppose the phase-difference is $\frac{1}{8}$, so that when the glass plate or the dot A is in the middle of its path and moving upward B has been moving to the right for $\frac{1}{8}$th of its period. To find the initial position of B draw

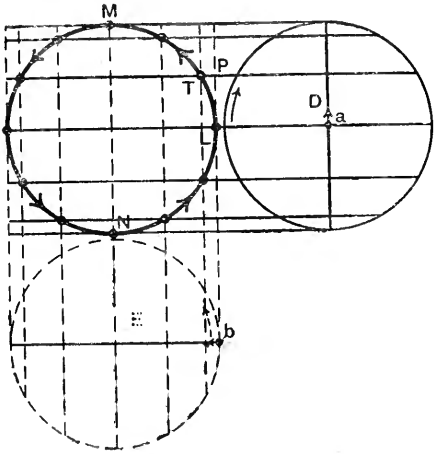


Fig. 16.

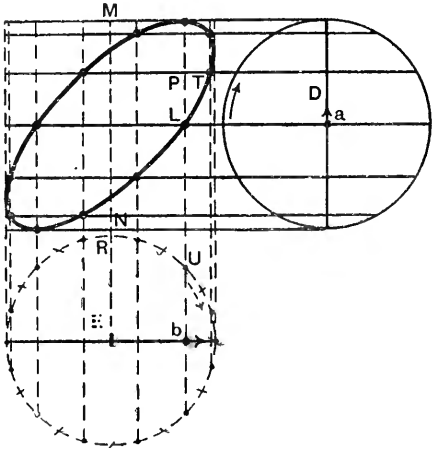


Fig. 17.

a line perpendicular to the middle of the path of B to cut the E-circle in R, and mark off RU equal to $\frac{1}{8}$ the circumference of the E-circle (Fig. 17). Then b the foot of the perpendicular drawn from U to the horizontal diameter of the E-circle is the initial position of B. To get the points on the circle giving the position of B at intervals of $\frac{1}{12}$th of the period after leaving b, divide the circumference of the E-circle into 12 parts starting from U. Plot the

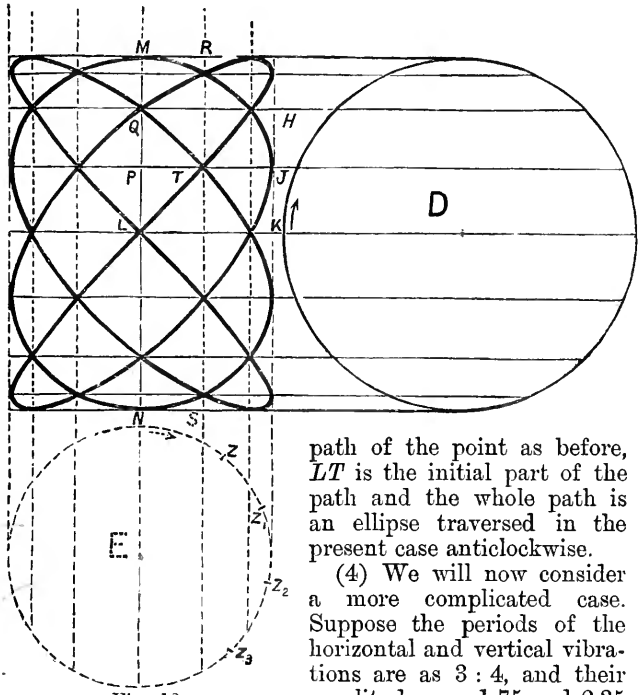


Fig. 18.

path of the point as before, LT is the initial part of the path and the whole path is an ellipse traversed in the present case anticlockwise.

(4) We will now consider a more complicated case. Suppose the periods of the horizontal and vertical vibrations are as 3 : 4, and their amplitudes are 1·75 and 2·35 cms. respectively, the vibrations starting in the same phase. Since the periods are as 3 : 4, $\frac{1}{16}$ of the period of the plate is equal to $\frac{1}{12}$ of the

period of B, hence we draw the circle D (Fig. 18) 2·35 cms. in radius and divide it into 16 parts, and the circle E 1·75 cms. in radius and divide it into 12 parts. Draw horizontal and vertical lines as before. The curve $LTQNLMJL$ is the resultant path of B.

If there is a phase-difference, say $\frac{1}{3}$, between the vibrations, and the horizontal vibration is in advance, then (since $\frac{1}{3}$ of the G.C.M. of the periods represented by 3 and 4 is $\frac{1}{3}$ of the shorter or $\frac{1}{12}$ of the longer) at the moment when A is over L, B will have arrived at a vertical line on the glass passing through Z, to a point $\frac{1}{3}$ of the circumference of E from N, and will be where this line cuts LK. If we divide E into 12 parts, beginning at Z, and number these Z_1, Z_2, etc., then B will pass through the point on PJ, which is exactly above Z_1, the point on QH which is exactly above Z_2, and so on, and a different curve will be traversed.

The circles might have been divided into any other numbers of parts in the ratio 4 : 3, instead of into 16 and 12.

These curves are known as *Lissajous' figures.* Their forms, for some of the most important ratios and phase-differences, are shown in Fig. 19, where the numbers at the left hand show the ratio of the period of the horizontal to that of the vertical vibration. As the figures stand on the page, they correspond to the phase-differences from 0 to $\frac{1}{2}$ as indicated above them; if the page is inverted, the same figures will correspond to the phase-differences from $\frac{1}{2}$ to 1, as shown by the fractions which will then be above them. The first, second, and third figures in the top line of Fig. 19, and the first figure in the fourth line, we have already drawn in detail; the others are readily obtained by the same method.

If n horizontal vibrations take very nearly but not exactly the same time as m vertical ones, the curve traced will be very nearly one corresponding to the ratio $m : n$, but when m vertical vibrations are finished, rather more or less than exactly n horizontal ones have been performed, and the phase-difference is not exactly the same as at starting. A slightly different curve, corresponding to a different phase-relation, is therefore traced in the next cycle, and as the phase-difference alters during each repetition of the figure, a series of curves corresponding to gradually increasing or diminishing phase-differences are traced in turn.

If the horizontal and vertical vibrations are nearly equal in period and are at first in the same phase, the curve traced in successive vibrations will gradually change, assuming in turn the forms of the first line of Fig. 19 from left to right and back again, and returning to its original form when one movement has been repeated once more than the other. This gives a very accurate method of determining the difference between the frequencies of two vibrations. The practical details are given in Chapter V.

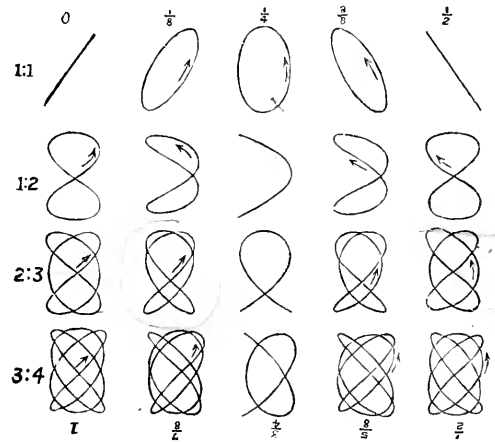


Fig. 19.

To deduce the ratio of the periods, count the number of times the curve cuts any horizontal line drawn across it, and the number of times it cuts any vertical line, and take the ratio. Thus taking the first figure in the third row of Fig. 19, the curve cuts a horizontal line four times, and a vertical line six times; so the ratio is 4 : 6 or 2 : 3. In the middle figure it cuts a horizontal line twice and a vertical line thrice, giving same result.

Even if the component vibrations are not simple harmonic the curve can be traced in the same way provided that the particular time-scales of movement involved are used.

13. **Methods of producing Lissajous' Figures.**—It is usual in text-books on sound to mention several mechanical and optical devices for showing such curves as we have just described. A simple device is **Wheatstone's Kaleidophone**, a good form of which consists of a straight strip of steel, such as a piece of clock spring, twisted at the middle, so that the planes of the two halves are at right angles (Fig. 20). If the lower end of the rod is fixed in a vice, the upper end can vibrate either parallel to the jaws, in which case the lower part remains straight, and the time of vibration depends on the length and stiffness of the upper half; or at right angles to the jaws, and in this case only the lower part bends, and the time of vibration depends on the length of that part.

Fig. 20.

The vibrations of a spring fixed at one end soon become harmonic, as will be explained in Chapter XII., and by adjusting the position of the rod in the vice the periods of the two vibrations may be made to have any desired ratio. If the ratio is made a simple one, and the top of the rod displaced obliquely to the plane of the jaws, the extremity will describe the curve corresponding to the ratio. If a globule of mercury is attached by a little grease to the tip

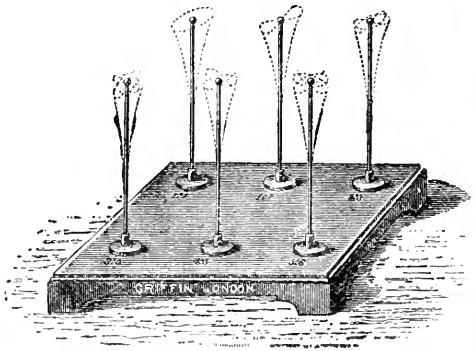


Fig. 21.

of the rod to form a brilliant point, the whole curve traced will be visible at once, by persistence of vision, if the vibrations of the rod are rapid enough.

Another form of the kaleidophone is shown in Fig. 21, which consists of six rods of equal lengths in which the dimensions of the cross sections are in different ratios. The dotted figures at the top of the rods show the paths of vibration of the end of the rods.

A simple modification of the kaleidophone can be quickly made by drawing out a piece of glass tube very fine in the blowpipe flame and then making a kink in the middle of it.

If two tuning-forks are fixed so that the vibrations of one take place in a vertical, and those of the other in a

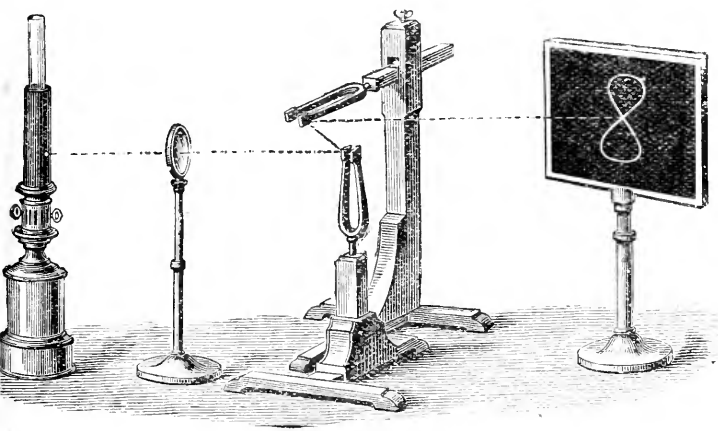


Fig. 22.

horizontal, plane (Fig. 22), and if a small mirror is fixed either on the side or the end of one prong of each fork, so that, when the prong vibrates, the surface of the mirror is tilted through a corresponding angle, and if a convergent beam of light falls on one of the mirrors, is reflected from it to the other mirror, and from that to a screen, then, if

only the first fork vibrates, the spot of light on the screen will vibrate harmonically right and left; if both vibrate together, the movement of the spot will be the resultant of these two vibrations, which, if their periods have a simple ratio, will be the corresponding Lissajous' figure. It was in this way that Lissajous produced the figures.

Instead of projecting the spot on a screen, we may watch, with the eye or a telescope,* the image of a bright point formed by successive reflection in both mirrors; in this case the figure will be visible even when the vibrations of the forks are very minute.

This method of illustration affords an exact method of tuning to unison or to certain intervals.

In these two methods, the track described by the spot is only visible by persistence of vision, and the vibrations must therefore be rapid. **Blackburn's Pendulum** is a contrivance for leaving a permanent trace, so that the vibrations may be much slower. A funnel *F*, filled with sand (Fig. 23), fits into a conical hole in a lead mass *M*, which is suspended by three strings from a ring *K*, supported by a string *PACKCBP*. The string passes through holes in the upper bar of the framework, and its ends are fastened to a screw peg *P*, which can be turned to regulate the length below *A* and *B*. By means of a ring, a portion *CKC*, of the string is separated off from the portion *ACB*, so that if the funnel is set swinging in the vertical plane which contains *A* and *B*, only the part *CM* swings, and the period is that of a pendulum of length *CM*, but if the motion of the funnel is in the plane perpendicular to *AB*, the whole system of strings

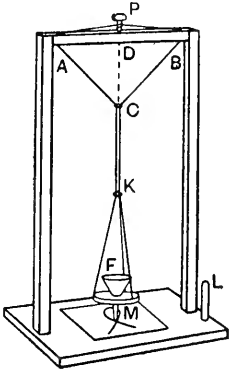


Fig. 23.

*In this case the lens shown between the lamp and the first tuning-fork will not be required.

swings together about the line AB in a period of a pendulum of length DM, where D is the middle point of AB.

By adjusting the peg and position of the ring the length of CM is altered, while DM can be kept constant to keep the funnel the right distance from the base. The ratio of the periods is thus given any desired value. M is then displaced obliquely, tied to the upright L, and when the sand begins to run out smoothly the string binding it to L is burnt and the sand traces out one of Lissajous' figures on a piece of paper placed below the funnel.

The period of a simple pendulum is given by the formula,

$$t = 2\pi\sqrt{\frac{l}{g}},$$

where t is the period, l the length from the point of suspension to the centre of mass of the bob, and g the acceleration of gravity.

The period t is therefore proportional to the square root of l; hence if we want the periods to be as $1:2$ we must make

$$\frac{CM}{DM} = \frac{1}{4},$$

and generally if we want the periods to be as $m:n$ we must make

$$\frac{CM}{DM} = \frac{m^2}{n^2}.$$

Tisley's Harmonograph is a much better device. It consists of two heavy pendulums, which swing in planes at right angles to each other, and whose rods extend a few inches above their centres of suspension. In one form, shown in Fig. 24, two horizontal strips of wood, each of which rests at one end on the pointed top of one of the pendulum rods, are hinged together at the other by a glass pen, which passes through both strips. Either pendulum, swinging without the other, would give the pen a very nearly harmonic rectilinear motion; when they swing together, the pen traces one of Lissajous' figures.

In all these methods, the periods of vibration in the two directions are independent, and, therefore, they can never be adjusted quite exactly to any simple ratio; the curve

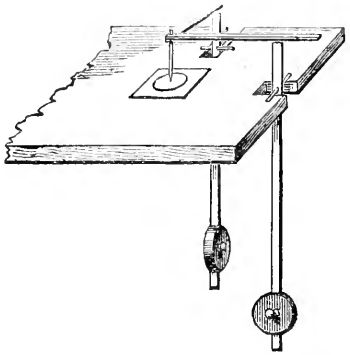


Fig. 24.

described, therefore, always changes slowly. In addition, owing to damping, the amplitudes gradually decrease, thus the curves interlace and very beautiful patterns are obtainable. It is easy, by means of rods connected to toothed-wheels working in one another, to give to a pencil harmonic motions at right angles, which have exactly a simple ratio, and so to trace always the same curve, but it is not usual to describe such methods in works on Sound.

14. Harmonic Vibrations in the same line.—Another important case is that in which the same point executes at the same time two or more harmonic vibrations in the same line. For instance, suppose that B vibrates harmonically with respect to the paper, along XY, while A vibrates harmonically, with respect to B, in the same direction. The motion of A, with respect to the paper, will be rectilinear, and its character can therefore only be shown by some device such as a displacement curve.

If O is the middle point of B's vibration the distances of B from O at the successive instants represented by 1, 2, etc., will be equal to the ordinates at those points of some harmonic curve, such as CD (Fig. 26), the length of a double bend of which represents the period of B's vibration. And the distances of A from B at the same instants will be equal to the ordinates of another harmonic curve,

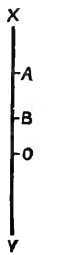


Fig. 25.

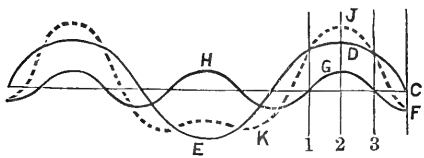


Fig. 26.

such as FGH, the length of a complete double undulation of which represents the periods of A's vibration. As the distance of A from O is always the algebraic sum of the distance of B from O, and that of A from B, the displacement curve of A's vibration with respect to the paper will be found by drawing at each point ordinates equal to the algebraic sum of the ordinates to CDE and FGH at the same point, and drawing the curve through the ends of these ordinates. The dotted curve in Fig. 26 shows the displacement curve of the vibration compounded of the two harmonic vibrations whose displacement curves are CDE and FGH.

It is evident at once that this is not a harmonic curve, so that a point which executes two harmonic vibrations at once may be vibrating non-harmonically. The importance of this fact will appear in Chapter VI.

A spot of light may be made to execute on a screen a vibration which is the resultant of two or more harmonic vibrations in the same direction, by successively reflecting a beam of light from mirrors carried by tuning-forks in

exactly the same way as described for Lissajous' figures, except that all the forks must vibrate so as to displace the spot in the same direction, *e.g.* all in vertical planes. The vibrating spot will, however, simply appear as a straight line of light; there will be nothing to show the character of its movement. The displacement curve corresponding to the vibration may be demonstrated by placing between the last fork and the screen a mirror rotating continuously on a vertical axis; the beam, being reflected from this after reflection by all the forks, will sweep round and round the room, and, vibrating vertically while it traverses the screen horizontally, will trace on the screen the displacement curve of its vibration, so rapidly that the whole curve will be visible at once (see Art. 74).

There are many mechanical methods of tracing the displacement curves of such resultant non-harmonic vibrations, but it is not usual to describe them in elementary text-books.

EXAMPLES I.

1. Describe what is meant by *simple harmonic motion.* Define the *period* and *amplitude* of a point moving in simple harmonic motion.

2. A particle of mass 10 grammes fastened to the end of a string 1 metre long is swung uniformly round a circle of radius equal to the length of the string in a period of 1 second. Find the acceleration of the particle and the force in the string.

3. Show that when a particle is moving in Simple Harmonic Motion its velocity at a distance $\frac{\sqrt{3}}{2}$ of its amplitude away from the central position is half its velocity in the central position.

4. A heavy particle is supported by a fine inextensible string. It is pulled aside from the vertical and then released (Simple Pendulum). Show that its motion is Simple Harmonic, and express its period in terms of the length of the string and the acceleration due to gravity. Set up such a simple pendulum by suspending a small leaden ball by a piece of thread. Time its oscillations and thus verify your formula. Make the length of the string 99·4 cms. What is the period now?

5. A heavy particle is supported by a spring. Show that when displaced in a vertical direction the particle vibrates in Simple Harmonic Motion.

6. Explain what is meant by *isochronism.*

7. Define *phase* and *frequency* in relation to vibration.

8. The frequency of vibration of a vibrating body is 200. Find its period.

9. Two bodies are vibrating in Simple Harmonic Motion. Their periods are 10 and 20 seconds respectively; their amplitudes are 20 and 10 cms. respectively. Compare their average velocities.

10. Take a curve similar to that given in Fig. 12 as the displacement curve of a particle in vibratory motion and construct from it a figure showing the position of the particle in its path of vibration at intervals of one-twelfth of a period during a complete vibration.

11. Taking times as abscissae and displacements as ordinates plot a curve showing the successive positions of a particle vibrating in Simple Harmonic Motion during a complete vibration.

12. Two particles are vibrating in Simple Harmonic Motion of the same period along the same line. One is at the end of its path while the other is at the middle of its path. Find the difference in phase.

13. Calculate the phases of a harmonically vibrating body (1) when its velocity is only half its maximum velocity, (2) when its displacement is half its amplitude.

14. Show that the maximum velocity of a particle vibrating with Simple Harmonic Motion is equal to 1·57 times its average velocity.

15. Let t = the period of a harmonically vibrating point. Show that its velocity at any instant is $\frac{2\pi}{t}$ of the displacement which it will have a quarter of a period later, and that its acceleration at any instant is $\frac{2\pi}{t}$ of the velocity it will have a quarter of a period later.

16. Show that the energy of a body vibrating with Simple Harmonic Motion is sometimes wholly potential, sometimes wholly kinetic.

17. A particle moves with a motion compounded of a Simple Harmonic Motion East and West and a Simple Harmonic Motion North and South ; these motions have the same period and amplitude. Plot its path

(1) when there is no difference in phase ;

(2), (3), (4) when the difference of phase is $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ respectively.

18. If the amplitudes of the two motions of the last question are unequal, plot the path when the difference of phase is $\frac{1}{4}$.

19. Find the resultant path of the particle in the four cases of Question 17 if the second motion is a uniform up and down motion instead of simple harmonic.

CHAPTER II.

PROGRESSIVE UNDULATION.

15. Transmission of Condensations and Rarifactions through Air.—Imagine a flat plate, held straight in front of you, vertically, with its edge towards you, and then suppose it rather suddenly displaced a very short distance towards your left. As the plate moves, the layer of air immediately to the left of it will, of course, be driven to the left with a velocity equal to that of the plate, and the air in that layer will also be compressed, the increase in pressure and in density being, in the early stages of the movement, proportional to the velocity of the plate (see Chapter III.). But though the velocity and the additional density of this layer of air both depend on the velocity with which the plate is displaced, these conditions are rapidly transferred from the layer of air in contact with the plate to a layer a little further off, and from that to the next, and so on, with a velocity which is practically independent of the velocity of the plate, and depends only on the properties of the air. In just the same way, a layer of air immediately to the right of the plate will move with it to the left, and at the same time its pressure and density will be diminished; and these conditions of motion to the left, reduced pressure, and reduced density, are transferred from that layer to other layers of air more and more distant from the plate with the same velocity as the compressed condition on the other side.

The simplest way of explaining this process is to suppose the air divided, by purely imaginary surfaces, into small blocks, or *particles*, in contact with one another; these

particles being, not molecules, but portions of air each containing many millions of molecules at least. Such a particle of air behaves, as regards compression, very much like an elastic solid; like a sponge, or ball of wool, to use Boyle's illustration; and the air of a room transmits displacements and condensations very much as a mass of balls of wool in contact might be imagined to do. The way in which a series of elastic bodies transmits a condensed or compressed condition to a distance, while the bodies themselves are only very slightly displaced, is well illustrated by the following experiment.

Exp. 9.—Place a number of equal glass or ivory balls in a groove in contact with each other. Withdraw the end one and roll it down upon the row. At once a ball is shot off from the other end of the row, while the others remain motionless. A pulse has been transmitted along the row by the forward impact of the first ball.

In the above experiment the transmission is too quick to be followed by the eye. We now consider another arrangement, in which both eye and ear enable us to follow the changes which occur. Suppose that a number of railway carriages are standing with their buffers in contact, and an engine at one end. We will call the carriages in order, from right to left, *A*, *B*, *C*, etc. (Fig. 27, line 1), and suppose that the engine suddenly moves a short distance, say a foot, towards *A* (2). The buffer-springs of *A* are driven in, so that the carriage (if we measure to the ends of the buffers) is shortened, or, as we may call it, condensed, and the force due to the compressed springs makes *A* move with increasing velocity towards *B*. When it reaches the half-way point between the engine and *B*, the pressures at the two ends of *A* will be equal (3), but, owing to its inertia, it will not stop at that point, but will continue to move to the left till the increasing pressure between it and *B* brings it to rest. This will happen when *A* has moved just one foot, so that the distance between it and the engine is the same as at first (4).

Meanwhile *B* has begun to move to the left, and exactly repeats the movements of *A*, and when it has been displaced one foot it stops at its original distance from *A* (5),

and the buffers between A and B cease to extend. The whole of A is then at rest, including the buffers, and it has its original length. C repeats the movements of B, just as B repeated those of A, and so a condition of condensation and movement is transferred from carriage to carriage all along the train.*

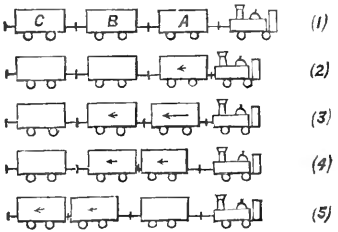


Fig. 27.

In this process several points are noteworthy. First, that though a condition, that of compression or condensation of the carriages, travelled continuously along the train from right to left, yet each carriage moved only a short distance, and then stopped, so that it is a *condition*, and not any material substance, which travelled along the train. Secondly, not only this transfer of a condition, but the existence of the condition, depends on the fact that each carriage repeats the movement of one next it *a little later*; if they all moved together there would be no condensed condition anywhere, and the direction in which the condition travels is from the earlier-moving to the later-moving carriages. Thirdly, the condensed condition is to be found always, and only, in carriages which are moving; when any carriage comes completely to rest (buffers included) it is exactly its original length. Fourthly, the condition may travel much faster than the carriages themselves move even at their quickest; in our example, the condensed condition is transferred from one carriage to another in the time in which the carriage itself has moved only a foot. In all these particulars the transmission of a condensation through air or any elastic substance resembles that through a row of railway carriages.

We shall find, as we proceed, that many different conditions can be transferred besides that of condensation,

* In this description the movement of the carriages has been slightly modified to make it illustrate more exactly the movements of the air. The carriages would not move just as described.

and the four statements just given are true of all of them. In the case described, we may notice also that the movement of the carriages themselves was in the *same* direction as the movement of the condition, but this is only true when it is a condition of condensation which is transferred.

If instead of a momentary impulse the engine gives to A a push lasting for an appreciable time, several carriages will have begun to move before A comes to rest, and as the condition travels, there will always be several carriages moving at once. But it will still be true that each carriage moves exactly like the one next nearer the engine, but arrives at each stage of its movement a little later; and also that each carriage is compressed as long as it is moving, and recovers its original length as it comes to rest. The movement may be considered as due to a succession of momentary impulses given to A one after the other, and each passed on in turn.

In the case of a row of carriages, each communicates its movement to one of equal mass, and the movement of an elastic substance does not correspond exactly to that of the carriages except when the portions which are successively

Fig. 28.

in motion are equal. This is not the case when a plate is waved in free air, so it will be simpler first to consider a column of air confined in a tube. Let AB, Fig. 28, be a long tube full of air, and D a piston. We may imagine the air in AB divided by imaginary planes, transverse to the tube, into equal discs or layers, and we will suppose these planes to move with the air, so that the air between the same two planes is always the same air.

If D is displaced a short distance to the left, the first of these layers, that next to D, becomes compressed and moves to the left, and then the next, and so on, just like

the railway carriages. As with the carriages, the movement may have extended to a number of layers before *D* comes to rest, so that a number are moving at once; but, in any case, each moves exactly like the one next nearer to *D*, but begins to move, and reaches each stage of its movement, a little later. As with the carriages, also, every layer is condensed while it is moving, and comes to rest as it is restored to its original length. The only differences to be noticed (and these are only apparent) are, first, that the air-discs become condensed all through, instead of shortening only at the ends like the carriages, and, secondly, that, however thin we make our imaginary layers, each will always be a little later in its movements than the one next nearer *D*, so that no finite length of the air-column begins to move absolutely at the same instant, as a whole railway carriage appears to do.

If we suppose the carriages *A*, *B*, *C*, etc. (Fig. 27), all held compressed between an engine to the right of *A* and the end of a siding to the left of *Z*, then, if the engine moved one foot away from *A*, allowing the buffers at that end of *A* to expand, *A* would be, for the moment, longer than the other carriages, or rarefied, as we may call it, and would begin to move towards the engine. As this would diminish the force on the end of *B* next *A*, *B* would follow and become rarefied, while *A* would stop when it reached its original distance from the engine, and would have been compressed to its original length again when *B* stopped after moving one foot in its turn. In this way a condition of *rarefaction* passes from *A to Z*, while each carriage in turn moves a foot in the direction *from Z towards A*. In every travelling rarefaction the movement of the matter is in the *opposite* direction to the movement of the rarefied condition.

Air is always in a compressed condition, for there is no limit to the extent to which it expands if pressure is removed: and particles of air move like compressed railway carriages. If the piston *D* was displaced to the right, first the air in the layer just to the left of *D*, and then that in layer after layer further to the left, would become rarefied and move to the right, each in turn coming to rest at the same moment at which it was restored to its original

thickness. The air to the right of D is of course condensed by the movement of D to the right, and a condensation travels away on that side, but for the present we consider only the air to the left of D.

Whether we move D to the right or to the left, then, the movement is repeated, first, by the air in contact with D, and then, layer after layer, by the air further and further off, so that the condition of being in motion travels away from D in either case. When D moves towards the air on the side we are considering, it condenses the air, and each layer moves in turn away from D, and is condensed while it is moving; when D moves away from the air on that side, it rarefies the air, and each layer (beginning with the one close to D) moves in turn towards D, and is rarefied while it is moving.

If we move D backwards and forwards, in any manner, exactly the same movement is performed by each layer, but later and later the further the layer is from D, so that D and layers near it may have begun to move one way, while more distant layers are still moving the other. All the layers which are moving away from D are condensed, and all those which are moving towards D are rarefied.

16. Progressive Undulation.—This process, in which a condensed or rarefied condition is transmitted through the carriages of a train, or air in a tube, is an instance of what is called *progressive undulation*, which may be defined as the **continuous transference in the same direction of a condition of altered relative position of adjacent particles by similar movements performed successively by consecutive particles.** In the instances we have so far considered, whether the condition transmitted is one of condensation or rarefaction, the movements of the particles are in the line along which the condition is transferred (though they may be in the opposite direction along that line). When this is the case, the progressive undulation is called *longitudinal*. In longitudinal progressive undulation the condition that is transferred is always one of altered *distance* between successive particles, *i.e.* of condensation or rarefaction; it is practically the only kind which occurs in air

and gases. In Chapter XII. we shall treat of other kinds of progressive undulation in which the condition transmitted is an altered relative *direction* instead of an altered relative distance.

Since the sensation of sound is usually caused by the arrival through the air of condensations and rarefactions transmitted by the process of longitudinal progressive undulation, this process is often conveniently called *sound;* travelling condensations and rarefactions are called *sound waves*, and the velocity with which such waves are transmitted in any substance is called the *velocity of sound* in the substance. The term *pulse* is often used to denote either a condensation or a rarefaction, and avoids the frequent repetition of both terms.

Undulation and Vibration.—It is important to distinguish clearly between undulation and vibration. When the same point repeats the same movements over and over again, the point is vibrating. When different parts of the same substance perform a similar movement one after the other, the substance is undulating. If it happens that the movement which the parts perform in turn is a vibratory movement, then the substance is both vibrating and undulating. This is what is usually happening when sound passes through the air. But there is no necessary connection between the two; a substance may undulate without vibrating, like the train after a push, or it may vibrate without undulating if all the parts of it move at the same time, like a pendulum.

17. Relation between Displacement, Condensation and Velocity in a Longitudinal Wave System.—Let $ABC...S$ (top line, Fig. 29) be a number of equally spaced particles in a uniform medium. When longitudinal waves pass through this medium, these particles oscillate about their normal positions. Let the second line represent the positions of the particles at a given instant. The positions of the particles can also be represented by a displacement curve $abc...s$, where the ordinates of $a, b, c...s$ are proportional to the displacements of $A, B, C...S$, and are above the axis if the particles are displaced forward and below the axis if the

displacements are backward. For simplicity the displacements are assumed to obey the sine law.

To get the velocity curve and pressure curve from the displacement curve, consider the displacement curve at two instants very close together. If the wave is moving from left to right let $a'b'c' \ldots s'$ be the displacement curve at the

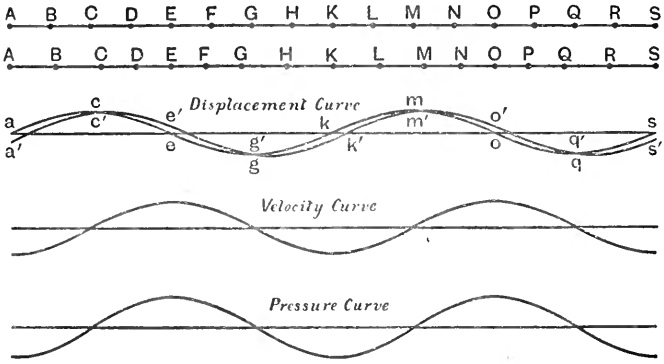


Fig. 29.

second instant. The velocity of a particle is proportional to the rate of change of its displacement. It is obvious, from a study of the two curves, that the particles which are moving fastest are those which are least displaced, namely, A, E, K, O, S, while those which are practically stationary are those whose displacements are a maximum, namely, C, G, M, Q. Also the particles between C and G, M and Q are moving forward, and particles between A and C, G and M, Q and S are moving backwards. The velocity curve can be drawn easily from these facts, velocities forward being represented by ordinates above the axis and velocities backwards by ordinates below the axis.

To get the pressure curve study the displacement curve again. The particles B, C, D are displaced towards E, so are also the particles F, G, H, hence E is subjected to great pressure. Similarly the pressure is a maximum at O and

a minimum at A, L and S. In the immediate neighbourhood of C the displacement varies little, hence the pressure at C is normal, similarly the pressure is normal at G, M and Q. The pressure curve can now be roughly drawn, the ordinates being above the axis when the pressure is above the normal and below the axis when the pressure is below the normal.

Since density is proportional to pressure the density curve has maxima and minima at the same time as the pressure curve.

It will be shown in the next chapter that in any longitudinal progressive undulation there is a constant ratio, depending only on the nature and condition of the substance conveying the undulation, between the velocities with which different parts are moving, and the amounts by which the densities of those parts differ from the average density. If we call the difference from the average density the "condensation" or "rarefaction,"* the velocity at any point is proportional to the condensation or rarefaction there.

This is, of course, true close to (Fig. 28) D as at any other point, and the velocity of the air close to D is the velocity of D; the degree of condensation which exists close to D (and therefore the excess of pressure on the side towards which D is moving) is always proportional to the velocity with which D *is moving*, and does not depend at all on how D has previously moved.

18. Wave-form.—Suppose that when D has been moving, in any manner, for some time, we draw a horizontal line, OX Fig. 30 (1), along the tube, and from it draw ordinates at different points, proportional to the velocities of the air at those points, at some given moment, drawing the ordinates above OX to represent velocity in the direction in which

* Strictly, "condensation" means not the difference between the actual and average densities, but the ratio of this difference to the average density. But these are proportional to one another, so that the velocity with which the air is moving is proportional to the "condensation" in either sense. Rarefaction may be conveniently included in the term condensation, and considered merely as the negative variety.

the waves are moving, and below for velocity in the opposite direction. Then the same ordinates are also proportional to the degrees of condensation or rarefaction at the

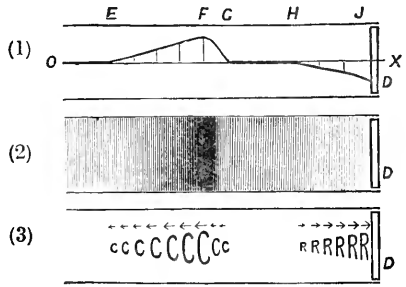


Fig. 30.

same points, ordinates above OX representing condensation. So that the curve in Fig. 30 (1) denotes the same condition which is represented by shading in (2), and by the relative heights of the letters C and R (Condensation and Rarefaction) in (3). In (3) the relative velocities are represented by the lengths of the arrows over the letters; this is unnecessary in the case of a progressive undulation, as the velocities are proportional to the condensation or rarefaction, but we shall find it useful in other cases.

A curve like that in Fig. 30 (1), whose ordinates, at equidistant points along a horizontal line, are proportional to the velocities with which the air is moving at a number of equidistant points, is said to show the *wave-form* of the undulation, the wave-form meaning the way in which the velocity of the air varies from one part to another of the wave. It will, of course, be understood that waves of condensation and rarefaction in air have not anything which can be termed *form* in the ordinary sense, as waves of the sea have; the term wave-form is not to be taken literally.

In air, condensations and rarefactions are transmitted with a speed of about 1100 feet per second. If the wave-form shown in Fig. 30 (1) represents the velocities of the

air at different points of the tube at a moment denoted by T, then the ordinate which represents the velocity at the point 1 foot to the right of any point, F for instance, represents the velocity which will exist at the point F at a moment $\frac{1}{1100}$ second later than T [written, the moment $(T + \frac{1}{1100})$], and so on. The same curve which represents the simultaneous velocities of the air at different points in a wave which is travelling from right to left is therefore also a velocity curve in which the velocities of the air at any one point, at successive instants, are represented by the successive ordinates read from left to right. The same length along OX, which represents one foot when the curve is taken as representing the simultaneous velocities at different points in the air, represents $\frac{1}{1100}$ second when the curve is taken as representing the successive velocities of the air at the same point. Successive ordinates of the part of the curve to the left of any point represent the successive velocities which have, in turn, existed at that point. As the velocity of the air close to D is the same as that of D itself, the wave-form of the air in the tube is a velocity curve of the movements already performed by D.

The difference between the actual pressure at a point in the air and the average pressure (a difference which we shall call the *pressure-difference*) is proportional in all waves to the condensation or rarefaction, and therefore, in travelling waves, to the velocity of the undulating substance (Art. 17). The successive ordinates of the wave-form, read from left to right, may therefore be taken as representing the successive pressure-differences, as well as the successive velocities, which exist at the same point at successive instants. This is the most useful of the many meanings of the wave-form, as will appear in Chapter VII.

19. Wave-length.—If D *vibrates*, or repeats the same movements at regular intervals, the same series of conditions will be produced at its surface over and over again, and travel along the tube. Suppose, for instance, that it repeats its whole movement in $\frac{1}{1100}$ second, then the air, 1 foot, 2 feet, etc., from the surface of D is in the same condition as at the surface, and if we divide the tube into

lengths of 1 foot, the air in any of these sections, at any instant, is moving in the same way as the air in any other section. One of these sections is called a *complete wave*, and its length is called a *wave-length*. It is evident that the wave-length is the distance which a given condition travels before D comes round to the same stage of its vibration again; in other words, it is the velocity of the undulation multiplied by the period of the vibration. If V denotes the velocity of sound and λ the wave-length, $\lambda = Vt$, or $\lambda = \frac{V}{n}$.

20. Harmonic Waves.—If D vibrates harmonically, the wave-form of the undulation in the tube will be a harmonic curve. Such waves are called *harmonic waves*. Two complete harmonic waves travelling from right to left are represented in Fig. 31, the heights of the letters C and R being proportional to the condensation or rarefaction. The corresponding wave-form is shown below.

Fig. 31.

The way in which a condition of condensation or rarefaction can move always in the same direction while the air itself moves backwards and forwards is illustrated by successive rows, 1, 2, etc., in Fig. 32. In this figure the successive rows represent successive stages of a movement in which each of the vertical lines vibrates harmonically to right and left of a fixed point, from which it never departs more than $\frac{1}{8}$ of an inch, a vibrating about a point under A, and so on; but each line of a row, in order from right to left, is a little behind the line to the right of it in its movement, and the result is, as seen, that though the mean positions of the lines are equidistant, the lines themselves are at any one time closer in some places and further apart in others, and that these conditions are found further to the left at

each successive stage of the motion. The construction for the harmonic motion of a few lines is given, and it will be seen that in any one row, each line is $\frac{1}{12}$ of its period behind the one to the right of it, and that in each row all the movements are $\frac{1}{12}$ of a period further advanced than in the previous row. The thirteenth row is the same as the first, every line having returned to its original position.

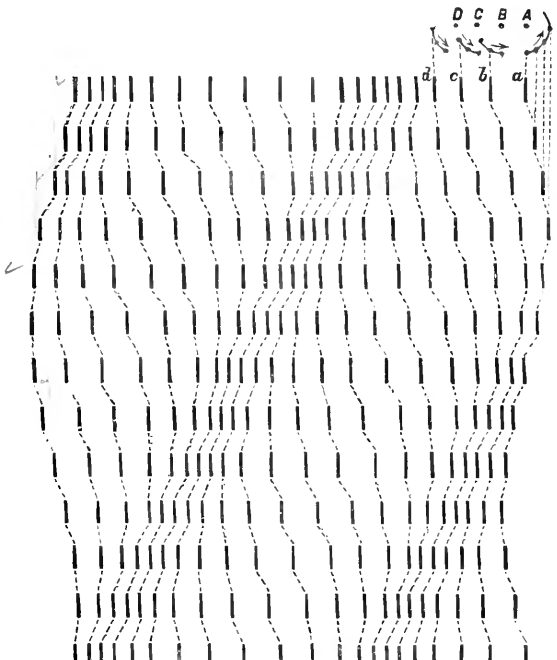


Fig. 32.

If any line is traced (by the dotted lines) from one row to another, it will be seen that it is when it is moving to the left (*i.e.* in the same direction as the conditions of

condensation and rarefaction) that it is nearest to its neighbours, and when it is moving fastest to the right that it is in the most rarefied region.

If we imagine an equal mass of air confined between each line and the next, and moving with the lines so that the *same* air always remains between the same two lines as they move, but alters in density as the distance between the lines changes, this will correspond to the actual movement of the air.

21. **Crova's Disc.**—A good and simple device for illustrating the movements of the air or other substance when a harmonic progressive undulation is passing through it is

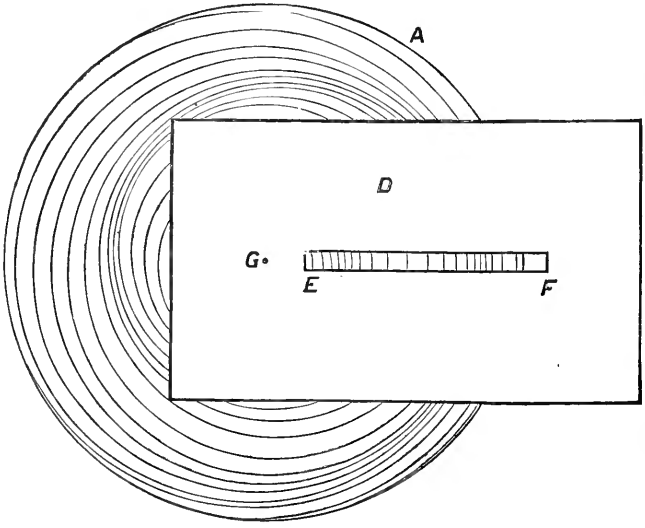


Fig. 33.

Crova's disc (Fig. 33) ; this the student can easily construct, and should do so if he finds any difficulty in realising the motion.

Exp. 10.—*Make a Crova's disc, and by means of it study progressive undulation.* In the middle of a circle of cardboard, A, 8 inches in diameter, draw a small circle of $\frac{1}{8}$ inch radius, and divide its circumference into twelve equal parts. With each of these points of division in turn as centre, describe a circle in ink, making the radius of the first $\frac{3}{4}$ inch, of the second $\frac{7}{8}$ inch, and so on, increasing $\frac{1}{8}$ inch each time. When you get to the last of the centres go on with the one you used first, and so round the circle again, making twenty or more circles in all. In another piece of card cut a slit, EF, 3 inches long and $\frac{1}{8}$ inch wide, and make a pinhole at G in a line with EF and 1 inch from E. Push a pin exactly through the centre of the disc, and put its point through G; then, holding D in one hand, rotate the disc behind D by means of the pin.

Note that the portions of the circles visible through the slit are practically straight lines, close in some places, and wider apart in others, like those in Fig. 33; as the disc rotates these condensations and rarefactions travel continuously along EF. Now rotate the disc slowly and carefully follow the motion of any one of the lines; note that it simply moves harmonically to the right and left, and never departs more than $\frac{1}{8}$ inch from its mean position. Also verify the fact that where the lines are near together they are moving in the same direction as the condensations and rarefactions, but not so fast, but that where the lines are far apart they are moving in the opposite direction, and that the quickest motion of the lines is to be found where there is the greatest degree of condensation and rarefaction.

22. Spiral Wire.—The same motion can be shown by means of a spiral wire.

Exp. 11.—Get about 20 yards of thick copper wire (14 gauge is suitable) and wind it in a close spiral round a tube or cylinder $2\frac{1}{2}$ to 3 inches in diameter. Stretch this spiral till it is about 2 yards long, so that the turns are about an inch apart. Suspend the highest point of each turn by two threads not less than 2 feet long (the longer the better) from two horizontal wooden bars about 18 inches apart, as shown in Fig. 34. This is Weinhold's Wave Machine.

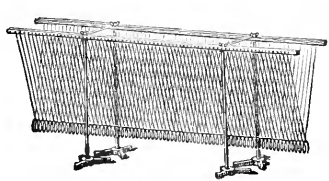


Fig. 34.

If a slight push is given to one end of the spiral, in the direction of the length of the spiral, some of the turns at that end will be pressed closer together, and this condition will be transferred from them to the next turns, and so on, a condition of unusual closeness of turns

travelling continuously to the other end of the spiral, while each turn of wire, successively, moves a little way and then stops. The condition of unusual closeness leaves one turn as it includes another, so that it always includes the same number of turns; it does not extend so as to include more than when it was originally produced.

Again, if the end of the spiral is quickly pulled outwards, the end turns are separated, and a distended condition travels away from the hand along the spiral, each turn moving slightly towards the disturbed end. The two conditions occurring side by side form one wave; or, when several equal waves follow each other, a wave-length is the distance from one part of the spiral to the nearest which is in the same condition, whether of condensation or rarefaction. Thus

Fig. 35.

in Fig. 35 the portions denoted by A and C are in extension and B in condensation. The distance from A to C may be taken as one wave-length, these parts being in like condition of extension. When the end of the spiral is first pushed in, and then immediately pulled out, a complete wave is transmitted. It consists of two parts, the first of which is seen to be a condensation, the second a rarefaction.

23. Some difficulty is sometimes found in understanding how it can be true that the air in a condensation is always moving away from the source, and yet true that the air through which sound is passing moves alternately towards and from the source, and never departs more than a very small distance from its mean position. To understand this it must be remembered that the movement of the air is usually extremely slow compared with that of the condensed and rarefied conditions. Consider a cubic centimetre of air which we will call A quite at the left-hand end of the tube in Fig. 31; it is at rest, and neither condensed nor rarefied But a condensed condition is advancing to the left, and when it reaches A, A will diminish in volume and at the same time begin to move to the left. But the air moves no faster than the piston moved, while the condensed condition rushes on with the speed of a rifle ball, so that before A has moved far to the left, the condensed condition has passed it, and the following rarefied condition has got to it. A now expands to more than its original volume,

and begins to move slowly to the right, but has only got back to its starting point when the next condensation comes up, and so on. *A* is always moving the same way as the waves when it is in a condensed condition, but it is only in a condensed condition a little while at a time, and when it is rarefied it moves the other way.

Another form of the difficulty is to understand how the air in a rarefaction and that in the following condensation can always be moving towards each other (see Fig. 31). As before, the explanation is that the air in the same condensation is not always the same air. The air that is now rarefied will an instant hence be condensed, and the air which is now condensed with then be rarefied, so that the parts which are now approaching each other will then be receding, so that no great change in the distance of two portions of the air occurs.

24. Energy Transmitted by Progressive Undulation.—We have already pointed out that in progressive undulation there is no continuous transference of anything *material*, but it would perhaps hardly be correct to say that *nothing* except a condition travels from one place to another. Usually, progressive undulation is a process by which *energy* is transferred from one place to another, and it is now becoming quite usual to speak of energy as a *thing*, rather than a condition. When a succession of particles perform similar movements in turn, it is nearly always because the movement of each *causes* the movement of the next, and one body can cause the movement of another only by transferring to that other some of its own energy.

In the case of air, through which waves of condensation and rarefaction are travelling, we can easily see that the wave as a whole—the condensed and rarefied portions taken together—contains more energy than an equal volume of undisturbed air. For both the air in the rarefied portion and that in the condensed portion is moving relatively to the earth, and work could be done by stopping this motion. When this energy was exhausted, and all the air at rest, we could still get work done by allowing the air in the

condensed portions to expand through a suitable engine into the rarefied portions;* this work would be done by using part of the heat of the compressed portions. The air would then be in its ordinary condition, so that it must, while undulating, have contained more than the usual amount of energy.

It is usual to distinguish the part of this energy which depends on the motion of the air relatively to the earth as its kinetic energy, and the part which depends on the differences of pressure in different parts as the potential energy. If we make this distinction, it can be shown that half the energy of a complete wave is kinetic and half potential. The kinetic energy is greatest in the most condensed and most rarefied portions of the wave; the regions where there is average density are also at rest with respect to the earth, and do not differ in any way from undisturbed air, so that there is no extra energy in them.

We shall see later that this is rather an artificial way of looking at the energy of a sound-wave in air, since the whole energy is really kinetic energy of the movements of the molecules But it is convenient for the present to regard the air simply as an elastic substance, and to divide the extra energy due to its undulation into kinetic and potential portions, as we are obliged to do in the case of solids and liquids where we do not know the real nature of the "potential" part of the energy.

It will be seen that though sound is a special mode of transmission of energy, it is not a distinct kind of energy, for part of the energy of a sound-wave is ordinary mechanical kinetic energy of air in motion, and the other part is potential energy of air strained by compression or rarefaction.

25. Sound Waves in Free Air. The statement at the end of Art. 16, that in a tube any movement of one layer of air is exactly repeated by those more distant from the source, is only roughly true if applied to considerable

* It is not, of course, meant that this is a practicable experiment; that it is an imaginable one proves the existence of energy in the wave.

distances, for owing to friction between the air and the sides of the tube, and still more owing to exchanges of heat between the air and the sides of the tube, the energy of a wave travelling along a tube is rather rapidly converted into heat, and left behind in that form in the air through which the wave has passed and in the walls of the tube, and so the movements of the air become less and less as we get further from the source. This will be more fully explained in the next chapter. If a body vibrates, not in a tube but in free air, no transformation of the energy of the waves after they have once started can occur from these causes, and though the energy is still ultimately converted

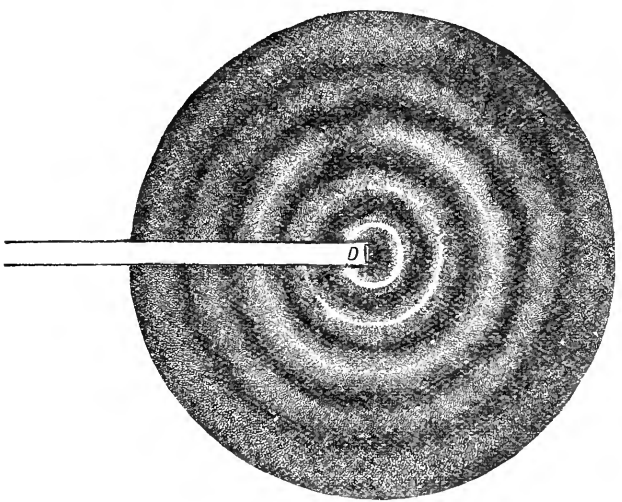


Fig. 36.

into heat, the transformation is almost incomparably slower.

In some other respects, also, waves produced in free air differ from those sent along a tube. Suppose that D, Fig. 36, is, as before, a piston vibrating at the end of a

tube,* but this time we will consider the free air to the right of *D*. First we must notice that though any movement of a piston would send a wave travelling along a tube, waves are not produced in free air, to any appreciable extent, unless the vibrating surface is large, or the vibrations of high frequency. If neither of these conditions is fulfilled, the air in front of the moving surface simply slips away, parallel to the surface, till it gets to the edges, instead of becoming compressed.

Exp. 12.—Stretch a steel wire about a metre long and a millimetre in diameter between two practically immovable blocks of stone or metal. Pull the middle of the wire to one side and then release it. No sound is heard, although it is evident to the eye that the vibration is large. Vary the experiment by fastening one end of the wire to the panel of a door (or use a sonometer, Art. 167). A loud sound comes from the panel of the door when the string vibrates, although the movements of the panel cannot be detected by the eye. The air in front of the moving string simply slips round it to the back, without being appreciably condensed, but the air in front of the moving panel has not time to get to the edge of the panel while the panel moves one way, and must become condensed and rarefied alternately.

Even if the surface is large enough, or its frequency high enough, to produce waves, it does not communicate nearly as much energy to the air in each vibration as it would in a tube. Secondly, when the condensations and rarefactions succeed each other rapidly enough to be transmitted, they spread through the air in *all directions*, so that when the piston has finished a few vibrations, the air near it will be in the condition represented in Fig. 36, alternate spherical shells of air being condensed while the others are rarefied. As in the tube, wherever the air is condensed it is moving away from the piston, and wherever it is rarefied it is moving in towards the piston; the movement of the air is everywhere at right angles to the shells.

* A moving plate surrounded on all sides by free air would produce condensation on one side and rarefaction on the other, at the same time, and these would spread through the *same air*, and the resulting effect depends on principles to be explained in Chapter V. The plate is, therefore, described as moving at the end of a tube so that only the effect of the condensations and rarefactions produced on one side of it need be considered.

The air in each shell keeps transferring its condition to the air outside it, so that the condensed and rarefied conditions alternately spread continuously outwards from shell to shell of air, like the circular wave-crests produced when we throw a stone into a pond. Thirdly, as the movement and energy of one shell of air are always communicated to a larger one, the extent of the movement and the degree of condensation and rarefaction become less and less as we go outwards from the piston, or the wave-form becomes flatter. Fourthly, in the immediate neighbourhood of the piston very complex actions occur, with the result that the movement of the air a little distance from the piston may differ considerably in character, as well as in amplitude, from the previous movements of the air close to the piston, so that the wave-form changes as the wave spreads, and at a little distance is not the same as the velocity-curve of the movement of the piston.

One noticeable instance of this gradual change of character is that, although a single movement of a piston in a tube, to left or right, not followed by an opposite movement, would send a condensed or a rarefied condition, not followed by its opposite, along the tube, a single movement of a surface in the open air produces a complete wave, consisting of a condensed and a rarefied shell. In fact we can easily show that the two principles of conservation of energy and conservation of momentum require that a wave, when expanded to many times its original radius, shall consist of two opposite layers, the excess of air in one of which is nearly equal to the defect of air in the other; more and more nearly equal the larger the wave. So that a source may produce nothing but condensations, like a harmonium or a siren (Arts. 150, 71), and yet the waves which arrive at a point a little way off will be alternately condensed and rarefied.

Though a single movement of the source produces a complete wave, a double movement only produces one complete wave, not two waves, at least in ordinary cases. If the movement of the piston is harmonic and very small, the movements in all the surrounding air are harmonic, differing only in amplitude, and not in character, from those of

the source. This is an example of a universal principle, that if any physical quantity varies harmonically, and if its variations are small enough, then all variations which depend on it are also harmonic.

All the above statements about the waves produced by a piston like *D* are roughly true about the waves produced by any surface vibrating in free air, such as a spring or tuning-fork. The slight modifications necessary are explained in Chapter IV.

Though the movement of the air near a vibrating body such as a spring is a vibration similar to that of the spring itself, the causes of these similar movements are very different. When we bend a spring and let it go, it returns of itself to its original position, passes it, returns again, and so on, each movement being a consequence of its previous movement; it vibrates in a particular way, and with a particular period. But if we displace a part of the air, either in an infinitely long tube, or in open space, it shows not the slightest tendency to return to its original position.

The air, therefore, cannot be made to vibrate, like a spring, by simply displacing a part of it and then leaving it free; each movement that it makes is the result, not of its own previous movements, but of a previous movement of something else, transmitted to it through the air between. The air vibrates only as a pump-handle vibrates; it requires a separate push from outside for every movement. It therefore vibrates just as easily in one way as another, and in one period as another; in fact, it simply copies the movements of some other vibrating body.

We shall see in Chapter XI. that in a tube of finite length the air may vibrate merely through having been displaced, without any other vibrating body to cause each movement.

At different distances from the piston the air will be in different stages of vibration, but it will be possible to draw round *D* continuous surfaces, in each of which all the air is moving in the same way at the same moment; these will be, approximately, spherical surfaces, having *D* as their centre. Such **a continuous surface, at every point of which the air is in the same stage of its vibration at the same moment, is called a wave-front.** In the case of the waves

produced by a piston in a tube, the wave-fronts are planes transverse to the tube: in the open air, at a distance from the source, they are generally roughly spherical, as far as they are complete. The form of these wave-fronts must not be confused with, or called, the wave-form, a term which, as explained above, is used in an entirely different sense.

We may now define a *wave-length* rather more accurately as the distance, measured at right angles to the waves, between two consecutive wave-fronts in which the air is at the same stage of its movement. If the movement is harmonic, the wave-length is the least distance between two surfaces in which the air is in the same *phase*.

26. Law of Inverse Squares.—If, through any point in the air, we suppose an imaginary surface drawn at right angles to the line along which the air is vibrating, then the amount of energy which passes, in each second, through a square centimetre of this surface round the point, is called the *intensity of sound* at that point. It is not usual to measure it in absolute units, but only to compare the intensities of the same sound at different places. If sound is freely travelling outwards in all directions from a source P, the intensities at two points S_1, S_2 must be inversely as the squares of their distances from P. For, suppose imaginary spherical surfaces drawn through S_1, S_2 (Fig. 37). Then, if R_1, R_2 are the distances of S_1, S_2 from P, the areas of these spherical surfaces are as R_1^2 to R_2^2. And the sound energy which starts from P in (say) a second, takes a second in passing $S_1S_1S_1$, and also a second in passing $S_2S_2S_2$. Therefore the amounts of energy which pass in a second through *one square centimetre* of $S_1S_1S_1$ and $S_2S_2S_2$

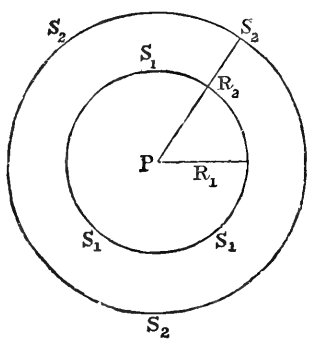


Fig. 37.

respectively must be inversely as the areas of these surfaces, that is, as $\frac{1}{R_1^2}$ is to $\frac{1}{R_2^2}$.

In this proof we have assumed that all the sound-energy which starts from P passes $S_1S_1S_1$ and $S_2S_2S_2$ unchanged in form. This is nearly true; the energy of each shell is passed on to the next almost unaltered in total amount for a very long distance, and though a very little remains behind (in the form of heat) in every part of the air through which the wave has passed, yet, many miles from the source, by far the greater part of it is still travelling on. It is because of the diminution of the intensity of the vibration, due to the distribution of the energy over larger and larger shells of air, rather than because it has been converted into heat, that we cease, at a great distance, to be able to detect the vibration; the whole of the energy is, however, ultimately converted into heat.

The term *intensity* of an undulation is used by some writers to mean the quantity of energy which flows per second through a square centimetre parallel to the wave-fronts, and by others to mean the quantity of energy in a cubic centimetre of space at a given moment. We have adopted the former definition, which has the high authority of Lord Rayleigh; but the other seems to be coming more into favour.

Let E be the energy per cubic cm., and V the velocity in cms. per second with which sound is transmitted. Consider a centimetre cube, of which one face is on the wave-front. The thickness perpendicular to the wave-front, 1 cm., is described by the sound in $1/V$ secs. Consequently E is the energy that flows through 1 sq. cm. in $1/V$ secs., or EV is the energy per second, and therefore is the intensity as defined above. Now V is constant, and EV is proportional to $1 \div (\text{distance})^2$, hence E also is proportional to $1 \div (\text{distance})^2$.

27. Intensity and Amplitude.—At any two points in the air, where the air is vibrating with the same frequency and in the same manner, but with different amplitudes, the intensities are proportional to the squares of the amplitudes.

Let a be the amplitude, n the frequency, T the period. It was shown in § 8 that the greatest velocity is $2\pi a/T$, and the average value of the square of the velocity is $2\pi^2a^2/T^2$. If D be the density,

D is the mass per cubic centimetre, therefore the energy per cubic cm. is $E = 2\pi^2a^2D/T^2$, which $= 2\pi^2a^2n^2D$, since $nT = 1$. The intensity, as we defined it above, $= EV = 2\pi^2a^2n^2VD$.

Since *when sound spreads freely in all directions* the intensity varies inversely as the square of the distance from the source, and since, whether the sound spreads in all directions or not, the intensity is directly proportional to the square of the amplitude, it follows that **when sound spreads freely in all directions the amplitude of vibration of the air must vary inversely as the distance from the source,** so that at 100 yards from a bell the air is moving backwards and forwards half as far as at 50 yards.

The circumstances which determine the intensity of sound at a point in the air can be stated in various ways, which sometimes are not obviously equivalent. For instance, we may state the intensity at a point P entirely in terms of what is going on at that point. It would be a complete statement to say that it depended on the frequency and amplitude of the vibration of the air at the point P, and the density of that air, for if these were given we could calculate the intensity at P without any other data. So that if we give these as the circumstances on which the intensity depends, we must not include any others, such as the distance from the source.

We could say, with equal correctness, that the intensity at P depends on the frequency and amplitude of the vibrating source, on the density of the air in which the source vibrates and the dimensions of the vibrating surface in contact with it, on the distance of the point P from the source, and on whether the sound has spread equally in all directions or not. This again would be a complete statement, and we cannot include it in any other conditions, such as those which were included in the other statement, without making it incorrect. There are many other ways in which we could make a complete statement different from either of these, the essential point being that the statement must include sufficient data to calculate the intensity, and no more.

The illustrations given to explain the nature of progressive undulation, being chosen because of the obvious

movement of the undulating substance, are liable to suggest an exaggerated idea of the extent of the movements of air conveying sound. There is no theoretical limit either to the largeness or smallness of the movements which can be passed on through the air, all are passed on in the same way, and (unless *very* large) with practically the same velocity.

But, as was found by Rayleigh (see below), the backward and forward swingings of the air by which ordinary conversation is transmitted to us are of very small extent, usually less than a thousandth of an inch, and often less than a millionth. Thus, though the movement of the air is reversed hundreds or thousands of times per second, its velocity, even at its quickest moments, may be very small; the sound of a whistle is quite audible at a place where the air at its quickest moment is not moving as fast as 2 inches an hour. The velocity with which the conditions travel may thus be millions of times as great as the velocity of the movements of the matter which transmits them. The difference in density between the condensations and the rarefactions is correspondingly small.

In the experiment quoted above, Rayleigh supplied a measured amount of power to blow a whistle fixed out in the open, and then found the greatest distance at which the whistle was just audible.

When the whistle (of frequency 2730) was blown with an air blast of power $1{\cdot}85 \times 10^6$ watts he found that the sound was just audible at a distance of 820 metres from the whistle.

The waves of sound travel out in hemispherical shells; the area of the shell at the distance of 820 metres

$$= 2\pi (82000)^2 \text{ sq. cms.}$$

As shown above the intensity of the sound $= 2\pi a^2 n^2 VD$, so that the energy sent through the shell per second $= 4\pi^3 a^2 n^2 VD(82000)^2$ ergs.

Assuming, for the present, the value of the velocity of sound, namely, 34000 cms. per second at ordinary temperatures when the density is about ·0013 gms. per c.c. (Art. 43), this becomes

$$4\pi^3 a^2 (2730)^2 \ (34000) \ (\cdot 0013) \ (82000)^2 \text{ ergs.}$$

But this is numerically equal to the power,

$$\therefore \quad a = \sqrt{\frac{1{,}850{,}000}{4\pi^3 2730^2 \times 34000 \times \cdot 0013 \times 82000^2}}$$

$$= 8{\cdot}2 \times 10^{-8} \text{ cm.}$$

The maximum velocity of a vibrating particle occurs when it is passing through its central position, and equals $2\pi na$ (Art. 8). Substituting for n and a this becomes ·0014 cm. per sec.

28. Sound Waves in Liquids and Solids.—So far we have spoken only of sound waves travelling in air. But condensations and rarefactions are transmitted through liquids and solids in exactly the same way, by exactly similar movements. If one end of a rod of wood or metal is pushed or pulled, successive layers move, and are compressed or rarefied, in turn, just like the layers of air in a tube, and if any point of a solid block is made to vibrate, condensation and rarefaction spread through it in widening spheres, as they do through air.

It must not be supposed that this handing-on of condensations and rarefactions by successive movements of different portions is a process specially connected with audible sound; it is the only way in which a pull or a push can be transmitted at all. When a signalman moves a distant signal by pulling at a handle, which is connected with the signal by a rod or wire, the movement of the signal appears to be simultaneous with the movement of the lever, but this is only because of the great velocity with which pulses travel in iron, not because any pull or push can be transmitted in a different way or with a different velocity from sound. When the man begins to move the lever, he produces an elongation or rarefaction of a piece of the wire at his end, and this state of rarefaction travels along the wire at the rate of about three miles a second, and it is only when the rarefaction reaches the other end of the wire that the signal begins to move.

29. Experimental Verification.—From the nature of the case, there is little direct evidence that the air in sound-waves moves, and alters in density, in the way described in this chapter. That there is no continuous movement of the air may be inferred from the fact that the sound from any source passes through a cloud of smoke without causing any visible movement of the smoke. That the air does move backwards and forwards seems proved by the

fact that a light membrane in it moves backwards and forwards. That the movement of the air is in a line passing through the source seems proved by the fact that an acoustic pendulum is very little or not at all affected when it is edgewise to the source. That the passage of sound waves through the air is accompanied by changes of pressure at any point is shown by the fact that if a hole, cut in one side of a tube, has a thin membrane stretched over it, against which a pith-ball hangs, sound waves sent along the tube make the pith-ball dance; a mere movement of the air along the tube, without any change of pressure, could not produce a movement of the membrane, since the air moves parallel to the membrane.

There is no experiment to show that changes of density occur, but we cannot alter the pressure of a gas suddenly without changing its density, and the changes of pressure must be sudden, since they make the membrane vibrate hundreds of times a second Also it can be shown (by acoustic pendulums in a long tube) that the air near the source moves sooner than that at a distance, and it is obvious that if the air in one part of a tube moves in the direction of the length of the tube, while the air in another part does not, the air between must change in density.

Lately, however, it has been found possible by utilising the change in the refractive index of the air, due to the change of density, to see, instantaneously, the sound waves from an electric spark as they travel through the air.*

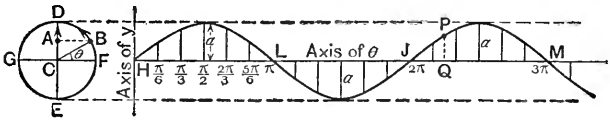


Fig. 37*a*.

29*a*. Equation of a Sine Curve, the Displacement Curve of Simple Harmonic Motion.—Let a point *A* vibrate up and down the line *DE* (Fig. 37*a*) with simple harmonic motion,

* See Wood's *Physical Optics* (Macmillan).

C being the central point and E and D the limits of vibration. If a line AB is drawn perpendicularly to DE to cut the circle described on DE as diameter in B, then as A continues in vibration B moves uniformly around the circle.

Draw GCF through C perpendicular to DE, then if we denote the amplitude of vibration by a, the displacement of A from C by y and the angle FCB by θ we get

$$y = CA = CB \sin FCB = a \sin \theta \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The curve which represents the changes of y as θ changes is called the sine curve (cf. Art. 9). It may be plotted from the numbers in the accompanying table. The first column gives θ in degrees, the second in circular measure. It is more usual to work with angles in circular measure, and after this we shall always use this notation. The curve is shown in Fig. 37*a*. A certain length HJ is taken to represent the angle 2π, and this is divided into convenient sub-multiples as indicated in the table. The curve repeats itself after θ exceeds 2π.

θ.		y.
Degrees.	Circular Measure.	
0	0	0
30°	$\frac{\pi}{6}$	$\frac{a}{2}$
60°	$\frac{\pi}{3}$	$\frac{a\sqrt{3}}{2}$
90°	$\frac{\pi}{2}$	a
120°	$\frac{2\pi}{3}$	$\frac{a\sqrt{3}}{2}$
150°	$\frac{5\pi}{6}$	$\frac{a}{2}$
180°	π	0
210°	$\frac{13\pi}{6}$	$-\frac{a}{2}$
	and so on.	

The length taken to represent 2π is therefore the *wave-length* of the curve.

The points H, J, etc., where the curve rises above the axis are called *ascending nodes*, and the points L, M, etc., where the curve dips below the axis *descending nodes*. The ordinate PQ at any point Q is evidently equal to $a \sin \frac{HQ}{HJ} \cdot 2\pi$. This is also equal to $\frac{JQ}{HJ} 2\pi$ where JQ is

the distance that Q is beyond the nearest ascending node to the left of Q.

Now the angle θ increases uniformly with time and is equal to ωt where ω is the angular velocity of rotation of the radius vector BC, and t is the time which has elapsed since B was at F. Equation (1) may therefore be written

$$y = a \sin \omega t \ldots\ldots\ldots\ldots\ldots (2)$$

and the same curve may be plotted from the numbers given in the table adjoining. In this case the wave length of the curve is equivalent to the time $\frac{2\pi}{\omega}$.

t.	y.
0	0
$\frac{\pi}{6\omega}$	$\frac{a}{2}$
$\frac{\pi}{3\omega}$	$\frac{a\sqrt{3}}{2}$
$\frac{\pi}{2\omega}$	a
and	so on.

If t is not measured from the instant when B was at F but from an instant which occurred a time α before this it follows that $(t - \alpha)$ is

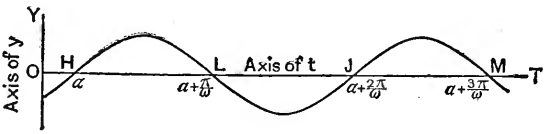


Fig. 37*b*.

the time which has elapsed since B was at F, and therefore we have now

$$y = a \sin \omega (t - \alpha) \ldots\ldots\ldots (3)$$

The accompanying table gives the data from which the curve represented by this equation and shown in Fig. 37*b* has been plotted.

t.	y.
α	0
$\alpha + \frac{\pi}{6\omega}$	$\frac{a}{2}$
$\alpha + \frac{\pi}{3\omega}$	$\frac{a\sqrt{3}}{2}$
$\alpha + \frac{\pi}{2\omega}$	a
and	so on.

29*b*. Equations of Curves to represent Wave Motion.—So far we have limited our attention to the consideration of the change of

displacement of a single vibrating particle with change of time or vectorial angle. It is now necessary to find out how the phenomena attached to travelling waves may be represented by travelling sine curves. If we confine our attention to particles at points equally spaced along the line of motion we know, since the wave velocity remains constant, that the vibration of each particle lags a certain definite time behind that of the next particle nearer the source, so that a sine curve may be used to give the displacements of all particles at a given instant. In Fig. 37*c* the wave motion is considered to be travelling to the right, ordinates above the line represent positive displacements (*i.e.* forward displacements in longitudinal motion), ordinates below the line negative displacements, and *HJ* represents the distance between two particles in

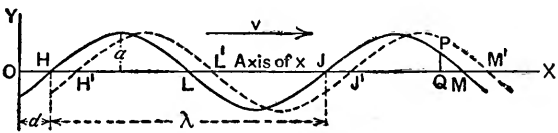


Fig. 37*c*. Curves showing the displacements of particles at two given instants.

the same phase, *i.e.* *HJ* is equal to the wave length. If the origin of coordinates is taken at a point *H* (Fig. 37*c*) where at the instant considered the particle at *H* is in its equilibrium position, the displacement of *Q* any particle along *HX* is given by

$$y = a \sin \frac{HQ}{HJ} \cdot 2\pi,$$

$$= a \sin \frac{x}{\lambda} \cdot 2\pi = a \sin \frac{2\pi}{\lambda} x \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where x is the abscissa of the point and λ is equal to *HJ*, the wave length of the motion. If, however, we take our origin at a point 0, a distance d to the left of *H*, and measure our x from this we see that

$$y = a \sin \frac{2\pi}{\lambda} (x - d) \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

This equation gives us the displacements of all particles at any given instant in terms of the distances of the particles from the chosen origin.

We want now to obtain an equation which shall also show us how the displacement of a particle changes with the time.

Let v be the velocity of propagation of the wave motion through the medium and let $HLJM$ and $H'L'J'M'$ (Fig. 37c) be the displacement curves at the initial instant and after an interval t respectively.

Then reckoning from an origin at 0 we have:—

Equation of curve $HLJM$, $$y = a \sin \frac{2\pi}{\lambda}(x - OH).$$

Equation of curve $H'L'J'M'$, $$y = a \sin \frac{2\pi}{\lambda}(x - OH').$$

$$= a \sin \frac{2\pi}{\lambda}(x - OH - HH').$$

$$= a \sin \frac{2\pi}{\lambda}(x - OH - vt) \ldots (6)$$

If we had reckoned from an origin at H, which was an ascending node at the time $t = 0$, equation (6) would have been

$$y = a \sin \frac{2\pi}{\lambda}(x - vt) \ldots\ldots\ldots\ldots (7)$$

29c. Applications of the Equations obtained in the last Article.—By giving different values to t it is easy to plot a curve showing how the displacement of a point at a certain distance, x from H, varies with the time.

For example, take a point at L where $HL = \frac{\lambda}{2}$. For this point we have by equation (7)

$$y = a \sin \frac{2\pi}{\lambda}\left(\frac{\lambda}{2} - vt\right) = a \sin \left(\pi - \frac{2\pi vt}{\lambda}\right) = a \sin \frac{2\pi vt}{\lambda}.$$

It facilitates plotting if we choose values of t which make the values of $\frac{2\pi vt}{\lambda}$ multiples of a simple submultiple of π, *e.g.* multiples of $\frac{\pi}{6}$.

Having drawn up a table as shown we can plot the curve (Fig. 37*d*).

Note that the initial increase of positive displacement

t.	y.
0	0
$\frac{\lambda}{12v}$	$\frac{a}{2}$
$\frac{2\lambda}{12v}$	$\frac{\sqrt{3}a}{2}$
$\frac{\lambda}{4v}$	a
and	so on.

of L given by Fig. 37*d* agrees with that indicated by the dotted curve of Fig. 37*c*.

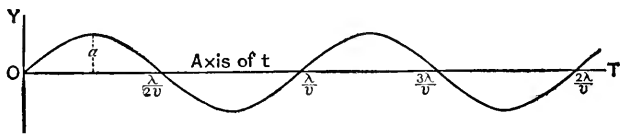


Fig. 37*d*. Curve showing how the displacement of the particle at L (Fig. 37*c*) varies with the time.

If we wish to see how the displacement curve moves along as t is varied we note that the displacement curve at a time t is the initial displacement curve moved to the right a distance vt. Thus the displacement curve at the time $t = 0$, $HLJM$ (Fig. 37*e*), being given by

$$y = a \sin \frac{2\pi}{\lambda} x,$$

the equation of the displacement curve at a time $t = \frac{\lambda}{4v}$ (*i.e.* at a time a quarter of a period later) is

$$y = a \sin \frac{2\pi}{\lambda} \left(x - v \frac{\lambda}{4v} \right)$$

$$= a \sin \left(\frac{2\pi x}{\lambda} - \frac{\pi}{2} \right) = a \cos 2\pi \frac{x}{\lambda}.$$

This gives us the dotted curve $H'L'J'M'$ (Fig. 37*e*) which is simply $HLJM$ shifted forwards a distance $v \times \frac{\lambda}{4v}$ *i.e.* $\frac{\lambda}{4}$ (= a quarter wave length).

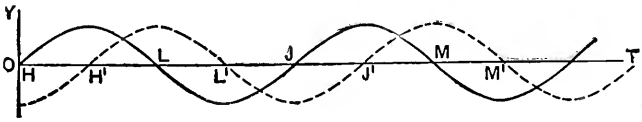


Fig. 37*e*. Diagram showing how the displacement curve moves along with change of time. The dotted curve gives the displacement curve a quarter of a period later than the instant represented by the full curve.

29*d*. Definitions of Epoch and Phase.—In the curve represented by the equation $y = a \sin \frac{2\pi}{\lambda} (x - d)$ the abscissa of the first ascending node is at a distance d from the origin. Either d or its angular equivalent $\frac{2\pi d}{\lambda}$ or its time equivalent $\frac{d}{v}$ is called the *epoch* of the wave train.

The *phase* of a vibrating particle is the fraction of the period which has elapsed since it last passed through its central position in the positive direction. The phase of the particle when its displacement is PQ (Fig. 37*a*) is $\frac{JQ}{HJ}$; the phase of the particle at Q (Fig. 37*c*) is $\frac{JQ}{\lambda}$.

29*e*. Converse of the Treatment in Articles 29*a-c*.—It is possible to reverse the above arguments and readily show that Equation (7)

$$\text{viz.} \quad y = a \sin \frac{2\pi}{\lambda} (x - vt)$$

represents a train of waves of amplitude a and wave length λ moving along the x axis with velocity v.

(i) The maximum value of $y = a$. $\therefore$ a is the amplitude.

(ii) The ordinate at the point whose abscissa is x_1 is

$$y_1 = a \sin (x_1 - vt).$$

If we go beyond x_1 to a point a distance λ ahead, we find that the ordinate of the second point is given by

$$\begin{aligned} y_2 &= a \sin \frac{2\pi}{\lambda} (x_1 + \lambda - vt) \\ &= a \sin \left\{ \frac{2\pi}{\lambda} (x_1 - vt) + 2\pi \right\} \\ &= a \sin \frac{2\pi}{\lambda} (x_1 - vt). \end{aligned}$$

Thus $y_2 = y_1$. This is sufficient to show that λ is the wave length.

(iii) To show that v is the velocity of propagation of the wave motion, construct the equation of the wave at a time given by $t + \frac{\lambda}{v}$. It is

$$\begin{aligned} y &= a \sin \frac{2\pi}{\lambda} \left\{ x - v \left(t + \frac{\lambda}{v} \right) \right\} \\ &= a \sin \left\{ \frac{2\pi}{\lambda} (x - vt) - 2\pi \right\} \\ &= a \sin \frac{2\pi}{\lambda} (x - vt). \end{aligned}$$

The new equation is the same as the former, showing that the curve has advanced exactly one wave length. Therefore the time $\frac{\lambda}{v}$ is the periodic time of the vibration from which it follows that v is the velocity of propagation of the wave motion.

EXAMPLES II.

1. Explain what is meant by *longitudinal wave motion* in a medium. Describe the process of propagation of longitudinal wave motion in a free medium.

2. What arguments would you bring forward to prove that longitudinal waves travel with uniform velocity through an isotropic free medium?

3. Explain how to get a strain diagram from a displacement diagram of longitudinal wave motion. If the displacement diagram is a sine curve show that the strain diagram is also a sine curve.

4. Show that the strain at a point in a medium through which longitudinal waves are passing is proportional to the velocity of that point, also that a particle which is in compression is moving away from the source while a particle which is in rarefaction is moving towards the source.

5. Describe the construction of Crova's disc and show that it very closely exhibits longitudinal wave motion.

6. The wave front due to a body of irregular shape vibrating in a free medium is irregular in shape near the body but is spherical far away. Explain this.

7. A body vibrating with a constant frequency sends waves 10 cms. long through a medium *A* and 15 cms. long through a medium *B*. The velocity of the waves in A is 90 cms. per second. Find the wave velocity in B.

8. Define the *intensity* of the wave motion at any point and prove that the intensity at a point is inversely proportional to the square of the distance of that point from the source.

9. The amplitude of vibration of a particle 100 metres from a source is 1 cm. Find the amplitude at 200 metres. Compare also the intensities at these two distances.

10. In what way does the intensity of the wave motion depend upon the density of the medium surrounding the source?

11. Explain the energy changes which occur during the vibrations of a vibrating body. What happens finally to the energy of vibration?

12. Show that the energy of a vibrating body is proportional to the square of the amplitude of vibration. A particle of mass 20 gms. is vibrating 10 times per second with an amplitude of 10 cms. Find (1) its maximum velocity, (2) its energy.

CHAPTER III.

VELOCITY OF SOUND.

30. Elasticity.—Elasticity is a general name given to that property of bodies by which they resist, and recover from, change of shape or volume. All substances resist changes in volume, and so we have what is called **volume or bulk elasticity.** Only solids, however, possess **elasticity of shape**: no fluid can offer a permanent resistance to change of shape. The deformation, whether of shape or volume, which a body experiences when forces act on it is termed a strain. The equilibrating system of forces which produces the strain is called a stress. Experiment proves that when the strains are small they are proportional to the stresses which produce them, thus the stretching or bending of a rod, the deflection of a fiddle-string, the compression of a gas, etc., are all proportional to the forces causing them.

The ratio of stress to strain is called the **modulus of elasticity.**

31. Young's Modulus.—This modulus applies only to solids. Suppose that a metal wire of length L cms. and cross section a sq. cms. hangs from the ceiling and carries a hook at the bottom. If a mass is attached to the hook its weight will stretch the wire. Suppose that a weight of F dynes produces an elongation of l cms. Then the strain is defined as $= \frac{\text{elongation}}{\text{original length}} = \frac{l}{L}$, and the stress is defined as $= \frac{\text{force applied}}{\text{area of cross section}} = \frac{F}{a}$.

The ratio of stress to strain for such a case is called Young's Modulus, therefore, denoting the Young's Modulus for the material of the given wire by Y, we get

$$Y = \frac{F/a}{l/L} = \frac{LF}{al} \text{ dynes per sq. cm.}$$

EXAMPLE.—A steel wire 585 cms. long and radius of cross-section ·333 mm. was stretched ·568 cm. by a load of 7·27 kilos. Find Young's Modulus for steel. (Given that 1 gm. = 980 dynes.)

$$\text{Strain} = \frac{\cdot 568}{585}.$$

$$\text{Stress} = \frac{7270 \times 980}{\pi \times (\frac{1}{30})^2}.$$

$$\therefore \text{ Young's Modulus} = \frac{7270 \times 980 \times 900}{3 \cdot 14} \times \frac{585}{\cdot 568}$$

$$= 2 \cdot 1 \times 10^{12} \text{ dynes per sq. cm.}$$

Young's Modulus may be looked upon as the force which, if applied to a rod of unit cross section, would double its length, provided this extension did not exceed the limits of elasticity and the cross section did not change.

32. Bulk or Volume Modulus.—This modulus applies to solids, liquids, and gases. If a hydrostatic pressure of p dynes per sq. cm. acting on a body of volume V cub. cms. reduces the volume by v cub. cms., then

$$\text{the volume strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{v}{V},$$

$$\text{the stress} = p,$$

therefore the bulk modulus of the material of which the body is composed

$$= \frac{p}{v/V} = \frac{pV}{v} \text{ dynes per sq. cm.}$$

The experimental determination of the bulk modulus in the case of solids and liquids is difficult, as these are not very compressible. In this case the bulk modulus can be obtained more readily by indirect calculations. Gases, however, submit easily to pressure, hence the bulk or volume elasticity of a gas is easily determined.

The bulk modulus for gases has very different values, depending upon the temperature conditions under which the experiment is carried out. There are two principal cases. If the temperature remains constant during the change, the modulus obtained from $\frac{pV}{v}$ is called the **isothermal elasticity.** If the change takes place so quickly that there is no time for heat to enter or leave the body, the modulus obtained is called the **adiabatic** elasticity. Perhaps the difference between these two moduli can be made clearer by an experimental illustration.

33. The two ways of Measuring Volume Elasticity.— If we could have a cubic centimetre of air confined under a piston *C*, Fig. 38, in a tube *AB* of unit sectional area, both piston and tube being of very low thermal conductivity, and if we applied to the piston a very small downward force of f dynes, the piston would instantly descend, diminishing the volume of the air by an amount x, which is numerically equal to the volume strain. At the same time a quantity of heat equal to the work done in the descent of the piston would be produced in the compressed air, and would raise its temperature. As the air cooled to its original temperature, the piston would descend further, till the volume of the air had been diminished altogether by a volume x'.

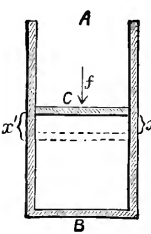


Fig. 38.

Reversing the process, if we applied an upward pull f to the piston, it would instantly rise till the volume of the air under it had been increased by x, and the temperature of this air would fall; then, as the air returned to its original temperature, the piston would rise still further till the total increase of volume was x'. The ratio $\frac{f}{x}$ is called the *adiabatic elasticity* (x being measured before any heat has entered or left the air), and $\frac{f}{x'}$ is called the

isothermal elasticity. As x' is greater than x, the isothermal elasticity is smaller than the adiabatic.

The experiment as described above is impracticable, as the gas loses or gains heat much too quickly to allow of determining x, but when a pulse travels along a tube, the air in each centimetre of the tube changes from its original to its altered volume in less than $\frac{1}{30000}$ of a second, which would be too short a time for it to lose or gain much heat, even if air were a good conductor and radiator instead of an extremely bad one, so that the volume strain in the air column is the same as the instantaneous volume strain which would be produced by the same pressure applied to the piston of our imaginary experiment.

34. Values of the Isothermal and Adiabatic Elasticities of a Gas.—In the case of a gas or vapour which is hard to liquefy the relation between pressure and volume at constant temperature is given by Boyle's Law. This law states that if PV, P_1V_1, P_2V_2 are successive values of corresponding pressures and volumes of a constant mass of gas at constant temperature,

$$PV = P_1V_1 = P_2V_2 = \ldots = \text{a constant.}$$

To find the value of the isothermal elasticity suppose that, starting with a volume of gas V at initial pressure P the pressure is increased to $P + p$, and the volume isothermally changed to $V - v$, where p and v are small.

Applying Boyle's Law we have

$$PV = (P + p)\ (V - v).$$

Multiplying out the terms on the right hand we get

$$PV = PV - Pv + pV - pv.$$

Now both p and v are small, therefore their product pv is negligibly small compared with the other terms,

$$\therefore Pv = pV,$$

$$\text{or } P = p\frac{V}{v} = \frac{p}{v/V}.$$

But p is the stress and $\frac{v}{V}$ is the volume strain. Therefore **the isothermal elasticity of a gas is equal to the pressure.**

The value of the adiabatic elasticity of a gas is not so readily obtained from first principles, but it may be obtained by the following direct and instructive method.

Take unit mass of gas, and let P, V, T denote its original pressure, volume and absolute temperature. Let c_v denote its specific heat at constant volume, and c_p its specific heat at constant pressure. If its volume be kept constant and a small quantity of heat h be given to it, its temperature rises by an amount h/c_v, and therefore its pressure (which is always proportional to the absolute temperature, since the volume is constant) becomes

$$P\left(T + \frac{h}{c_v}\right)/T,$$

therefore the rise of pressure is the small quantity $p = Ph/Tc_v$.

If the pressure had been kept constant, and quantity of heat h taken away, the temperature would fall by h/c_p, and the volume would become

$$V\left(T - \frac{h}{c_p}\right)/T,$$

therefore the change of volume would be the small quantity $v = Vh/Tc_p$.

Since h is quite small, we can take these changes as simultaneous; then there is no heat given or taken away, and the changes of pressure and volume are

$$p = Ph/Tc_v,\ v = Vh/Tc_p.$$

Hence

$$\frac{p}{v} = \frac{Pc_p}{Vc_v}.$$

Hence the elasticity of this adiabatic change is

$$E = p \div \frac{v}{V} = p\frac{V}{v} = P\frac{c_p}{c_v}.$$

The ratio of the two specific heats $\frac{c_p}{c_v}$ is generally denoted by γ; hence the adiabatic elasticity is equal to γP,

i.e. **adiabatic elasticity $= \gamma \times$ isothermal elasticity.**

The difference between isothermal and adiabatic elasticity is too small to be considered in the case of liquids and solids, also the isothermal elasticity is not equal to the pressure, and is practically unconnected with it.

35. Velocity of a Pulse along a Rod.—If we take a long uniform rod AB of any elastic substance, and apply at one end A a force of f dynamical units, directed towards B, a portion of the rod close to A shortens, till it exerts a force f on the next portion, and so the compressed condition extends along the rod, each portion shortening, in turn, by a certain fraction, l, of its original length. This fraction is called the *longitudinal strain.* It will now be shown that if m is the mass of a piece of the rod of unit length, the velocity V with which the compressed condition extends along the rod is $\sqrt{\frac{f}{lm}}$.

When the force of f dynes acts on the end A on the rod the first centimetre of the rod shortens till it exerts a force of f dynes on the next centimetre, and then this shortens till it exerts the same force on the next, and so on. As the process is exactly the same for each successive centimetre, the compressed condition extends, with uniform velocity, along the rod. Let this velocity be V, and let the mass of each centimetre be m, and let the amount by which each (original) centimetre is diminished in length, when there is a force f at each end of it, be l. As each centimetre becomes compressed in its turn, all the centimetres between it and A, which are compressed already, advance towards B through a distance l. As V fresh centimetres become compressed in each second, Vl is the velocity with which the portion already compressed moves towards B. In each second V fresh centimetres are added to this portion, and each of these, when compressed, has a momentum Vlm. So V^2lm is the increase of momentum of this portion per

second. But rate of change of momentum is equal to the force producing it.* Hence

$$f = V^2 lm \text{ or } V = \sqrt{\frac{f}{lm}}.$$

This is true whatever the circumstances under which the compression takes place, and whether the "rod" is solid or fluid, but only if f is measured in the *dynamical* units based on the units of length, mass, and time employed in the other measurements. Thus, if m is in grammes, f must be in *dynes*, and the velocity will be in centimetres per second; or, if m is in pounds, f must be in *poundals*, and the velocity will be in feet per second.

For moderate forces, l is proportional to f, so that $\sqrt{\frac{f}{lm}}$ is the same whether the force is smaller or larger, if not very large. Also, if we double the sectional area of the rod, we require twice the force to produce the same change in the length of a centimetre of it, but a centimetre has twice the mass, so that $\sqrt{\frac{f}{lm}}$ is the same for a thick rod as for a thin one of the same material.

Thus we need not know the force applied or the diameter of the rod to calculate the velocity with which the condition extends along the rod, for it would be the same for any diameter and any force. If we suppose the rod of unit sectional area, f is numerically equal to the longitudinal pressure† applied, and if the rod is solid so that it is quite free to expand sideways, $\frac{f}{l}$ is equal to Y, the dynamical measure of Young's Modulus for the substance

* Force = Mass × Acceleration.
Acceleration = Rate of change of Velocity.
Momentum = Mass × Velocity.
∴ Force = Rate of change of Momentum.

† In this book "pressure" always means stress, *i.e.* the ratio of the force to the area on which it is exerted or the force per unit area. Some writers call the force itself the "pressure," and the ratio of force to area the "intensity of pressure," but this notation should be avoided.

of the rod,—Young's Modulus for any substance meaning the ratio of longitudinal stress to longitudinal strain in a rod of that substance to which no other forces are applied except lengthwise. Also, as the rod is of unit sectional area, each unit of length of it contains unit volume, and m is numerically equal to the density, D, of the substance. So that for such a rod, and therefore for any rod,

$$V = \sqrt{\frac{\text{dynamical measure of Young's Modulus}}{\text{density}}} = \sqrt{\frac{Y}{D}}.$$

If the force at A is a pull instead of a push, the same reasoning applies, except that "lengthens" must be substituted for "shortens." If the pull is not very violent, the longitudinal strain it produces is equal to that produced by an equal push, so that the stretched or rarefied condition extends at the same rate as the condensed condition.

The longitudinal strain produced by a given *change* of stress is nearly the same whatever the actual stress may be, and the density of a solid is only slightly altered by any ordinary stress. The velocity of a condensation or rarefaction along a rod or wire, therefore, depends hardly at all on whether the rod or wire is stretched or not.

36. Velocity of a Pulse along a Fluid in a Tube.—If instead of a solid rod we had a fluid column in a tube, and applied a force of f units to a piston at one end, the condensation would extend along the tube with a velocity $\sqrt{\frac{f}{lm}}$ just as before. If the tube were of unit sectional area, we should be applying an increase of pressure f to the end of the condensed portion of the fluid, and (from the properties of fluids) to every side of it as well; and as in this case there is no change of cross section, the volume of each portion changes in the same ratio as its length changes, so that l is the ratio of the change of volume of each portion to its original volume, a quantity which is called the *volume strain*. So $\frac{f}{l}$ would be the ratio of a

change of hydrostatic pressure to the volume strain produced by it, a quantity which is called the *volume elasticity** of the fluid. The velocity with which a condensed or rarefied condition spreads along a liquid or gaseous column is therefore

$$\sqrt{\frac{\text{dynamical measure of volume elasticity}}{\text{density}}} = \sqrt{\frac{E}{D}}$$

where E is the dynamical measure of the volume elasticity.

37. The following method of finding the velocity of propagation of longitudinal wave motion in a fluid is very instructive:—

(1) **Relation between wave velocity and particle velocity for longitudinal wave motion in a fluid medium.**—

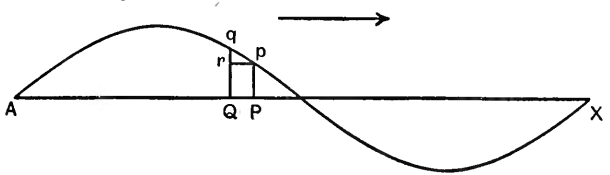


Fig. 39.

Let the curve AX (Fig. 39) represent the curve of displacement for wave motion travelling from A to X, outwards into the medium. The velocity with which the motion is propagated along AX is evidently the velocity

* Strictly, Young's Modulus is defined not as the ratio of change of stress to longitudinal strain, but as the limit which this ratio approaches when the change of stress is indefinitely diminished; and, similarly, the volume elasticity is the limit of the ratio of change of hydrostatic pressure to volume strain, when the change of pressure is indefinitely diminished. So that the formulæ $V = \sqrt{\frac{\text{Young's Modulus}}{\text{density}}}$ and $V = \sqrt{\frac{\text{volume elasticity}}{\text{density}}}$ are only absolutely true for indefinitely small changes of stress. But, even for much larger changes of stress than those which occur in sound waves, the ratio of change of stress to change of length or volume is practically the same as for indefinitely small changes of stress.

with which any particular displacement travels along AX. Let Q and P be two points taken very close together on the line AX. When the displacement Pp, at P, has increased until it is equal to the displacement Qq, at Q, the displacement at Q is said to have "travelled" from Q to P. This implies that the displacement at Q "travels" from Q to P in the *same time* as the particle at P increases its displacement (along AX) from a distance represented by Pp to one represented by Qq, that is, by a distance rq, the difference between Qq and Pp. Hence, if V denote the velocity of the wave motion, t the time in which any given displacement travels from Q to P, and v the average velocity of the particle at P during this time, we have

$$Vt = QP \text{ and } vt = rq.$$

Hence we get the relation

$$v/V = rq/QP \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If the point Q is supposed to be very near, infinitely near, P, then v denotes the velocity of the particle at P in the path of its vibration. Now, for any two layers close together, such as P and Q, the strain in the medium between them is determined by the difference of the ordinates at P and Q. Both layers are here displaced onwards, but the layer at Q is displaced further than the layer at P. The medium between the layers is therefore *compressed*, and the degree of compression or *strain* is measured by the ratio rq/PQ. Now, if E denote the modulus of volume elasticity for the medium,

$$E = \frac{\text{stress}}{\text{strain}}.$$

The stress at any point would be measured by the *excess* of the pressure at that point over the normal pressure. Hence, if p denote this excess of pressure at P, we get

$$E = \frac{p}{rq/rp} \text{ or } rq/rp = p/E.$$

Substituting this in relation (1) above we have

$$v/V = p/E \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This is an important general relation between the quantities involved at any point on the line of transmission.

Dynamics of the motion.—If we now consider the motion of any very thin plane layer of the medium taken at right angles to the line of transmission AX, it will be evident that the force *per unit area* causing the motion of the layer backwards or forwards along AX at any instant is the difference of the pressures at the two faces of the layer.

Hence, if we take the layer between the points P, Q on AX (Fig. 40), and if p_1 and p_2 denote the excess of pressure in the medium at the points Q and P respectively

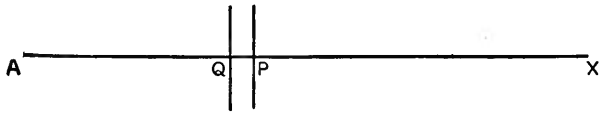


Fig. 40.

at a given instant, then $p_1 - p_2$ is the force per unit area causing the motion of the layer. If x denote the thickness of the layer and D the density of the medium, then the *mass* acted on by the force $p_1 - p_2$ is evidently Dx, and, applying the usual relation between force, mass, and acceleration, we have

$$\frac{p_1 - p_2}{Dx} = a,$$

where a denotes the acceleration of the layer.

If Q is supposed indefinitely close to P, a will denote the acceleration at P. But if v_1 denote the velocity of a particle at Q and v_2 the velocity of a particle at P at the given instant, then v_2 will change to v_1 in the time the motion takes to travel from Q to P. This time being x/V we at once get

$$\frac{v_1 - v_2}{x/V} = a,$$

where a again denotes the acceleration at P if the distance QP is indefinitely small.

Equating these two values of the acceleration at P we get

$$\frac{p_1 - p_2}{Dx} = \frac{v_1 - v_2}{x/V}$$

or

$$\frac{p_1 - p_2}{v_1 - v_2} = VD.$$

But, from the relation expressed by (2) above,

$$\frac{p_1 - p_2}{E} = \frac{v_1 - v_2}{V}$$

or

$$\frac{p_1 - p_2}{v_1 - v_2} = \frac{E}{V}.$$

Hence we get

$$VD = \frac{E}{V} \text{ or } V^2 = \frac{E}{D}.$$

That is,

$$V = \sqrt{E/D}.$$

38. **Relation between wave velocity and particle velocity for longitudinal wave motion in a rod.**—Using the same notation we get

$$v/V = rq/QP = f/E$$

just as for a liquid (f replacing p). Of course E is now Young's Modulus and not the volume elasticity.

Let m be the line-density of the rod in its normal state, then m is the mass of unit length. When compressed, unit length is shortened by l, and becomes length $1-l$; hence its line density becomes $m/(1-l)$. Hence

$$\frac{\text{Line density of compressed portion}}{\text{original line density}} = \frac{1}{1-l} = 1 + l \text{ nearly},$$

$$\therefore \quad \frac{\text{change of line density}}{\text{original line density}} = l, \text{ nearly.}$$

But

$$E = f/l, \quad \therefore \ v/V = f/E = l.$$

Hence

$$\frac{\text{Change of line density}}{\text{Original line density}} = \frac{\text{Velocity of compressed portion}}{\text{Velocity of wave}}.$$

Since the denominators are constant, the velocity of a vibrating portion of substance $\propto$ difference between actual and average line-density, *i.e.* $\propto$ change of density.

39. Velocity in Free Media.—The velocity of sound along a rod which is free to expand sideways is much less than it is when the transverse expansion is prevented, the ratio $\frac{f}{l}$ being much larger in the latter case. In a column of air which was free to expand sideways sound would not be propagated at all, as in that case the ratio $\frac{f}{l}$ would be zero.* But in open air the velocity, except within a very short distance from the source, is the same as along the air in a tube. For as the condensed condition travels outwards in all directions from the source, the condensed air does not move except in the line of propagation of the sound, since in all directions at right angles to this line it is in contact with air condensed as much as itself. The velocity with which the condensation travels is therefore the same as if transverse expansion were prevented by a rigid tube. For the same reason the velocity of a sound wave through a large mass of a solid substance, a cliff for instance, is much greater than along a rod.

The kind of elasticity f/l which is called into play when a solid rod is shortened or lengthened, while lateral motion is prevented, is called the **Compression Modulus.** Denoting it by E, we can apply the same methods to this case as to that of a liquid; and find that the velocity is

$$\sqrt{E/D}.$$

It can be shown that

E = modulus of volume elasticity $+ \frac{4}{3}$ modulus of rigidity.† From this it follows that the velocity of sound in an unlimited medium is about 1·1 times the velocity in a rod or wire.

* This does not apply to a column of air in our atmosphere for the "sideways" expansion of such a column is resisted by the pressure of the surrounding atmosphere.

† See Wagstaff's *Properties of Matter*, Arts. 116, 117, for definition of modulus of rigidity, and Barton's *Sound* (Macmillan), Arts. 77, 78, for proof of the formula.

40. Application of the formula in special cases.—The velocity of longitudinal wave motion in a fluid medium is therefore given by the relation

$$V = \sqrt{E/D},$$

where E denotes the modulus of volume elasticity and D the density of the medium. In applying this relation it must be remembered that E is the modulus of elasticity which properly applies to the strain set up in the medium. In a free unlimited fluid medium, where the strain is a pure volume strain, the modulus applicable is the modulus of volume elasticity. If the conditions of the strain are isothermal or adiabatic, then the modulus of isothermal or adiabatic elasticity must be taken. Since the strains of compression and rarefaction which accompany longitudinal wave motion are very suddenly produced and of very short duration, they must, in a medium of low thermal conductivity, take place under practically adiabatic conditions, and the modulus of elasticity involved is therefore the modulus of adiabatic elasticity of volume. The density of the medium, represented by D, is the density under the conditions that exist at the time of propagation of the wave motion.

In a solid medium E is replaced by a quantity which involves the modulus of volume elasticity and the modulus of simple rigidity or form elasticity. In the case of solids and liquids the difference between the adiabatic and isothermal elasticities is very small, and as the latter is the one usually determined experimentally it is used in the formulae.

In the case of the transmission of longitudinal wave motion along the length of a thin solid rod the conditions of strain are those to which Young's modulus applies ; this modulus must therefore be taken to determine the velocity in this case.

In the case of a gas, the value of E is γP, as explained in Art. 34, and the velocity of longitudinal wave motion in a gas is therefore given by

$$V = \sqrt{\gamma P/D},$$

where P denotes the pressure of the gas, γ the ratio of its two specific heats, and D its density.

The value of γ depends, in the case of gases, chiefly on the number of atoms in a molecule; for one-atom molecules, like mercury vapour and argon, it is 1·66; for two-atom molecules, like oxygen and nitrogen, it is between 1·40 and 1·41; for three-atom molecules, like carbon dioxide and water vapour, it is between 1·30 and 1·31; for more complex molecules it is less, but does not depend so definitely on the number of molecules.

The value of γ is known most accurately for air. It is 1·404.

The velocity of sound really depends, of course, on the changes which actually take place in the air as the wave travels through it, not on the changes which would take place under quite different circumstances. The isothermal elasticity and the specific heats have therefore no direct connection with the velocity, for the air is neither compressed without change of temperature, nor heated without change of pressure, nor heated without change of volume, when a sound wave passes through it. But the physical properties of gas on which the velocity of sound depends (the chief of which, as we shall shortly see, is the velocity of its molecules) are also factors in determining its adiabatic and isothermal elasticities and its two specific heats, and therefore it is quite possible to express the velocity of sound in terms of these quantities.

41. Calculation of the Velocity of Sound in Air.—The velocity of sound in a gas is therefore given by the relation $V = \sqrt{\gamma P/D}$, where P expresses the pressure of the gas and D its density under existing conditions.

For the purpose of this formula P and D must evidently be expressed in appropriate and consistent units; P must be given in units of force per unit of area, and D in units of mass per unit volume. Thus, in the C.G.S. system, P would be expressed in dynes per square cm., and D in grammes per c.c., and in these units the value of V would be given in cms. per second. Similarly, in the English F.P.S. system, P should be in poundals per square foot, D in pounds per cubic foot, and V in feet per second. The pressure of a gas is usually specified in terms of the

height of the barometric column, so that in applying the above formula it is necessary to convert the pressure specified in this way into proper units. Thus the pressure indicated by a barometric height of 760 mm. (corrected to 0° C.) is appropriately expressed as $76dg$ dynes per square cm., where d represents the density of mercury at 0° C. and g the acceleration due to gravity at the place of observation. The value of d is 13·60 grammes per c.c., and g may be taken, with sufficient accuracy for most places in England, as 981 cms. per second per second.

Hence, in air at normal temperature and pressure P may be taken as $76 \times 13{\cdot}60 \times 981$ dynes per square cm., or $1{\cdot}0136 \times 10^6$ dynes per square cm., and

$$E = 1{\cdot}404P = 1{\cdot}404 \times 1{\cdot}014 \times 10^6 \text{ dynes per square cm.}$$

Also D, for *dry* air, is ·001293 gramme per c.c., and V is therefore given by the relation

$$V = \sqrt{\frac{1{\cdot}404 \times 1{\cdot}014 \times 10^6}{{\cdot}001293}} \text{ cms. per second}$$

$$= 33{,}200 \text{ cms. per second.}$$

That is, the velocity of sound in *dry* air at normal temperature and pressure is about 332 metres per second, or about 1090 ft. per second. This result agrees with that obtained by experiment, as detailed in Arts. 46, 47.

The velocity of sound in hydrogen, or any other gas, may be calculated in the same way. Thus, for hydrogen at normal temperature and pressure

$$E = 1{\cdot}404 \times 1{\cdot}014 \times 10^6,$$

$$D = {\cdot}00009001 \text{ gramme per c.c.,}$$

and V is given by

$$V = \sqrt{1{\cdot}404 \times 1{\cdot}014 \times 10^6 / {\cdot}00009} \text{ cms. per second}$$

$$= 126{,}000 \text{ cms. per second.}$$

It evidently follows from the relation $V = \sqrt{\gamma P/D}$ that, under the same conditions, the velocities of sound in any two gases for which the value of γ is the same are inversely proportional to the square roots of their densities. For in one gas we have $V_1 = \sqrt{\gamma P/D_1}$, and in the other

$V_2 = \sqrt{\gamma P/D_2}$, and therefore $V_1/V_2 = \sqrt{D_2/D_1}$. For example, in the case of air and hydrogen at normal temperature and pressure, from the data given above, we have $V_1/V_2 = \sqrt{90/1293}$, and from this relation either V_1 or V_2 may be calculated if the other is known.

The densities of simple gases are proportional to their atomic weights; hence the velocities of sound in these gases are (provided γ is the same) inversely proportional to the square roots of the atomic weights.

42. Influence of Pressure on the Velocity of Sound in Air.—Let D = density of a gas at 0° C. and any standard pressure H dynes per sq. cm.; then $\frac{D}{H}$ is a constant for the gas and does not depend on what the pressure H was at which D was measured. We will denote $\frac{D}{H}$ by d; it is the density which the gas would have at 0° C. and a pressure of 1 dyne per sq. cm.

Then the density of the gas at a pressure of P dynes per sq. cm. and a temperature t° C. is

$$Pd\left(\frac{273}{273+t}\right);$$

so the velocity of sound in the gas at this pressure and temperature is

$$\sqrt{\frac{\gamma P}{Pd\left(\frac{273}{273+t}\right)}} \text{ or } \sqrt{\frac{\gamma\,(273+t)}{273d}}.$$

It is therefore *independent of the pressure*, a fact which may be explained by saying that the adiabatic elasticity and the density are both proportional to the pressure, and that therefore their ratio, on which the velocity depends, is the same for all pressures; but this is a mathematical rather than a physical explanation, and the physical reason will appear later.

In the formulae just given note that d is not the density of the gas actually conveying the sound, but the density it would have at 0° C. at a pressure of 1 dyne per sq. cm.

43. Influence of the Temperature on the Velocity of Sound in Air.—Variation in temperature, however, causes change in the density of the gas, and therefore affects the velocity of sound in the gas. A rise of temperature causes the density, D, to decrease, and therefore increases the value of V; similarly, a fall of temperature increases D and decreases the value of V. That is, the velocity of sound in a gas increases with rise of temperature.

If V_1, V_2 are the velocities of sound in the *same* gas at two different temperatures t_1, t_2, we have

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{\gamma\,(273 + t_1)}{273d}}}{\sqrt{\frac{\gamma\,(273 + t_2)}{273d}}} = \sqrt{\frac{273 + t_1}{273 + t_2}},$$

or the ratio of the velocities is the square root of the ratio of the absolute air-thermometer temperatures. (*Higher Text-book of Heat*, Art. 35.) This is true whether the pressures are the same or not.

Let V_o be the velocity of sound in air at 0° C., and V_t be the velocity at t° C., then

$$V_t = V_o \sqrt{1 + \tfrac{1}{273}t}.$$

If we take the value of V_o as 33200 cms. per second,

$$V_t = 33200 \sqrt{1 + \tfrac{1}{273}t}.$$

If t is small, so that we may treat $\sqrt{1 + \frac{1}{273}t}$ by approximations, the formula reduces to

$$\begin{aligned} V_t &= 33200 \left(1 + \frac{t}{546}\right) \\ &= 33200 + \tfrac{33200}{546}t \\ &= 33200 + 61t. \end{aligned}$$

Similarly, if we take $V_o = 1090$ ft. per second, then $V_t = 1090 \sqrt{1 + \cdot00366t}$ or $V_t = 1090 + 2t$. That is, the velocity of sound in a gas increases with rise of temperature by about 61 cms. per second, or about 2 ft. per second for each degree (Centigrade) rise in temperature.

The following alternative method is useful.

Let P = pressure, v = volume of unit mass, t = Centigrade temperature, and T the absolute temperature = $273 + t$.

Then for a so-called perfect gas $Pv/T = R$, the gaseous constant.

For Hydrogen at normal temperature and pressure, 1 gramme has a volume of 11·11 litres or 11110 c.cs. Hence

$$Pv/T = 11110 \times 1{\cdot}014 \times 10^6 \div 273 = 4{\cdot}13 \times 10^7;$$

this is numerically so near the value of Joule's Equivalent of Heat that it is often denoted by the same letter J. Hence $Pv/T = J$ for a gramme of hydrogen, which molecular formula is H_2 and molecular weight 2. For other gases the constant is inversely proportional to the molecular weight x, and we can therefore put

$$Pv/T = 2J/x.$$

Now the velocity of sound $V = \sqrt{E/D} = \sqrt{\gamma P/D}$, but $D = 1/v$,

$$\therefore\ V = \sqrt{\gamma Pv} = \sqrt{2J\gamma T/x}.$$

This is a very convenient formula. For nitrogen, N_2, $x = 28$; for oxygen, O_2, $x = 32$. For air (4 parts of nitrogen to 1 of oxygen); the mean value of x is 28·8. Taking $\gamma = 1{\cdot}40$, $J = 4{\cdot}13 \times 10^7$, and $t = 0°$C., *i.e.* $T = 273°A$, we get

$$V = \sqrt{8{\cdot}26 \times 10^7 \times 1{\cdot}40 \times 273/28{\cdot}8}$$

$$= 33200 \text{ cms. per sec.}$$

44. Influence of the Presence of Moisture on the Velocity of Sound in Air.—The humidity or presence of water vapour in a gas affects the velocity of sound by altering the density. The density of water vapour relative to air at the same temperature and pressure is about ·62, so that in the case of air the presence of water vapour reduces the density, and the velocity of sound in moist air is therefore greater than in dry air.

From what has been said it is evident that in applying the formula $V = \sqrt{\gamma P/D}$ to determine the velocity of sound in a gas, it is only necessary in order to correct for variation of pressure, temperature, and humidity, to determine the correct value of D under the existing conditions, and to calculate the value of V given by substituting this value of D, and the specified value of P, in the formula. We are assuming that the air is not so moist as appreciably to affect the value of γ.*

* The value of γ for air saturated with water vapour is 1·401, so that very little error can arise on this account.

EXAMPLE.—Find the velocity of sound in air at 15°C., given that the dew-point is 10°C. and the barometric height (reduced to 0°C.) is 759·13 mm.

Under these conditions the density of the moist air is found to be 1·219 grammes per litre,* that is D = ·001219 gramme per c.c. The value of P is the pressure due to 759·13 mm. of mercury expressed in dynes per square cm.; that is, $1{\cdot}0125 \times 10^6$ dynes per square cm. From these data the value of V given by the relation $V = \sqrt{\gamma P/D}$, is $V = \sqrt{1{\cdot}404 \times 1{\cdot}0125 \times 10^6/{\cdot}001219}$ cms. per second, or $V = 34{,}200$ cms. per second.

The effect of aqueous vapour in the air is not fully corrected for by correcting the density only; moist air does not strictly obey Boyle's law, so that its isothermal elasticity is only approximately measured by its pressure. Also the ratio of the two specific heats for the mixture of air and water vapour is not exactly 1·404. Under ordinary atmospheric conditions, however, these sources of error in applying the formula $V = \sqrt{\gamma P/D}$ as above, to determine the velocity of sound in air under given conditions, may be neglected.

45. Laplace's Correction of Newton's Calculation of the Velocity of Sound in Air.—The calculation of the velocity of sound in air from the relation $V = \sqrt{E/D}$ was first made by Newton. In making the calculation, however, he assumed that the compressions and rarefactions of the waves were effected under *isothermal conditions*, and on this assumption deduced from Boyle's law, as in Art. 34, that the modulus of elasticity applicable to the case was measured by the pressure of the gas. Newton, therefore, stated that $V = \sqrt{P/D}$, where P denotes the pressure and D the density of the air. With data then available for air this formula gave the value of V in air about 979 ft. per second.

This value Newton recognised to be much below the value obtained by experiment, and he endeavoured to explain the difference in the results by assuming that sound travelled instantaneously through the air molecules, and that the 979 feet obtained by the formula as the distance travelled in one second was really only the sum of the distances between the molecules and not the total distance. He also assumed that the water vapour present in the air took no part in the transmission of the waves. These assumptions are, however, now known to be

* *Higher Text-Book of Heat*, p. 196.

inadmissible, but it was only in 1816 that the true explanation of the mistake made by Newton was pointed out by Laplace. Laplace showed that the very rapid compressions and rarefactions which take place in sound waves were not effected under isothermal but under adiabatic conditions, and that the value of E was not given by P, but by γP as already explained.

It is obvious that the isothermal conditions cannot hold with such rapid compressions and rarefactions, but it is not so obvious why the conditions are purely adiabatic and not somewhere between isothermal and adiabatic. Stokes, however, has shown that if the conditions were somewhere between isothermal and adiabatic, at each vibration a large quantity of the energy would be frittered away into heat and thus the waves would quickly die away. This, we know, does not happen, hence we assume that the conditions are wholly adiabatic.

46. Experimental determination of the Velocity of Sound in air.—Common observation shows that sound takes time to travel. In watching a cricket match the sound of a hit is always heard a little after the ball has left the bat; the report of a gun fired at a good distance from an observer is heard some seconds after the flash is seen; the explosion of a rocket shell is heard some time after the shell is seen to burst, and thunder is always heard a short time after the lightning flash is observed.

In all these cases the interval between the instant the production of the sound is *seen* and the instant the sound is *heard* is really the difference in the times taken by light and sound to travel from the source to the observer. The velocity of light is, however, so great that the time taken by it to travel over ordinary terrestrial distances is negligibly small, and the intervals referred to above are practi cally the times taken by the sounds to travel from the points of production to the observer. Careful observation will show that the magnitude of these intervals depends upon the distance of the source of sound from the observer: the greater the distance the greater the interval, and the shorter the distance the shorter the interval. Hence, if in any particular case the distance from the source of sound to the observer is known, and the interval of time between

the instant of seeing the sound produced and the instant of hearing the sound is carefully measured, the average velocity of sound in air over the observed distance can be at once determined.

Thus, if a cannon, distant 13,420 feet, is fired and the interval between the instant of observing the flash and the instant of hearing the sound is found to be exactly 11 seconds, then the average velocity of the sound in air over the observed distance is 13420/11 feet per second or 1120 feet per second.

From the beginning of the eighteenth century many determinations of the velocity of sound in air have been made by means of this direct method of experiment. The time taken by the report of a cannon in travelling between two stations a known distance apart was measured, and the velocity calculated from the data so obtained. In the earlier experiments the effect of wind and the influence of the temperature and hygrometric state of the air on the result were more or less neglected. The methods of measuring the time interval were also less accurate than those adopted in later experiments.

The effect of wind was first determined experimentally by Derham's experiments. In 1708 Derham found, after repeated experiments, that the report of a cannon travelled over a distance of $12\frac{1}{2}$ miles, between Blackheath and the tower of Upminster Church in Essex, in a time which varied between 55·5 and 63 seconds according as the strength and the direction of the wind varied. The shorter interval was obtained when the directions of the wind and sound were the same, the longer interval when these directions were opposite. As the result of this experiment Derham gave the velocity of sound in air as 1142 feet per second, but he made no record of the temperature or hygrometic state of the air, as he considered they had no influence on the result.

In later experiments of this type the effect of wind has usually been eliminated by the method of *reciprocal observation*. A gun is fired at *each* of the two stations of observation and the times taken by the report in travelling over the distance between the two stations in each direction

are carefully measured. By taking the means of these two times as the correct time of travelling over the distance the effect of the wind is practically eliminated, provided its velocity is constant during the time of the observations and not unusually great. The temperature and humidity of the air were also, in most cases, determined as far as possible, and the proper corrections applied to the results of the experiments.

Another error arises from the fact that we do not perceive either the flash or the report exactly at the moment when the light and sound reach us, the processes of perception requiring appreciable times, which are unequal for sight and hearing. This can be avoided in several ways, of which the best is perhaps to dispense with the observer altogether, and by an electrical method cause a pointer to make a mark on a revolving cylinder at the moment when the sound of the gun reaches, in turn, each of two acoustic pendulums at different distances from the gun. For this purpose the membrane and pith-ball are gilt to make them conducting, and form part of an electric circuit; the breaking of this circuit on the arrival of the sound releases the armature of an electro-magnet, which marks the cylinder. It is better to take the time between two points at different distances rather than from the gun itself to one of them, since near the gun, where the sound is very loud, the sound travels a little faster than usual.

By this and similar methods the time taken by the sound in travelling from one station to the other can be very exactly ascertained, but as it is impossible to determine at all accurately the average temperature of the air through which the sound passed, or to correct completely for the effect of wind, such experiments do not enable us to say what the velocity would be in still air at any given temperature.

The general result of the more reliable experiments gives the velocity of sound in dry air at 0° C. to be about 332 metres per second or 1090 feet per second. This agrees well with the result required by the formula $V = \sqrt{E/D}$.

A series of very careful determinations were made by Mr. Stone, of Cape Town Observatory, in 1871. The one

o'clock gun fired at Port Elizabeth was taken as the source of sound. Two observers were stationed on the line joining the gun to the Observatory and the time taken by the sound in travelling over the distance between the observers was carefully determined.

From these experiments Stone gives the velocity of sound in dry air at 0° C. as 1090·6 feet per second.

Echoes may also be employed to find the velocity of sound (see Art. 122). This was the method used by Newton.

The velocity of sound is not affected by the pitch of the note, for if it were and if the high note travelled faster, and if, further, a high note and a low one succeeded each other, the interval between them would be increased when they were heard at a distance. No such effect is noticed when music from a distant band is heard.

Theoretically if the displacement of the vibrating particles be so great that the law, that the force is proportional to the displacement, no longer holds good, the disturbance would travel more rapidly than when the displacement was less, hence very loud sounds should travel faster than fainter ones. It is probable that this effect is produced in the cases of sounds produced by dynamite cartridges, at all events near the source of disturbance.

47. Determinations of special interest.—Some of the experimental determinations of the velocity of sound in air were made under special conditions which give them special interest.

In 1845 two observers, Bravais and Martin, experimented between two stations on the Faulhorn *at levels differing in height* by about 2000 metres. The result of their experiments gave the velocity of sound in air at 0° C. as 332·4 metres per second. This result confirms the theoretical deduction that the velocity of sound in a gas obeying Boyle's law is independent of its pressure.

In 1889 Greely made a number of determinations of the velocity of sound in air in the Arctic regions at temperatures between — 10° C. and — 45° C.

As the result of his determinations he found that the velocity of sound in dry air at any temperature $t°$ C. is given in metres per second by the relation

$$V = 333 + \cdot 6t.$$

This result is in good agreement with the theoretical result obtained in Art. 43, and confirms the theory on which the temperature correction is made, by direct experiment.

In 1864 Regnault made a very careful determination of the velocity of sound in the open air, and the results he obtained as the means of a very large number of experiments over two distances of 1280 metres and 2445 metres gave the velocity of sound in dry air at 0° C. as 331·37 metres per second over the shorter distance and 330·7 metres per second over the longer distance. These results seem to show that the velocity of sound in air decreases as the intensity decreases, tending probably to a lower limit as the intensity diminishes. There is some experimental evidence in support of this conclusion, but no entirely satisfactory confirmation has yet been obtained.

Regnault also made a large number of determinations of the velocity of sound in pipes of diameters varying from a decimetre to a metre. He found as the general results of his experiments that in pipes or tubes the velocity of sound in air certainly decreases as the intensity diminishes, and also that the velocity increases as the diameter of the tube increases. For example, in a tube ·108 metre in diameter the velocity was found to be 324·25 metres per second, while in a tube 1·1 metre in diameter it was 330·3 metres per second.

48. Calculation of the Velocity of Sound in Water.—The velocity of sound in any liquid medium for which E and D are accurately known may be calculated from the formula $V = \sqrt{E/D}$, and the result obtained usually agrees satisfactorily with the result obtained by experiment. Thus in the case of water at 15° C. the adiabatic value of E is $2{\cdot}23 \times 10^{10}$ dynes per square cm., and the value of D

is approximately 1 gramme per c.c., so that the value of V from the formula is given by

$$V = \frac{\sqrt{2{\cdot}23 \times 10^{10}}}{1} \text{ cms. per second}$$

$$= 150{,}000 \text{ cms. per second.}$$

The experimental determinations of the velocity of sound in water give values between 1400 and 1500 in metres per second.

49. Experimental determination of the Velocity of Sound in Water.—In 1826 Colladon and Sturm determined the velocity of sound in water by direct experiment in the Lake of Geneva. A bell placed at a depth of about a metre in water was used as the source of sound. The arrangement for striking the bell (Fig. 41) was such that at the instant the stroke was made a charge of powder, p, was fired by a lighted fuse, m, so that the observer at the receiving station could mark the instant of striking by means of the flash of the discharge. The sound of the bell stroke travelled through the water to the observer at the distant receiving station, where it was heard by means of a long ear trumpet fixed with its receiving end in the water, this end being covered with a tight membrane to take up the sound vibrations.

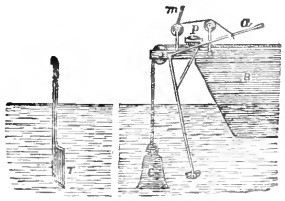


Fig. 41.

It was found during most of the experiments that the bell could be heard right across the lake, a distance of nearly ten miles. By these means it was possible to determine with considerable accuracy the time taken by sound to travel over a measured distance through water, and from data so obtained the velocity of sound in water was calculated. Colladon and Sturm gave 1435 metres per second, or about 4700 feet per second, as the mean result of their experiments.

50. Indirect Methods of Determination.—Indirect experimental methods of determining the velocity of sound in air and other media by measurements connected with the vibration of solid rods or fluid columns are dealt with later.

It may, however, be here noted that these methods are, in general, direct applications of the relation $V = n\lambda$ explained in Art. 19.

It is obvious that if the wave length λ for a note of known frequency n can be determined experimentally for any medium, then the velocity of sound *in that medium* is given by $V = n\lambda$. For example, if a note of frequency 300 has a wave length of 34 feet in wood as determined by the period of vibration of a wooden rod of known length, then the velocity of sound along the rod is 300×34 feet per second or 10,200 feet per second.

51. Relation of Velocity to Molecular Structure.—So far we have considered the air as a continuous elastic substance, of which every portion exerts a pressure on all the surrounding portions. This is not an incorrect way of regarding the matter, but it is rather a superficial one, for the transmission of sound through a substance is closely connected with its molecular structure. It is well known that air, for instance, is not a continuous substance, but consists of very minute detached bodies, called molecules, of which there are, very roughly, a million billions in a cubic centimetre. These fly about in every direction with an average velocity which depends on the temperature (more accurately, perhaps, the temperature depends on the average velocity), but which at ordinary temperatures is about 50,000 cms. (1,700 feet) per second.

Though the space actually filled by the molecules is less, and probably much less, than $\frac{1}{1200}$ of the whole space, the number of molecules is so great that the average distance traversed by a molecule before coming into collision with another is only $\frac{1}{2000000}$ of a centimetre, and though there is of course no regularity in these collisions, it must be a very rare thing for a molecule to move $\frac{1}{20000}$ of a centimetre ($\frac{1}{100}$ of a hair's-breadth) without an encounter which

quite changes its direction. Thus, though the molecules move as fast as cannon-balls, they do not make rapid continuous progress, and most of those which are in a particular cubic centimetre of the air at one moment would still be found in a compact group a second or two later, though the position of this group, relatively to the earth, may have changed considerably.

A "particle" of air is thus very comparable to a swarm of bees, which may be at rest, or moving in a constant direction, though every bee is moving much faster, and continually changing its direction. Where we have spoken in the preceding chapter of the velocity of the air, it is this swarm-velocity that is meant, and it is equal to the average velocity of the molecules reckoned algebraically, velocities in one direction being counted as of opposite sign to those in the other.

To simplify the explanation of the way in which the transmission of sound is related to molecular movements, we will suppose that all the molecules have the same mass (which is not the case in a mixed gas like air), and have the same velocity, which we will take as 50,000 cms. per second. We will also suppose that they all move in horizontal lines running east and west, and that all the collisions which happen are direct, so that two molecules still move after collision in the same line as before, and that the molecules behave like perfectly elastic equal balls in direct collision, so that each moves after the collision in the same direction and with the same velocity as the other did before collision.

Suppose a solid plate in the air moving broadside-on from east to west, with a velocity of 10 cms. per sec. Then all the molecules on the west side of the plate which strike it do so with a velocity, *relative to the plate*, of 50,010 cms., and rebound with the same relative velocity; that is, they move westwards with a velocity of 50,020 cms. Each of these soon encounters a molecule coming eastwards at 50,000 cms. per second, and sends this molecule off westwards at 50,020 cms. per second, while it returns eastwards at 50,000 cms. per second, till it meets the plate again, and so on. Meanwhile the molecule to which it has passed on

its velocity passes it on to one still further west, and returns eastwards at 50,000 cms. per second.

Thus in the air near the plate, but on the westward side of it, all the eastward-moving molecules have a velocity of 50,000 cms. per second, while the westward-moving ones have a velocity of 50,020 cms. per second, and this condition of the air is spreading westwards with the velocity of *the molecules themselves*. The swarm-velocity of this air is the average of + 50,020 and − 50,000, or 10 cms. per second, the same as that of the plate, and it is in the same direction as the condition is spreading. It is a condensed condition, for there is all the air between the plate and the furthest point to which the condition has extended westwards that there would have been if the plate had been at rest, and the distance is smaller than it would have been in that case. Also it is a condition of raised temperature, for the relative velocity of the molecules moving eastwards to those moving westwards is 100,020 while in the rest of the air it is 100,000, and the temperature of a gas is a function of the relative velocity of its molecules among themselves. Thus all that we have previously learnt about a travelling condensation is seen to be a natural result of the molecular constitution of the air. It will be obvious how the description must be altered to make it apply to a rarefaction.

Roughly speaking, then, the velocity of sound in a gas is the velocity of the molecules, but in an actual gas the condensed or rarefied condition spreads outwards only about two-thirds as fast as the average velocity of the molecules. This is chiefly because most of the collisions which happen are oblique, so that each molecule rebounds in a different direction to that in which the other one was going before the collision, and thus the extra velocity communicated to one molecule by the moving plate is handed on in a zig-zag manner. There are also other reasons, which cannot be explained satisfactorily in an elementary book.

The reason why the velocity of sound is independent of pressure may now be more clearly explained. The temperature of a gas is an expression for (though not simply

proportional to) the average kinetic energy of each of the molecules, and this depends on their velocity and mass, not on their number. So that for any one gas the velocity of its molecules, and therefore the velocity of sound in it, is always the same at the same temperature, however other conditions may vary.

It will now be seen that the two statements that "the air in a condensed region is moving in the same direction as the waves," and that "the air in a condensed region is at a higher temperature than the undisturbed air," are two ways, each incomplete, of stating the fact that in a condensed region the molecules which are moving in the direction of the sound are moving with more than usual velocity, while those which are moving in the opposite direction have their ordinary velocity. It is only this particular kind of molecular movement which transfers itself from one part of the air to another, so that raised temperature is an essential part of the condition of travelling condensation, and not merely a cause increasing the velocity with which it travels. The same is of course true of the lowered temperature of a travelling rarefaction.

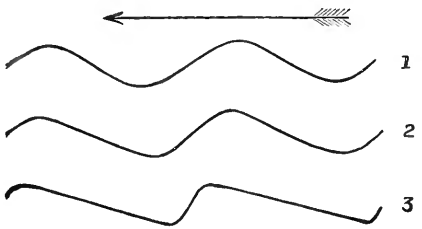


Fig. 42.

From the molecular point of view it seems as if the condensed condition ought to travel a little faster than the rarefied one, for each condition is handed on by the molecules which travel in the same direction as the condition, and these have more than the average velocity in a condensation, less in a rarefaction. The formula of Art. 43 suggests the same thing, since the condensations are at a

higher temperature than the rarefactions. The matter is not nearly so simple as it appears, and the real explanation cannot be given here, but it is a fact that condensation does travel faster than rarefaction. So that, in a wave-system, the wave-form gradually changes as the waves get further from the source, just as sea-waves change in form when, reaching shallow water, the crests travel perceptibly faster than the troughs. Some stages in the transformation of waves whose wave-form was originally harmonic are shown in Fig. 42; the waves are supposed to be travelling from right to left. The greater the degree of condensation and rarefaction the greater the difference in the velocities, and the more rapid the change in wave-form.

If the sound is very loud, the most intense part of the first condensation may get right to the front, and leave the less condensed and the rarefied parts far behind. As it is the velocity of the first disturbance to arrive which is measured by most methods in use, loud sounds are found to travel quicker than faint ones, though there is no reason to think that the mean velocity of a condensation and a rarefaction is greater in loud sounds.

VELOCITY OF SOUND.

	$\frac{\text{ft.}}{\text{sec.}}$		$\frac{\text{m.}}{\text{sec.}}$
Air at 0° C.	1,090	...	332
Coal Gas at 0° C. ...	1,680	...	515
Carbon Dioxide at 0° C.	850	...	259
Hydrogen at 0° C. ...	4,190	...	1,280
Oak from	12,620	...	3,850
,, to	14,430	...	4,400
Deal	16,000	...	5,000
Glass (flint)	13,000	...	4,000
,, (crown)	16,000	...	5,000
Steel	16,000	...	5,000
Brass	12,000	...	3,700
Silver	8,600	...	2,640
Water at 8° C. ...	4,700	...	1,435
Brine (sat.) at 18° C. ...	5,130	...	1,560
Alcohol, absolute, at 10° C.	4,130	...	1,260

CALCULATIONS.

The most important formulae for the velocity of sound are—

(1) For a solid rod

$$V = \sqrt{\frac{Y}{D}}$$

where Y is Young's Modulus and D the density of the material of which the rod is composed. (Art. 35.)

(2) For a liquid or gas

$$V = \sqrt{\frac{E}{D}} \qquad \text{(Arts. 36, 37.)}$$

where E is the adiabatic value of the volume elasticity and D is the density of the gas.

(3) For a gas (2) reduces

$$\text{to } \sqrt{\frac{\gamma P}{D}},$$

where γ is the ratio of the specific heats, P is the pressure and D the density. (Art. 40.)

(4) For air

$$V = \sqrt{\frac{1{\cdot}404\,(273 + t)}{273d}} \qquad \text{(Art. 42.)}$$

where t is the centigrade temperature, and d the quotient obtained by dividing the density of the gas at 0° C. and any pressure by that pressure expressed in dynes per sq. cm.

(5) For the same gas at different temperatures, t_1, t_2,

$$\frac{V_1}{V_2} = \sqrt{\frac{273 + t_1}{273 + t_2}}. \qquad \text{(Art. 43.)}$$

(6) For different gases, both having the same number of atoms to the molecule, at the same temperature,

$$\frac{V_x}{V_y} = \sqrt{\frac{\text{atomic weight of } y}{\text{atomic weight of } x}}.$$

(Art. 41.)

EXAMPLES III.

1. A certain mass of air occupies 101 c.cms. at a pressure of 1,000,000 dynes per sq. cm. The pressure is increased slowly to 1,010,000 dynes per sq. cm. and it is found that the volume becomes 100 c.cms. Find the isothermal elasticity.

Under what conditions is the modulus of volume elasticity of air measured (*a*) by its pressure, (*b*) by 1·41 times its pressure?

2. Newton showed that the velocity of sound in a gas $= \sqrt{\frac{P}{D}}$. What was his error? Explain Laplace's correction.

3. Show that sound travels faster in moist air than in dry air under the same conditions of temperature and pressure.

4. Describe any careful experiments made to find the velocity of sound in air and in water.

5. What effect does a rise in temperature produce in the velocity of sound? When the temperature is at the freezing point a sound passes from A to B in 10 seconds. Find the temperature if the sound could pass in 9·652 seconds.

6. A tube 1000 ft. long is filled with oxygen. Find how quickly a sound will travel from one end to the other, it being given that the density of oxygen is 16 times as great as that of hydrogen, and that the velocity of sound in hydrogen is 4200 ft. per second.

7. State how the velocity of sound depends upon the pressure, density, and temperature of the air. Find the velocity of sound in ft. per sec. at 27° C.

8. Calculate approximately the speed of sound in ft. per sec. in air at the following temperatures Centigrade: 20°, 15°, 30°.

9. Find approximately the temperature of the air when the speed of sound is 1000, 1120, 1200, and 980 ft. per sec.

10. Calculate the distance of a lightning discharge when between the flash and the first sound of the thunder the following intervals of time elapsed—$\frac{1}{2}$ sec., 4 secs., $2\frac{1}{2}$ secs.: the temperature in the first two cases being 10° C., and in the third 18° C.

11. Find at what temperature the velocity of sound in air is double the velocity of sound in air at the temperature of the freezing point of water.

12. Taking 1120 ft. per second as the speed of sound in air, find the number of vibrations which a C fork of frequency 264 must make before the sound is heard at a distance of 154 ft.

13. What is meant by a wave of sound, and by the length of a wave? Waves of sound, the frequency of which is 256, pass from a stratum of hot air to a layer of cold air. In the cold air the speed is 1120 ft. and in the hot air 1132 ft. Find the wave-length in each case.

14. A certain tuning-fork makes 256 vibrations per sec. Calculate the length of the wave it produces when the temperature of the air is 15°C.

15. If the same fork were sounded in hydrogen at the freezing point, how long a wave would it produce?

16. One wire of a piano is known to produce a wave 4 ft. long when the temperature of the air is 12° C. How many vibrations per sec. does the wire make?

17. A tuning-fork making 300 vibrations per sec. produces, when sounded in a certain gas, a wave 3 ft. long. Calculate the speed of sound in the gas.

18. The same fork sounded in air gives a wave 3·7 ft. long. Calculate the temperature of the air.

19. At what temperature is the velocity of sound in nitrogen (atomic weight, 14) equal to its velocity in oxygen at 10° C. (atomic weight, 16)?

20. At what temperature is the velocity of sound in air twice as great as in air at 20° C. ?

21. The velocity of sound in air is 34,000 centimetres per second when the thermometer is at 13° C. and the barometer at 78 centimetres. What will be the velocity if the thermometer rises to 22° and the barometer falls to 72 centimetres?

22. The velocity of sound along a tube full of air is found to be 1100 feet per second at 0°. What will be its velocity in the same tube (1) if the air in the tube is raised to 20° without letting any escape from the tube; (2) if the air in the tube is raised to 20°, and the pressure kept constant?

23. A cylindrical rod is 2 metres long when laid down, and weighs 2000 grammes. When it stands upright, with one end on the ground, it is $\frac{1}{10}$ millimetre shorter than when laid down. What is the velocity of sound in the substance of the rod?

24. Calculate the velocity of sound in hydrogen at 20° C. and 1,100,000 dynes pressure, given that the density of hydrogen at 0° and 1,000,000 dynes pressure is ·0000884, and that the ratio of the specific heats of hydrogen is 1·408.

25. Given that the velocity of sound in air at 0° is 33,240 cm. per second, and that the density of air at 0° and a pressure of 10^6 dynes is ·001275, find the adiabatic elasticity of air at a pressure of 10^6 dynes.

26. The velocity of sound in air at 0° C. is 33,200 cms. per sec. Find the velocity of sound in carbon dioxide at 100° C. (Given that at the same temperature and pressure the density of carbon dioxide is $\frac{22}{14{\cdot}4}$ times the density of air.)

27. Find the temperature at which sound travels in hydrogen with the same velocity as in oxygen at 1000° C.

CHAPTER IV.

SUPERPOSITION.*

52. Principle of Superposition.—Suppose that we have two sources of sound waves, two tuning-forks, for instance; we will call them A and B. Consider a region of air C, so situated that waves of quite small amplitude pass through it if either A or B vibrates alone. Let A and B be vibrating together, and suppose that we know what the condensation and velocity of the air would be at each point of the region C, at a given moment T, if A was vibrating as it actually is but B was not vibrating, and that we also know what the condensation and velocity at each point would be, at the same moment T, if B was vibrating as it actually is but A was not vibrating. Then it can be shown from the principles of dynamics that the actual condensation and velocity of the air at each point of the region C, at the given moment T, are very nearly the same as would be found by adding the condensation and velocity which there would be at that point if only A was vibrating, and the condensation and velocity which there would be if only B was vibrating.

In this addition, rarefaction and condensation must be counted of opposite signs and added algebraically, and velocities must be added in the only way in which quantities

*The name *interference* is applied by some writers to all cases of superposition, and limited by others either (*a*) to those cases in which the actual distribution of energy is very different from that of the imaginary systems, or (*b*) to those cases in which the vibration due to the sources together is less intense than that which would be due to one of them alone. We adopt (*a*) as being usual in England, though (*b*) has the high authority of Lord Rayleigh.

which have direction as well as magnitude can be added, by finding the diagonal of the parallelogram whose sides represent in magnitude and direction the velocities whose sum is required.

Similarly, the displacement of any particle of the air from its mean position is the same as would be found by adding the displacement which it would have if A was vibrating alone and the displacement which it would have if B was vibrating alone, displacements being added like velocities.

The principle that, when the condensation, velocity, and displacement of the air are small, they are equal to the sums of the condensations, velocities, and displacements which there would be if each of the sources was vibrating without the others, is called the principle of **superposition.** It is exactly true only when the amplitudes of the vibrating sources are indefinitely small, a case of no practical importance, but it is nearly true in all ordinary cases. It may be noticed here that the principle applies only to quantities which can have either a plus or a minus value (velocity, condensations, displacements); it does not apply to energy or intensity.

In the ordinary physical sense of the terms, neither the condensations and rarefactions which A would produce by itself, nor those which B would produce by itself, exist in the region C when A and B vibrate together. For instance, at a point where there would be a certain degree of condensation if A was vibrating alone, and an equal degree of rarefaction if B was vibrating alone, the principle of superposition shows that the air is at its average density, and is neither condensed nor rarefied. But the easiest way of finding the actual condition of the air in the region C is to *imagine* the condensations and rarefactions which there *would be* there if A was vibrating alone, and then those which there would be if B was vibrating alone, and then add these imaginary conditions to find the real condition of the air.

It is usual, and convenient, to speak of these imaginary wave-systems as "existing" at the same time in the region C, and to call the wave-system actually existing their

resultant. This should not cause any confusion, because there cannot, in any physical sense, be two wave-systems at once in the same air, any more than there can be two different winds at once at the same place; the same air cannot have two densities at once. But it is important to remember that, whenever we speak of two wave-systems in the same air, these wave-systems are imaginary, and that what really exists in the air is one wave-system different from either of the imaginary ones.

When the imaginary wave-systems in a given region travel in directions which cross each other, the real movements of the air are very complicated, each particle moving in curves something like Lissajous' figures. It is more important to consider the real condition of the air when the imaginary wave-systems travel along the same line, either in the same or in opposite directions.

53. Wave-systems in the same Direction.—Suppose, as before, that two bodies, A and B, are vibrating together. Let A and B be so situated that, if either of them vibrated

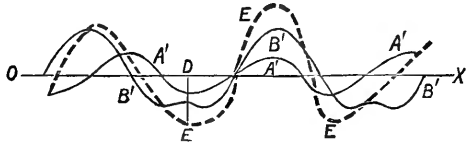


Fig. 43.

alone, waves would travel in the same direction through a given region, say the direction XO (Fig. 43). Let $A'A'A'$ be the wave-form of the waves which would be travelling through the given region if A was vibrating alone; $B'B'B'$ that of the waves which would be travelling through the same region if B was vibrating alone. Then at any point, such as D, the actual condensation is proportional to the algebraic sum of the ordinates from D to $A'A'A'$ and $B'B'B'$, so that, if we draw a curve EEE whose ordinate at each point is the algebraic sum of the ordinates to $A'A'A'$ and $B'B'B'$, the ordinates of EEE will be proportional to the actual condensations.

Since the two wave-systems which there would be if A and B vibrated separately would both travel in the direction XO, ordinates to either curve above the axis OX indicate velocities which the air would have in the direction XO, and ordinates below the axis indicate velocities which the air would have in the direction OX. The actual velocity of the air at any point is therefore proportional to the algebraic sum of the ordinates to $A'A'A'$ and $B'B'B'$ at that point, *i.e.* it is proportional to the ordinate to EEE, and it is a velocity in the direction XO where the ordinate to EEE is above the axis.

The actual condition of the air, then, is a condition in which both the condensation and the velocity are proportional to the ordinates to EEE, and are therefore proportional to each other. Now this, as we have seen, is the distinctive feature of progressive undulation. The actual condition of the air, then, is a progressive wave-system, travelling in the direction of XO, and having a wave-form EEE. A progressive system having this wave-form might be produced by suitable movements of a single body, such as the piston in Chapter II., and there would be no physical difference whatever between such a wave-system and the one which A' and B' produce when they vibrate together.

Whenever there are two or more vibrating bodies, each of which, vibrating by itself, would produce in a given region a progressive undulation *in the same direction*, there is in the given region a progressive undulation whose wave-form is the sum of the wave-forms of the undulations which there would be if the bodies vibrated separately, and such a progressive undulation bears no physical trace of having originated from several sources, but is exactly like an undulation which might be produced by a single vibrating body.

If, on the other hand, the undulations which the bodies would produce separately would travel in different directions through any region, the actual condition of the air in that region is one in which condensation and velocity are not proportional to each other, and is therefore a condition which could not be produced by any possible vibrations of a single source.

54. Harmonic Waves of Equal Length.—If the imaginary wave-systems (those which would actually exist if the different sources vibrated separately) are all harmonic, and all in the same direction, it does not follow that the actual wave-system is harmonic, for the sum of two harmonic curves is not always a harmonic curve (Art. 14). But it can be shown that the curve obtained by adding the ordinates of harmonic curves *whose bends are of the same length* is always a harmonic curve with bends of that length, whatever the relative positions of the bends.* Since harmonic curves whose bends are of the same length are the wave-forms of harmonic wave-systems of equal wave-length, it follows that, where there are several imaginary harmonic wave-systems of *equal wave-length*, travelling in the same direction, the actual undulation is a harmonic one, and of that wave-length. There are several important special cases of this.

(1) If the imaginary wave-systems are in the same phase (that is, if the wave-system which would exist at a given moment if one source was vibrating alone has its maximum condensations in the same positions as the maximum condensations of the wave-system which would exist at the same moment if another source vibrated alone), the maximum ordinates of the actual wave-form are the sums of the maximum ordinates of the imaginary wave-systems, and the actual maximum velocity is therefore the sum of the imaginary maximum velocities. The amplitudes of harmonic vibrations of equal frequency are proportional to their maximum velocities (Art. 8), so that the actual amplitude of vibration is the sum of the amplitudes of the vibrations which the air would execute if the sources vibrated separately.

* This can be shown easily by a graphical method. The two sets of curves may be compounded as in Art. 14. Analytically, the two sets of curves may be represented by $y_1 = a_1 \sin p(x-a)$, $y_2 = a_2 \sin p(x-\beta)$. The resultant curve is given by $y = a_1 \sin p(x-a) + a_2 \sin p(x-\beta)$ which reduces to $y = a \sin p(x-\gamma)$, where

$$a^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos(a-\beta),$$

and

$$\tan p\gamma = \frac{a_1 \sin pa + a_2 \sin p\beta}{a_1 \cos pa + a_2 \cos p\beta}.$$

This case is shown in Fig. 44, where the continuous curves are the wave-forms of two imaginary wave-systems in the same phase, and the dotted curve, obtained by adding the ordinates of the others, is the wave-form of the actual condition of the air.

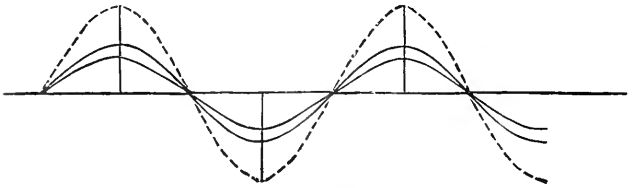


Fig. 44.

(2) If the imaginary wave-systems are in opposite phases (that is, if the maximum condensations of one imaginary system coincide with the maximum rarefactions of the other), the actual amplitude of the vibrating air is the difference of the amplitudes which it would have if the sources vibrated separately. (Fig. 45.)

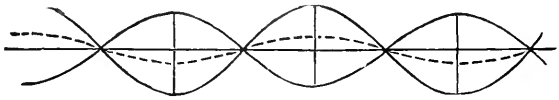


Fig. 45.

(3) In case 2, if the imaginary systems are equal, the real wave-form is a straight line; the air is of uniform density, and there is no real wave-system. The condition of the air is exactly as if the sources were not vibrating.

(4) If the condensations of one imaginary system do not coincide in position either with the condensations or with the rarefactions of the other imaginary system, the amplitude of the real vibration is less than the sum and greater than the difference of the amplitudes of the imaginary vibrations.

In all these cases the intensity is of course proportional to the square of the amplitude.

55. Illustrative Experiments.—In order to verify by experiment the principle of superposition in these different cases, we shall require some instrument for roughly comparing the intensities of sounds; the most convenient is the ear. All that we require to know about its indications at present is that vibrations of the air of the same frequency but different intensity cause sensations of different loudness, the more intense vibrations producing the louder sensations.

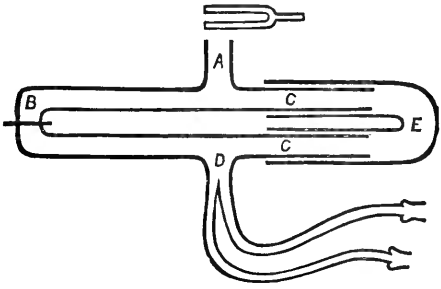


Fig. 46.

All the cases may be illustrated by means of an instrument, shown in section in Fig. 46, which is slightly altered from a form devised by Prof. Quincke. A tube A divides into two branches B and C, which re-unite at D. The length of C can be altered by drawing out the sliding portion E (on the trombone principle). D ends in a flexible pipe, or, better, in two flexible pipes, one of which is placed in each ear. In the branch B is a sliding door by which the branch can be closed.

If the branch CE is adjusted till it is the same length as B, and a vibrating fork held opposite the opening at A, it is evident that the condensations which there would be in D if waves travelled by one branch would be in the same places as the condensations which there would be, at the same instant, if waves travelled only by the other branch. The actual vibration in D is therefore greater than if waves travelled only by one branch, as is shown by the diminution of loudness when the sliding door in B is closed.

The same occurs if the difference between the lengths of *ABD* and *ACECD* is made any exact number of wave-lengths of the waves from the fork.

If *ACECD* is made half a wave-length longer than *B*, the condensations of the waves which there would be in *D* if waves travelled only by *CE* would be in the same places as the rarefactions of the waves which there would be if waves travelled only by *B*, and the amplitudes of these two systems are nearly equal. There is, therefore, little vibration of the air in *D* when the sliding door in *B* is open, and hardly any sound is heard. When the sliding door is closed, the sound is heard clearly.

The same occurs if the difference between the lengths of *B* and *CE* is made any odd number of half wave-lengths.

This apparatus may evidently be used to determine the length of the waves from a fork, but it is not a good method, as there is no point of absolute silence, and the points of faintest sound are not easy to determine exactly. For absolute silence it would be necessary, not only that the maximum condensations of one imaginary system should exactly coincide with the maximum rarefactions of the other, but that the condensation of one system should be equal *at every point* to the rarefaction of the other. If the wave-forms of the two systems are alike (as in the experiment just described), this requires that the wave-form should be exactly symmetrical, the part below the axis being exactly like the part above, inverted. Harmonic waves would, of course, fulfil this condition, but neither perfectly harmonic waves nor waves of any other perfectly symmetical form are easy to produce.

Exp. 13.—Fit a whistle into the middle of the bottom of a box lined with felt (to prevent reflection from and transmission through the sides of the box), and make two holes in the top of the box at equal distances from the whistle. When the whistle is sounded, similar pulses start simultaneously from these openings. Stretch a thin membrane (gold-beater's skin) on a ring and hold it horizontally at points in the air above the box. At points which are equidistant from the openings the membrane is strongly affected and sand scattered on it is instantly thrown off. It is also easy to find points nearer to one hole than to the other where the membrane is very little affected; these are, of course, points whose distances from the two holes differ by an odd number of half wave-lengths.

In these instances the two imaginary wave-systems are produced in the same phase, and the difference in their phases, at the points where the air is undisturbed, is due to the different distances they have travelled. When the imaginary wave-systems in a region are in opposite phases, it is usually either for this reason, or because they were originally produced in opposite phases by parts of the same vibrating body which move in opposite directions at the same time. This is a very common cause, for nearly every vibrating body has parts which move simultaneously in opposite directions, like the prongs of a tuning-fork or opposite sides of a bell; and, even in the simple case of a spring held in a vice, one side of the spring is producing a condensation while the other side is producing a rarefaction. A good illustration is furnished by the following experiment.

Exp. 14.—Fix a circular plate of glass or metal *A* (Fig. 47) in a horizontal position to a stand by a screw through the middle, but free everywhere else. Draw a violin bow across the edge of the plate. The plate vibrates in an even number of sectors—four, if it is not touched anywhere except by the bow—of which at a given moment half are moving upward and the alternate ones downward. Therefore, at the same moment condensations start from one set and rarefactions from the other. At a little distance above the plate, where the effects due to each of these wave-systems are about equal, there is practically no change of density at all.

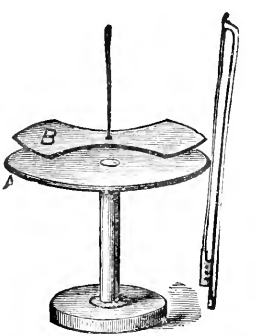


Fig. 47.

Cut a circle of cardboard of the same size as the plate, divide it into the same number of sectors that the plate is vibrating in, cut away alternate sectors nearly to the middle, and hold the remaining part of the card *B* so as to cover, but not to touch, alternate sectors of the plate. Note that the sound at a few feet above the plate is much louder than before, since the waves from one set of sectors are absorbed by the card and those which pass through the openings are all in agreement.

To ascertain the number and position of the sectors of the plate scatter a little sand on it; the sand quickly collects on the lines which divide the oppositely vibrating sectors.

Hopkin's Forked Tube (Fig. 48) may also be used in conjunction with a vibrating plate to illustrate the principle of interference. The ends of the two branches are open, but a piece of gold-beater's skin is stretched over the end of the main stem. The tube is held in an inverted position (as in the figure) so that the membrane is at the top, and a little sand is scattered on the membrane. If a plate like that shown in Fig. 47 is made to vibrate, and the tube held so that the two openings are over sectors which vibrate in the same direction, the sand is thrown off; but, if the openings are symmetrically over sectors which move in opposite directions, the sand is only slightly disturbed.

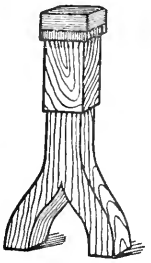


Fig. 48.

The prongs of a tuning-fork (Fig. 49) move outwards together; therefore they condense the air outside them while they rarefy the air between them, and condensation starts off in all directions from the outer surfaces A, A' at the same time that rarefaction is starting from the inner surfaces B, B'. The air in the direction of the line joining the prongs is, to a large extent, shielded from the waves from B, B by the prongs themselves, while along a line at right angles to this the effect of the waves from the outer surfaces A, A' is small, but in four directions which make angles of about 45° with the line joining the prongs the waves from the outer surface A of one prong are almost exactly balanced by the waves from the inner surface B' of the other prong, and in these four directions very little sound is heard.

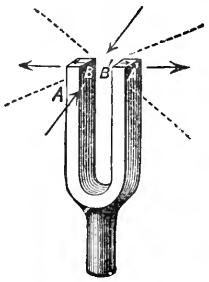


Fig. 49.

Exp. 15.—Sound a tuning-fork and twist the stem of it round between your fingers so as to make the fork present each side to your ear in turn; you will hear the sound swell out and die away four times in each rotation of the fork.

When your ear is in one of the four directions from the fork in which the sound is faintest, get someone to hold a paper tube so as to enclose, but not to touch, one of the prongs. The sound becomes much louder, showing that the waves produced by the two prongs vibrating together are of less intensity than either would produce by itself.

For reasons given above, it is very difficult to arrange an experiment in which the imaginary wave-systems are so exactly opposite that there is complete silence. A very near approach to success can be made, however, by mounting two equal organ pipes, open at the upper ends (as well as at the lower, or mouthpiece, ends), close side by side on the same wind-chest. When these are blown, they always start in opposite phases, so that a condensation leaves the mouth of one at the same moment that a rarefaction leaves the mouth of the other, and, as the waves from open pipes are almost perfectly harmonic, the condensation of one imaginary system is almost exactly equal to the rarefaction of the other at every point in the surrounding space. If there is any vibration of the air at all due to the two pipes, it cannot be heard among the rushing sound from the air at the mouthpieces. Yet, if we stretch a thin membrane on a ring, and lower this horizontally, with a little sand on it, into either of the pipes, the sand is at once thrown off, showing that the air-columns are vibrating.

56. Energy of Waves from two Sources.—When the condensations of two equal imaginary wave-systems agree in any region, the actual amplitude of vibration should, on the principle of superposition, be twice as great as either of the sources would produce separately, and, therefore, the actual intensity four times as great as the intensity of the sound which either of the sources would produce separately. There is, therefore, four times as much energy passing per second through the region as there would be if only one source vibrated. This looks at first like a creation of energy, but of course that never occurs. If the sources of the two imaginary systems are several wave-lengths apart, there are some regions in the surrounding space where the imaginary wave-systems are in the same phase,

and some regions where they are in opposite phases; in the former the intensity is greater, and in the latter less, than the sum of the intensities of the imaginary systems.

The total energy travelling away from the sources is the sum of the amounts which would be travelling away if the sources vibrated separately, though it is differently distributed; in some directions more energy travels away, and in other directions less, than the sum of the amounts which would travel in those directions if the sources vibrated separately.

If the sources are very close together, and vibrate in the same phase, the imaginary wave-systems are in the same phase in every part of the space round the sources; in this case either the actual amplitude must be less than the sum of the imaginary amplitudes (so that the principle of superposition does not hold), or the sources must give out energy faster than they would do if they vibrated separately. Usually it is the latter that occurs; each vibrating body, moving in air which is already moving in the same direction, gives up energy to it faster than it would to undisturbed air, just as a man pushing at a truck gives energy to it much faster when it has acquired some speed than he does when it has only just started and is moving very slowly.

57. Waves of Unequal Length.—So far we have considered only cases in which the imaginary wave-systems are of the same wave-length. Fig. 50 illustrates the actual condition of the air when there are in it two imaginary wave-systems, both harmonic and travelling in the same direction, but of different wave-lengths. As in previous figures, the complete lines are the wave-forms of the imaginary wave-systems, and the dotted curve, obtained by adding the ordinates of the others, is the wave-form of the actual wave-system.* Its bends are not harmonic in form, and their maximum ordinates are greater in some places

* For greater clearness, the imaginary and real wave-forms are here drawn on separate axis-lines, instead of as in the previous diagrams, but the three curves denote conditions of the air in the same region.

than in others, so that in some regions the condensations and rarefactions are intense, in others only slight. As the conditions indicated by this wave-form pass any point the air there moves backwards and forwards through a distance which alters at each movement, being greatest when the most intense regions of the wave-system are passing.

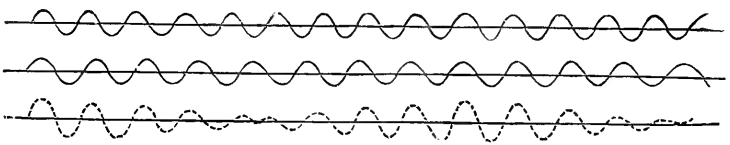


Fig. 50.

In this case the actual wave-system contains as much energy as the two imaginary systems would contain, but differently distributed; it is nearly all in the regions where the condensations and rarefactions are intense.

The sound heard by an ear at which such a wave-system arrives presents peculiarities which will be discussed later, but the most obvious is the alternation of increased and diminished loudness as more intense and less intense regions of the travelling system arrive at the ear.

58. Beats.—These changes in loudness are called *beats*. They are readily heard when two tuning-forks, or two organ-pipes, whose frequencies differ by three or four, are sounded at the same time.

If V is the velocity of sound, and n_1, n_2 the frequencies of two sources of sound, then there are n_1 complete waves of one imaginary system, and n_2 complete waves of the other, in a distance V, and, therefore, $n_1 - n_2$ places in that distance where the phases of the imaginary systems agree; these are the regions of maximum intensity. The distance between two regions of maximum intensity is, therefore, $\frac{V}{n_1 - n_2}$, and the number of such maxima which pass any given point in a second is $n_1 - n_2$. The number of beats heard per second is, therefore, the difference of the frequencies of the sources. Thus, if the frequencies are 254

and 258 respectively, the sources must evidently be in the same phase *four* times in each second, and a single beat must therefore occupy a period of one-quarter of a second. That is, when the difference of the frequencies is four per second there are four beats per second.

When the number of beats per second is more than ten it is very difficult to separate them, but their presence may cause discord. When the number per second exceeds thirty they cease to be distinguishable, and may have no disagreeable effect (see Art. 106).

These beats or variations in the *intensity* of the vibration at a point must be carefully distinguished from the condensations or variations of density at the point. The beats in fact are variations in the amount of variation of density; the mean density is the same in the more intense parts of the wave-system as in the less intense parts.

We see that it follows from the principle of superposition that in cases where the wave-lengths of two imaginary systems are equal or nearly equal, the energy of the actual waves may be very differently distributed from the sum of the energies of the imaginary systems. Such cases are called cases of *interference*. This name is appropriate when we are considering chiefly the energy or the intensity of the waves, but absurdly inappropriate if applied, as it sometimes is, to velocity or condensation of the air. For here the fact which is especially to be noticed is that no interference occurs, but that each source of sound produces the same difference in the condition of the air that it would do if the other sources were absent. As Lord Rayleigh says: "if this is interference, it is difficult to see what non-interference would be."

Mathematical Treatment.—It was shown in Art. 29*a* that the equation between displacement and time of a particle vibrating in simple harmonic motion may be written

$$y = a \sin \omega t.$$

If n is the frequency of vibration, $\omega = 2\pi n$. Hence

$$y = a \sin 2\pi n t.$$

Let there be two wave trains of equal amplitudes a, and frequencies n_1 and n_2 respectively, and suppose that at a particular point

through which they pass they are in zero phase when $t = 0$, then the resultant displacement-time curve of that point is given by

$$y = y_1 + y_2 = a \sin 2\pi n_1 t + a \sin 2\pi n_2 t,$$
$$= 2a \cos \frac{2\pi (n_1 - n_2) t}{2} \sin 2\pi \frac{n_1 + n}{2} t.$$

This represents approximately a periodic vibration of amplitude $2a \cos \frac{2\pi (n_1 - n_2) t}{2}$ and frequency $\frac{n_1 + n_2}{2}$. There is thus a periodic change of amplitude, its maximum value being nearly $2a$ and its maximum value 0, the interval between successive maxima and successive zero values being $\frac{1}{n_1 - n_2}$ sec. Hence the frequency of the beats is $n_1 - n_2$.

59. Experimental Illustrations of Beats.

Exp. 16.—*Compare the frequencies of two tuning-forks of nearly the same pitch, and adjust them to unison.* Sound the forks simultaneously, listen for the beats, and watch the seconds-hand of a clock. Count the number of beats in a given time and deduce the number per second. This will be the difference in the frequencies of the forks.

Usually the fork of higher pitch may be distinguished from the other by ear. Confirm by loading one of the prongs (bind a turn of fine wire round it): this will lower the pitch slightly. Hence (i) if the higher fork has been loaded the beats will be fewer, (ii) if the lower they will be increased. The loading (iii) may have been sufficient to reduce the pitch of the higher fork so much below the lower that rapid beats are produced. If this is suspected place the load about half-way along the prong and test again. The effect of the load in reducing the frequency is greatest when at the end of the prong.

To adjust the forks to unison. Method I.: Bind several turns of wire round each prong of the higher fork, adjust the wires along the prongs until the beats get slower and slower and finally disappear.

Method II.: Take a third fork, P, of nearly the same pitch as the two (Q, R) under adjustment. Load P so that there are, say, four beats per second when it is sounded with the lower (Q) of the two forks (Q, R). Next load R until there are four beats per second when it is sounded with P.

If a ray of light from an electric arc be reflected in turn from mirrors attached to the prongs of two vertical equal tuning-forks and then thrown on a screen, the spot of light will be drawn out to a line which would be of

constant lengths if the forks were in exact concord and if their vibrations were maintained. If, however, one of the forks be a little out of tune with the other, the motion of the prongs will sometimes be such that both throw the image up and down together. It is also sometimes such that one throws it up and the other down. The line will thus vary in length as time goes on and the interval between two maximum elongations corresponds to one beat.

This optical method enables us to detect beats so slow that they cannot be heard and thus serves as a delicate test of the tuning of the forks.

Exp. 17.—*Obtain beats between a fork and a wire.* Use a sonometer (Art. 167). Tune the wire nearly in unison with the fork. A low note is preferable, as beats may not be heard with high notes. Strike the fork, press its shank on the sonometer board, and pluck the wire. Beats will be heard as the sounds die away.

Instead of a fork and one wire, two wires may be used. Thus if two adjacent low notes on the piano are sounded and the pedal pressed down to free the wires beats will be heard.

Beats are often observed when listening to a single vibrating bell or metal bowl or basin. This is due to an irregularity in the sounding body. The main note given out is accompanied by one of slightly lower frequency. Hence beats are produced.

EXAMPLES IV.

1. Explain the principle of superposition. Show that two equal tuning-forks sounding side by side give up energy faster than when sounding at a distance.

2. Show how it is possible for two equal sounds of the same frequency to produce (1) silence, (2) a sound four times as intense as either.

3. Explain how a single tuning-fork may be used to illustrate interference.

4. If two sources of sound of very nearly the same frequency are sounding together, show that the frequency of the beats is equal to the difference between the frequencies of the sources.

5. Two notes are sounding and 10 beats per second are heard. If the lower note has a frequency of 530 what is the frequency of the higher note?

6. Draw a possible wave-form of a major fifth formed by two harmonic tones (ratio of frequencies = 2 : 3).

CHAPTER V.

FREQUENCY.

60. Musical Sounds.—It will be recognised that the sounds emitted by bodies in regular vibration are all of the particular type known as *musical sounds*. For example, the sounds produced by the vibration of the prongs of a tuning-fork, the strings of a violin, the steel wires of a piano, and the columns of air in the pipes of an organ are all musical sounds. This implies that the incidence on the ear of longitudinal waves of well marked periodicity and definite constant wave-length gives rise to the more or less pleasant sensation associated with what is called a musical sound or a musical note.

The essential element of a musical sound is its **periodicity.** Such sounds are usually produced by vibrating bodies, but any source capable of originating a *periodic* disturbance in the air may produce a musical sound. Thus if a card or thin strip of wood be held so as to be struck by the teeth of a toothed wheel rotating sufficiently rapidly at a uniform speed a distinct musical note is produced. The *hum* of a circular saw at work is an example of a sound produced in this way. The motion of the card produced by the rapid succession of taps from the teeth of the wheel is of a periodic character, and the waves originated by the card will be of constant wave-length and similar in type, although this type in the matter of sequence and gradation of states of compression and rarefaction may be of a somewhat irregular character. Similarly, if a stream of air from a nozzle is interrupted periodically, say by means of a rotating disc pierced with a circle of equidistant holes, the sound produced is a musical note. The puffs of air following each other at regular intervals originate waves of constant wave-length and fixed type, and produce a musical sound of a very characteristic quality.

Exp. 18.—Arrange a toothed wheel with equidistant saw teeth so that it can be rotated rapidly at a uniform speed, as shown in Fig. 51. Fix a card, as shown in the figure, so that it is lightly struck by the teeth of the rotating wheel. If the wheel is rotated sufficiently rapidly it will be found that a distinct musical note is produced.

A toothed wheel for this purpose was first used by Savart and is therefore commonly called Savart's toothed wheel.

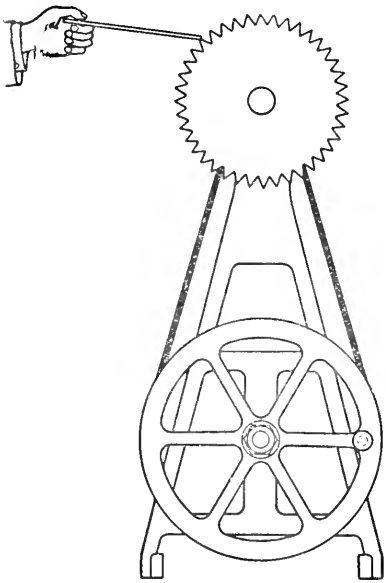


Fig. 51.

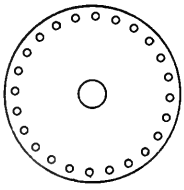


Fig. 52.

Exp. 19.—Take a circular plate of cardboard or metal, pierced with a circle of equidistant holes, as shown in Fig. 52, and mount it, like the Savart wheel in Fig. 51, so that it can be rotated rapidly round an axis passing through the centre of the circle of holes and at right angles to the plate. Direct a stream of air, forced through a nozzle against the ring of holes, as shown in Fig. 76. If the plates are rotated rapidly enough the rapid sequence of puffs of air through the holes in the plate sets up a periodic wave disturbance in the air and a musical sound is heard.

This simple apparatus for producing a musical sound is known as the *cardboard siren* or *Seebeck's siren.* It should be noticed that an important characteristic of this apparatus and of Savart's wheel is that the *frequency* of the originating disturbance can be calculated directly if the speed of revolution is known. Thus, if the number of teeth or holes is 50 and the speed of revolution is 10 per second, the frequency is evidently 500 per second.

61. Experimental Determination of Frequency.—The experimental determination of the frequency of vibration of a musical note may be made in three general ways:—

(1) By direct determination of the frequency of vibration of the source by suitable graphic or stroboscopic methods.

(2) By adjusting the frequency of a note produced by a source of known and adjustable frequency (such as the cardboard siren described above) in unison with the note whose frequency is to be determined.

(3) By comparing the frequency to be determined with that of a suitable standard source of constant known frequency, such as a standard tuning-fork.

The methods (2) and (3) are essentially the same and differ only in the character of the source employed as a standard. In both methods the adjustment or comparison of frequency is usually made by means of *beats.* When the two notes compared are nearly of the same frequency, beats can generally be distinctly heard, and the number of beats per second gives, as explained in Art. 58, the difference of the frequencies of the two sources, so that if the frequency of one is known the frequency of the other can be determined without further adjustment.

62. Electrical Maintenance of Vibration of a Fork.—It is sometimes necessary to maintain a fork in vibration for some time. Fig. 53 represents diagrammatically one way by which this may be done. The fork F is fixed on a firm support S which also supports an electromagnet C placed between the prongs of the fork and a metal collar supporting an adjustable screw B. A thin piece of platinum wire w, fastened to the lower prong of the fork, just touches the upper end of B when the prong is at rest. Electrical connections are made as in the figure.

Set the fork in vibration by a tap. When the lower prong moves upwards the contact at B is broken and the current ceases; when it moves downwards w touches B and the current flows through the circuit. This excites the electro-magnet which pulls the prongs together again. As the lower prong rises, contact is again broken, and thus the whole operation is repeated over and over again. During the inward motion of the prongs the electric battery does work on the fork and during the outward motion the fork does work on the battery. If these two quantities were

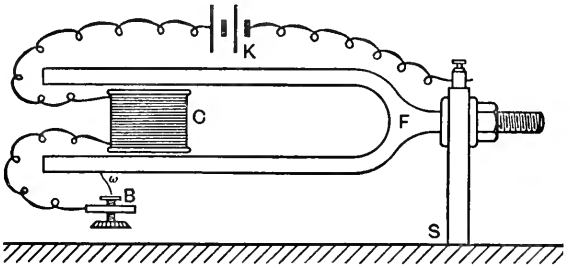


Fig. 53.

equal the vibrations would not be maintained, for the fork is constantly losing energy as sound and heat. Owing, however, to irregular contact and self-induction of the coil of the electro-magnet the fork gains more energy from the battery than it gives back to it, thus the vibrations are maintained, and if the current is small the period will be exactly the free period of the fork.

The phenomenon is one of resonance (Chap. VI.) in which the frequency of the driver (*i.e.* the electro-magnet) is regulated by the frequency of the driven (*i.e.* the tuning-fork).

If the intermittent current produced in such a way is sent around the coil of an electromagnet between the prongs of another fork of the same frequency, that fork will be set in vibration. The second fork will also be set in vibration if its frequency is a multiple of the frequency of the primary fork (see Art. 89).

63. Direct Determination of Frequency.—As already indicated, direct determinations of frequency are made either by a stroboscopic method, in which, by means of intermittent illumination or observation, the motion of the source can be accurately compared with that of a suitable standard, or by a graphic method, in which the vibrating source records its frequency by writing a trace of its motion on a suitably arranged moving surface on which definite time intervals are also recorded.

64. The Stroboscopic Method.—The stroboscope is a disc which can be made to rotate on its axis by means of a band passing round a small wheel on the axis and a larger wheel turned by hand. Radial slits are cut in the disc at equal intervals, and a counting mechanism records its rotations. If we fix a bright point to a vibrating part of the body and look at the point through the slits while we allow the disc to slacken gradually from a very high speed, we shall see the point as a line of light, except when the disc reaches certain speeds when the bright line changes for a few seconds into a number of separate points. When the frequency of the slits is only a little greater than that of the point, a single slowly moving point is seen, which becomes stationary when the frequencies are exactly equal.

The rotation is now maintained by a motor of some kind at such a speed that the point seems to remain stationary, and the counting mechanism set in action for a measured time; from the number of rotations recorded the frequency of the slits is easily found, and this is the frequency of the point. Care must be taken that the speed of the disc is the *highest* speed at which a single stationary point is seen, as the same appearance is presented if the frequency of the slits is an exact submultiple of that of the point. If the amplitude of the vibration is small, the bright point may be viewed through a microscope; the stroboscope disc revolving between the microscope and the point.

A variation of the above method is in common use to find the frequency of a tuning-fork. Two pieces of aluminium foil U and L (Fig. 54) are attached to the ends of the prongs A, B of a tuning-fork. A slit is cut out

of the lower piece and the higher piece is of such a length that when the prongs are at rest the slit is closed, and when the prongs are separated the slit is open. (The figure shows the slit open.) If the fork is set in vibration the number of times the slit will be opened and closed per second is equal to the frequency of the fork.

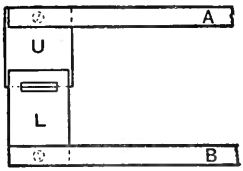


Fig. 54.

For the success of this method the vibration of the fork must be maintained. This is done either electrically (Art. 62) or by continual bowing; the former method is much more convenient. The stroboscopic disc used should be provided with one or more circles of equal-sized and evenly-spaced dots. The disc is rotated by a motor of adjustable and measurable speed, and is geared to a counting mechanism.

When the disc is set in rotation the dots give rise apparently to a uniformly dark circle. If now the fork is set in vibration and the time of one vibration is exactly equal to the time taken by the dots to move the distance between two spots the dots appear stationary when viewed through the slit. The frequency of the fork is then easily calculated. For if

n = the frequency of the fork,

m = „ „ of revolution of the disc, and

p = the number of dots in the circle on the disc,

then $$\frac{1}{n} = \frac{1}{p} \cdot \frac{1}{m}, \text{ } i.e. \text{ } n = mp.$$

The same appearance would be seen if the disc were rotating twice, or thrice, etc., as fast, for then each dot would move twice or thrice, etc., the distance between successive dots. Then $mp = \frac{1}{2}n, \frac{1}{3}n, \frac{1}{4}n$, etc. Care must be taken to distinguish these cases.

EXAMPLE.—A stroboscopic disc spaced with 15 spots was rotated and adjusted to agreement with a fork as described above. It made 3000 revolutions in 175·5 seconds. Find the frequency of the fork.

$$\frac{1}{n} = \frac{1}{15} \times \frac{175{\cdot}5}{3000}, \quad \therefore n = 256{\cdot}4.$$

65. Graphic Methods.—Graphic methods in which the vibrating body traces its movements are less accurate, since the necessary friction always affects the frequency of the vibration. Duhamel's **Vibroscope** (Fig. 55) is a good

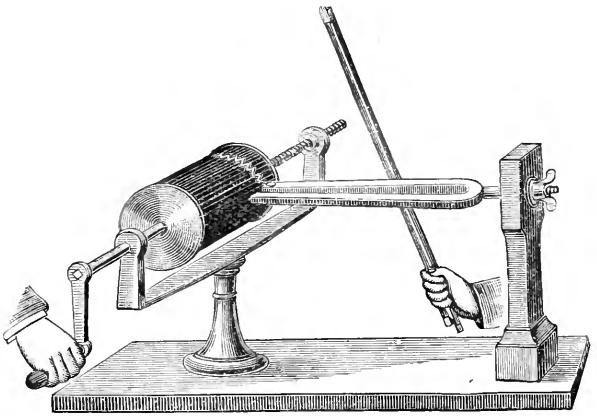


Fig. 55.

example of this method. The vibrating body is fixed so that the points whose movements are to be counted move near to the surface of the cylinder, and parallel to its axis. The cyclinder is covered with smoked paper, and a fine wire, attached with wax to the vibrating body, just touches the paper. As the handle is turned and the body vibrates, the wire traces a wavy line on the paper; this line forms a screw thread round the cylinder, which is made to advance in the direction of its length by a screw thread on the axle.

When this apparatus is used for measuring frequencies, the vibrating body is insulated and connected to one end of the secondary of an induction coil,* while the other end is connected to the revolving cylinder. The pendulum of a (seconds) clock is arranged to make contact, as it passes its lowest point, with a drop of mercury, and so to

* *Higher Text-Book of Magnetism and Electricity*, Art. 166.

close and break, once in each second, a circuit which includes pendulum, mercury drop, the primary of the induction coil, and a battery. The spark which passes between the wire and the smoked paper, each time this circuit is broken, knocks off a little spot of the soot at the point in contact with the wire at the moment; the number of double bends between two successive spots on the wavy line traced by the wire gives the frequency.

The cylinder may be rotated by hand or by clock-work; it is evidently not necessary that its motion should be uniform, for the time intervals determined by the clock pendulum are indicated on the trace by the spark marks. If, however, the motion is irregular the trace will also be irregular in character, being unduly drawn out at some points and unduly compressed at others. The speed of rotation evidently requires to be adapted to the frequency of vibration in order to give a satisfactory trace.

The apparatus here described is most generally used as a *chronograph* for the measurement of short intervals of time. It is evident that if the beginning and end of the interval to be measured are recorded on the trace by spark marks, *and if the period of vibration of the fork be known* then, with a fork of high frequency, very short intervals of time can be accurately measured.

Exp. 20.—*Find the frequency of a tuning-fork by the dropping plate method.* Use the apparatus shown in Fig. 56. The tuning-fork A is fixed in a nearly vertical position, and a bristle or pointed piece of thin sheet metal B is fastened with wax to one of its prongs. The prongs are pressed together till they are close enough to enter a square notch in the metal plate C, which holds them in this compressed position. A smoked plate of glass D is suspended from two pins E, F by a silk thread G hanging over them in two loops, one of which is cemented to the upper edge of the glass plate, while the other is caught under one of the points of the plate C.

When C is withdrawn, which should be done quite horizontally, the plate D falls freely (the friction of the silk thread on the pins being negligible), and at the same instant the prongs, suddenly allowed to fly apart, begin to vibrate. The point B thus traces on the falling plate a wavy line similar to H, the bends of which at the lower end are too close to be distinguished, because traced when the plate was falling very slowly, but widen out as we go up. In any part of the trace where the bends are quite clear and distinct

we mark three points X, Y, Z, such that there is some exact whole number a of double bends between X and Y, and the same number between Y and Z. Let the distance XY be d_1, and YZ, d_2. Then the *average* velocities of the plate while the parts XY, YZ passed the tracing point were $\frac{d_1 n}{a}$ and $\frac{d_2 n}{a}$ respectively (n being the frequency of the fork), and these were the actual velocities at the mid-instants of these intervals. So the plate gained a velocity $\frac{(d_2 - d_1)n}{a}$ while the pointer traced a waves, in $\frac{a}{n}$ seconds, so its acceleration was $\frac{(d_2 - d_1)n^2}{a^2}$. This $= g$,* which is 981 in England if centimetres and seconds are the units. Hence $n = a\sqrt{\frac{981}{d_2 - d_1}}$.

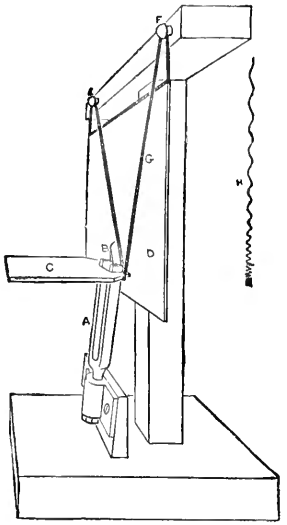


Fig. 56.

It is difficult to get on a freely falling plate a continuous trace representing more than $\frac{1}{6}$ sec., and this limits the accuracy with which n can be found. A better method is given below.

Exp. 21.—Fix the glass plate to the bob of a heavy seconds pendulum, so that the face of the glass is vertical and parallel to the plane of swing. The fork is held so that the bristle vibrates vertically and touches the plate. The trace on the plate is a wavy line, the bends of which are of unequal lengths, as the plate moves with different velocities in different parts of the swing. In any part of this trace where the waves are not too crowded to be distinct, mark off any three consecutive lengths containing each the same

* This experiment is also used as a method for finding g, the frequency of the fork being assumed known, but, as g is a quantity which can easily be measured many times more accurately than the frequency of the fork can possibly be known, this cannot be condesired as more than a very rough method for finding g.

number m of waves. Let d_1, d_2, d_3 be these lengths. The longer the portion of the trace occupied by them, the more accurate the result. Let θ be the angle whose cosine is $\frac{d_1+d_3}{2d_2}$, and T the time of a double swing of the pendulum. Then the frequency of the fork can be shown to be $\frac{360^\circ \times m}{\theta T}$.

Proof.—The glass plate on the pendulum moves with simple harmonic motion past the fork, but it will be more convenient for us to assume that the pendulum carries the fork past a stationary glass plate. Let AB (Fig. 57) be the actual distance moved by the fork relative to the glass plate, and d_1, d_2, d_3 the three consecutive lengths of m waves. (In Fig. 57 m is taken as 4; actually it would be many more.) On AB describe a semicircle. Erect perpendiculars at the extremities of d_1, d_2, d_3 to cut the circle in $P.Q.R.S.$ Join these points to O, the centre of the circle.

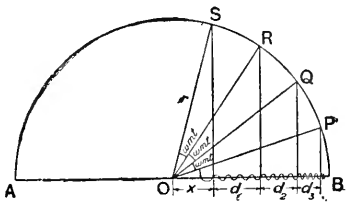


Fig. 57.

While the fork moves from A to B with simple harmonic motion an imaginary point may be supposed to move uniformly around the circle (Art. 8).

Let t = period of the fork,

r = radius of the semicircle = amplitude of vibration of the pendulum,

x = the distance from O to the near end of d_1,

and α = the angle POB.

The time occupied in tracing d_1 or d_2 or $d_3 = mt$. The angular velocity of the radius vector of the imaginary point moving around the circle $= \frac{360}{T} = \omega$ say; $\therefore$ the $\angle SOR = \angle ROQ = \angle QOP = \omega mt$.

Then
$$r \cos \alpha = x + d_1 + d_2 + d_3 \quad \ldots\ldots(1)$$
$$r \cos (\alpha + \omega mt) = x + d_1 + d_2 \quad \ldots\ldots(2)$$
$$r \cos (\alpha + 2\omega mt) = x + d_1 \quad \ldots\ldots(3)$$
$$r \cos (\alpha + 3\omega mt) = x \quad \ldots\ldots(4)$$

Adding (1) and (3) we get

$$r\{\cos a + \cos(a + 2\omega mt)\} = 2(x + d_1) + d_2 + d_3,$$

$$\therefore\ 2r\cos(a + \omega mt)\cos\omega mt = 2\left\{x + d_1 + \frac{d_2 + d_3}{2}\right\},$$

$$\therefore\ \cos\omega mt = \frac{x + d_1 + \frac{d_2 + d_3}{2}}{x + d_1 + d_2} \quad \text{.....(5).}$$

Similarly by adding (2) and (4) we get

$$2r\cos(a + 2\omega mt)\cos\omega mt = 2\left(x + \frac{d_1 + d_2}{2}\right),$$

$$\therefore\ \cos\omega mt = \frac{x + \frac{d_1 + d_2}{2}}{x + d_1} \quad \text{.....(6).}$$

Equating 5 and 6 we get

$$\cos\omega mt = \frac{x + d_1 + \frac{d_2 + d_3}{2}}{x + d_1 + d_2} = \frac{x + \frac{d_1 + d_2}{2}}{x + d_1}.$$

Taking differences of numerators and denominators of (5) and (6) we get

$$\cos\omega mt = \frac{d_1 + d_3}{2d_2},$$

from which the result above at once follows.

In all these graphic methods the attachment of a tracing point, by increasing the mass, diminishes the frequency of vibration, and this must be allowed for. It is best done by comparing the frequency of the vibrating body, before and after the attachment of the tracing point, with the nearly equal frequency of a fork which remains in the same condition in both experiments; the change due to the mass of the tracing point is thus determined. (See Exp. 16.) The same error applies to the stroboscopic method and may be corrected in the same way.

66. Comparison of Frequencies of Forks.—When the frequencies are very nearly equal the difference between them is most easily determined by means of the beats. This method has been described in Exp. 16.

The frequencies of forks may also be compared by a simple graphical method.

Exp. 22.—*Compare the frequencies of two forks.* Attach with as little soft wax as possible a short bristle to one prong of each fork. Slightly smoke a plate of glass by holding it over a luminous flame (burning camphor is best), lay it on the bench with the smoked surface uppermost. Take a fork in each hand, and after striking them against a piece of wood to make them vibrate hold them over the plate with the bristles close together and just touching the smoked surface. Quickly draw the bristles over the plate (or slide the plate under the bristles). Two wave lines will be traced on the plate. Draw two parallel lines (several inches apart) at right angles to the wave lines and count the number of undulations in each wave line between the parallels. The respective frequencies are in the ratio of the numbers of undulations. The relative frequency of a third fork may be found by comparing it with either of the above.

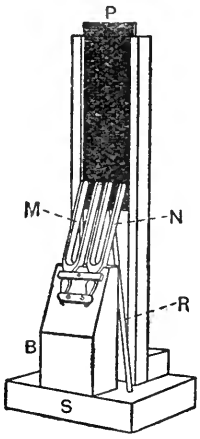


Fig. 58.

A more convenient method of carrying out the experiment is illustrated in Fig. 58. The diagram is self-explanatory.

The device of a third fork may be made to give very accurate results by employing Lissajous' figures instead of beats to determine the difference of frequency. In this method, if the nearly equal frequencies of *A* and *B* are to be compared, a third fork, *C* (Fig. 59), which has a convex lens *f* fixed to one prong, is adjusted by sliding movable masses along the prongs till its frequency is a little less than those of *A* and *B*. It is fixed so that its prongs, and their plane of motion, are horizontal. One of the forks to be compared, say *A*, has a small bright dot scratched with a diamond on the smoked end of one prong *m*, and is fixed vertically, with the dot immediately under the lens of *C*, and so that the movements of the dot are at right angles to those of the lens *f*. The dot is illuminated by a lens *c*.

If the forks *A* and *C* are both made to vibrate, the virtual image of the dot seen from above through the lens has a movement which is the sum of the movements of the dot

and lens; it describes, according to the relative phases of these movements, one of the forms of the top line of Fig. 19, the curve described (which, by persistence of vision, appears as a continuous bright line) assumes in turn all the forms of the series, going through the whole cycle in the period in which A gains one complete vibration on C. As the moments at which the curve becomes a straight line can be observed very exactly, the difference of frequency between A and C, which is the number of complete cycles per second, can be found within $\frac{1}{50}$ of a vibration per second. Similarly, B can be compared with C, and the difference between A and B deduced from the two results.

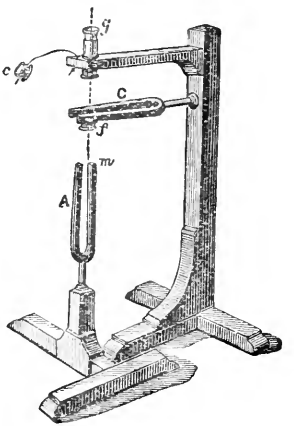


Fig. 59.

The superiority in exactness of this method to that of audible beats results from the great accuracy with which we can ascertain when one fork has gained an exact whole number of vibrations on the other, and the fact that the vibrations can be observed for a much longer time.

A still better arrangement is that shown in the figure. Here the lens f is placed at such a distance above m that a real image of m is formed, and this real image is examined through a fixed eyepiece g.

By means of this *vibration microscope** we can adjust a fork very exactly to a frequency which has any simple ratio to that of C. The fork can be made quicker by filing it near the points, or slower by filing it at the bend, and it is to be adjusted till a Lissajous' figure corresponding to the required ratio is seen, and changes only very slowly. Two forks can be adjusted to have a given ratio to each other by adjusting them, in turn, to any convenient ratios to the lens-fork which have the required ratio to each other.

* For another form of vibration microscope see Art. 183.

67. Other Determinations of Frequency.—The vibratory movements of the air caused by a vibrating body at any point which is always at the same distance from the body must agree in frequency with those of the body itself; the exact repetition of the same causes must produce the same effects. Frequency is indeed the only particular in which vibrations of the air necessarily agree with those of the body causing them. Any of the methods for determining the frequency of the movements of the air can therefore be used to determine the frequency, though not the character, of the vibrations of a solid body.

68. The Phonautograph.—To determine the frequency of the movements of the air at a point, Leon Scott invented

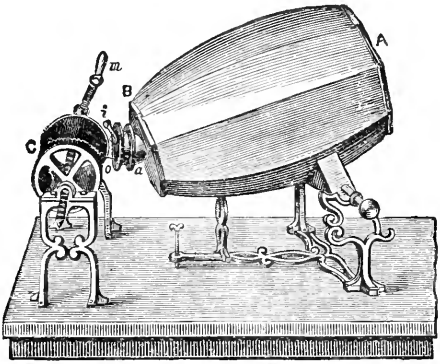


Fig. 60.

an apparatus called a Phonautograph (Fig. 60). It consists of a large funnel, usually made parabolic in form, and closed at the smaller end by a stretched membrane; below the centre of this membrane, at right angles to its surface, is fixed a short bristle. The funnel is fixed so that the membrane is parallel and very close to the surface of a smoked cylinder exactly like that of the Vibroscope. A spring, which touches a point of the membrane exactly above the centre,

insures the membrane always vibrating with a vertical nodal line through the centre; the part of the membrane which carries the style is therefore never displaced in the direction of the axis of the funnel, but merely alters its direction, vibrating on a fixed vertical axis. The point of the bristle therefore moves parallel to the axis of the cylinder, and always remains just in contact with the smoked surface, and its movements correspond in frequency, though not in any other respect, with those of the air.

Seconds can be marked on the trace left by the bristle by the electrical method described above for the vibroscope, or a tuning-fork of known frequency can be made to trace a wavy line on the cylinder by the side of that traced by the bristle, and the frequency of the bristle determined by comparing its movements with those of the fork.

Another method has only recently been rendered possible. The sound to be examined is received in a telephone, and produces variations in an electric current. These variations can be recorded in a deadbeat galvanometer [Duddell's Oscillograph] specially designed for the purpose; it is a mirror instrument, and the motions of its spot of light on a moving strip of photographic paper produce curves identical in character with those from the Phonautograph.

69 Phonograph.—Edison's Phonograph is similar to the phonautograph in principle, but a cylinder covered with wax is substituted for the smoked one, and a very narrow chisel for the tracing point. The centre of the membrane is in this case made an antinode,* and the point of the chisel therefore moves perpendicularly to the surface of the cylinder, not parallel to it, and so digs a trench of varying depth as the membrane vibrates. A section of this trench, parallel to its length and perpendicular to the surface of the cylinder, shows the form of the bottom, which depends on the quality of the sound, but is not the "wave-form" in any accurate sense of the term. It is, however, (like the trace of the phonautograph) a curve whose harmonic components are nearly the same as those of the waves. The

* For definitions of these terms see Art. 187.

curve can be examined by taking a longitudinal section of the trace and putting it under a microscope.

If the chisel is now replaced by a blunt point, and the cylinder replaced in its original position and turned again in the original direction, the blunt point presses on the undulating bottom of the trench already cut by the chisel, and the membrane to which the point is connected repeats the movements by which the trench was originally cut. It thus produces waves in the air similar to those which, by their arrival, caused the original movements of the membrane, though they are reversed, the condensations of the original waves being represented by rarefactions in those afterwards produced by the membrane. Though in this and other respects there is considerable difference between the wave-forms of the original sound and the sound reproduced by the membrane, the two have nearly the same harmonic components, and therefore seem to the ear so nearly of the same quality that, if the original sound was that of the speaking voice, the different vowels (which are only special qualities) are easily distinguished in the reproduction.

The following is the method adopted for making phonograph 'records." The master record made on the recording machine is suspended on a conveniently shaped cylinder, or mandrel, between two strips of gold foil. The whole is then covered with a glass bell-jar and the air exhausted by means of a pump. Over the bell-jar is suspended a magnet, which is made to rotate, and by so doing to rotate the mandrel. The gold foil is then connected to the negative electrode and the metal of the mandrel to the positive electrode of an induction coil. An electric discharge ensues and the gold foil sputters all over the contents of the jar. When sufficient gold has accumulated on the record surface the discharge is stopped and the record removed, the wax cylinder being then shrunk out by subjecting it to a gradually *decreasing* temperature. (By so doing the master record is not destroyed.) A gold shape which bears every mark appearing on the original master being thus secured, a heavy backing of copper is given to it (by electrolysis or otherwise) to prevent it losing shape. This mould is then filled with boiling wax (of very particular composit on) and placed in a room whose temperature is very gradually lowered until the wax has shrunk so much that it may be removed. The record, now a solid cylinder of wax, is next rendered into the familiar hollow cylinder and prepared for market.

It is worth noticing that it is the imperfection of the ear as a detector of differences in air waves that makes the phonograph and telephone possible. The waves they give out are in form often so different from the original waves that, if the ear had as great a power of distinguishing between different wave-forms as the eye has of distinguishing different outlines, it would be impossible to recognise any resemblance to the original sound.

70. The frequency of air waves can also be found by means of a suitably mounted Savart's wheel or Seebeck's siren (Art. 60). The note given by either one of these instruments could be adjusted in unison with the given note, the frequency of the wheel or siren source could be calculated from the observed speed of revolution and the number of teeth in the wheel or holes in the disc. It would, however, be extremely difficult to make the adjustment, and considerable elaboration of either instrument would be necessary in order to give anything like an accurate determination. Of the two instruments the siren is by far the most suitable for a purpose of this kind, and it was greatly improved and adapted for determinations of pitch both by Cagniard de la Tour and by Helmholtz.

71. The Siren.—A common but not very good form of counting siren is shown in Fig. 61. The lid of the box *C* and the revolving plate *D* contain the same number of holes, and a puff issues simultaneously from all the holes each time those in the plate coincide with those in the lid, so that there are as many successive puffs in each rotation of the disc as there are holes in the circle. The rotations of the disc are counted by wheels, similar to the counting

* Any instrument in which a sound is produced by puffs of air escaping at regular intervals through holes or notches in a rotating plate or cylinder, is called a *siren*. The name was given because most forms of such an instrument may be blown with water in water instead of with air in air; this is however not peculiar to the siren, but is equally true of nearly all wind instruments (Art. 153), and Homer's Sirens differed from the mechanical ones in this respect. A common form of whistle on the market works on the principle of the siren.

apparatus of a gas meter, which are driven by a screw on the axle of the disc, and can be made to begin counting by pressing a knob, and stopped by releasing it. Air is blown into the box A, and the angular velocity of the disc increased till the pitch of the note heard is nearly that of the sound whose frequency is to be determined.

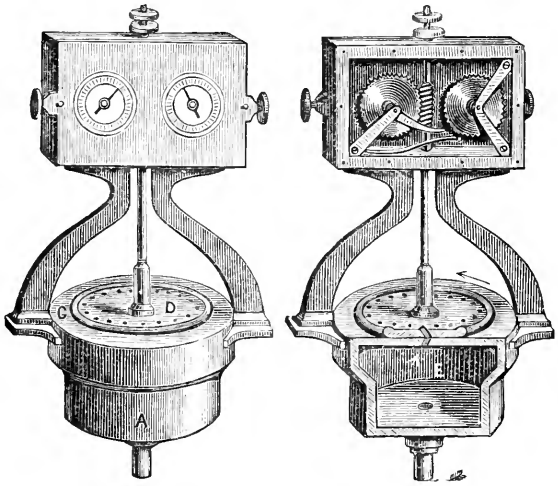


Fig. 61.

Now it is obvious that there is nothing harmonic about the process by which the waves from a siren are produced, and the waves, being not at all harmonic in form (Art. 93), have many harmonic components,* and can excite in the ear or other resonators vibrations of higher frequencies than that of the actual puffs, and it is only after much practice that it is possible to be certain whether it is the frequency of the puffs, or one of its multiples, which is equal to the frequency of the waves from the other source.

* The question of harmonic components may be postponed until Chapter VII. is read.

When we are quite certain that it is the lowest in pitch of the sensations due to the siren (which is often not the loudest) which corresponds to the lowest of the sensations due to the other source, the velocity of rotation should be slightly diminished till beats are heard at the rate of about four per second. The counting mechanism (see figure) is then set in action for a definite time, say thirty seconds, while the observer keeps count of the total number of beats heard during this interval, and regulates the rotation so that the beats do not become too rapid to count or too slow to distinguish. The total number of rotations recorded, multiplied by the number of holes in the disc, gives the number of waves from the siren during the observed interval of time, and this number, *plus* the number of beats counted, is the number of waves which left the other source during the same interval.

In the siren shown in the figure the disc is made to rotate by the air issuing from the holes, which are drilled obliquely in both lid and disc, but slope opposite ways, as shown in section at *E*. This is a very bad arrangement, for the speed can only be increased by blowing harder, which also increases the loudness of the sound, and beats are not very distinctly heard except between two sounds of nearly equal loudness. It is much better to drive the disc independently by an electromotor or other easily controlled mechanism.

It must not be forgotten that beats are also heard if any of the harmonic components of the sound from the siren nearly agree in frequency with any of the harmonic components of the sound from the other source, so that, if the waves from the other source are also very far from harmonic, the effect on the ear is that of several series of beats, of different frequencies, happening at the same time. This renders the counting of the beats of the fundamental vibration very difficult.

Helmholtz also invented a form of double siren. It is described on p. 205 and shown in Fig. 84.

A **stretched wire** or cord may also be used as an adjustable standard of frequency. The method of using it for this purpose is described in detail in Arts. 178, 179.

72. Determination of Frequency by comparison with a Standard of Constant Known Frequency.—The standard source of constant frequency in most general use is the tuning-fork (sometimes called a pitch-fork). It consists of a steel bar of rectangular section doubled into a U shape. From the bend a shank projects. Small forks, held at the shank by the hand, are made to vibrate by striking one of the prongs against a pad of india-rubber, or soft wood. Larger forks are screwed by their shanks to a box, called the *resonance box* (Fig. 62). This is made of thin wood. It considerably increases the intensity of the sound produced. A fork is made to vibrate by drawing a resined fiddle bow across the prongs, in a direction perpendicular to their length. A tuning-fork is a most satisfactory standard of frequency, and the note emitted is pure in quality (Art. 191), and the frequency is practically constant. Rise of temperature produces a very slight decrease of frequency, but the change is practically negligible.

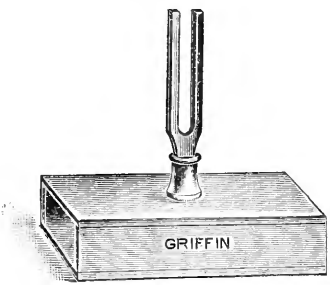


Fig. 62.

The frequency of vibration of a tuning-fork may be determined by any one of the previous methods. An approximation to its value may be calculated if the dimensions of the prongs and the Young's Modulus of the steel are known. The necessary formula is given in Art. 191.

The frequency of a note can be at once determined by selecting a standard tuning-fork giving a note of approximately the same frequency and counting carefully the number of beats produced in a given time when the two notes are sounded together. Then, if N denote the frequency of the fork, and n the number of beats heard per second, the frequency of the given note is $N \pm n$, according as it is sharper or flatter than the note of the fork.

73. The Tonometer.—For the purpose of determining frequency, Scheibler constructed a set of 64 forks covering the range of pitch between the middle *C* of the piano, of frequency 256, and its octave of frequency 512. The forks were adjusted so that the difference between the frequencies of any two consecutive forks of the set was exactly four. With this set of forks, which Scheibler called a *tonometer*, the frequency of any note within the octave could be accurately determined. For example, if the given note gives 102 beats per minute when sounded with the fork of frequency 300, 138 beats per minute with the fork of frequency 304, and 342 beats per minute with the fork of frequency 296, it is obviously sharper than the (300) fork and flatter than the (304) fork, and corresponds to a frequency of 301·7 vibrations per second.

Appunn constructed a tonometer made of reed instruments, but the frequency of vibration of a reed is altered by air movements caused by adjacent vibrating reeds, hence this form of tonometer is not very accurate.

It will be obvious that the method of determining the frequency of a note by comparing it with that of a standard fork is limited by the number and range of standard forks available.

74. Koenig's Manometric Capsules and Flames are useful for comparing the frequencies of notes in pipes and the open air. The flames indicate variation of pressure by the following means. The gas supply to the flame passes through the *manometric capsule* shown in section in Fig. 63 The gas enters by a tube into a shallow cylindrical cavity C, about two or three centimetres in diameter, and passes from this cavity through the exit tube B to the burner, which is simply a fine pin-hole burner giving a small steady flame. One side of the cavity is closed by a thin membrane *m*, so that any variation of pressure at the

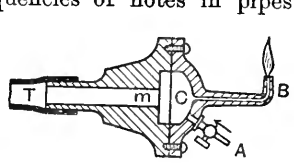


Fig. 63.

outer surface of this membrane is at once indicated by the "jumping" of the manometric flame. Periodic variation of the external pressure sets the membrane in vibration with the same period, and this, by its effect on the gas supply to the burner, causes the flame to jump up and down with corresponding frequency.

The motion of the flame is much too rapid to be observed directly, although it is clearly indicated by the elongated and flickering appearance of the flame. If, however, the flame is examined in a rotating mirror (Fig. 64), the successive images of the flame give a crooked band of light in the mirror, and the number of teeth seen in a given length of this band indicates the frequency of the flame's motion, and therefore of the periodic variation of pressure to which the motion is due.

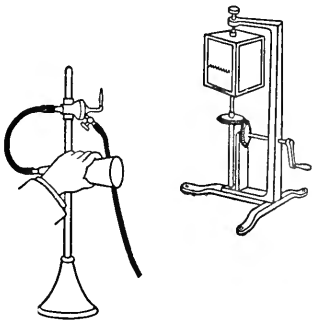


Fig. 64.

To compare the frequencies of two organ pipes apply a capsule to each pipe, and fix the two flames close together so that their images may be examined side by side in the rotating mirror. If it is found that n teeth in one image occupy the same length as n' teeth in the other image, the corresponding frequencies are evidently in the ratio n to n'.

75. Indirect Methods of Measuring Frequency.—The above are the most important methods of measuring frequency *directly*. If the velocity of sound in a medium is known, and the length of the waves is measured, the frequency can be at once determined, since

$$\text{frequency} \times \text{wave-length} = \text{velocity}.$$

These indirect methods will be described in Arts. 145, 158, 159.

EXAMPLES V.

1. Describe one good method of finding the frequency of vibration of a body.

2. A wheel with 33 teeth touches a card as it spins, and thereby emits a note two octaves above middle C, which has 256 vibrations per second. How many revolutions is the wheel making per minute?

3. A cog-wheel containing 64 cogs revolves 240 times per minute. What is the frequency of the musical note produced when a card is held against the revolving teeth? Find also the wave-length corresponding to the note if the velocity of sound in the air is 1126 ft. per second.

4. Describe the siren. How is it employed to find the frequency of vibration of an organ pipe?

5. The disc of a siren contains 40 holes. How many revolutions must it make a second to give a note of frequency 512?

6. In a dropping plate experiment the distance of a certain crest from the point on the plate marked by the style on the fork before the plate fell was 9·8 mms. The distance between this crest and the forty-third from it was 111·0 mms. Find the frequency of the fork.

7. How are *beats* used to effect exact unison?

8. How may a stroboscope be used to adjust the rate of revolution of an axle to uniformity?

9. Draw a diagram of the electrical connections of the vibroscope.

10. How may Lissajous' figures be used to compare two forks?

11. What is a tonometer and how is it used to find the frequency of a fork?

EXAMINATION QUESTIONS I.

1. Define a simple vibration. Show by a diagram how to exhibit the result of adding together two simple vibrations of different frequencies.

2. A stone is dropped perpendicularly into still water and produces a series of concentric waves. At the end of 5 seconds there are 50 concentric troughs and crests and the boundary of the outermost is a circle of 1·5 metres radius. Find the wave-length and velocity of propagation of the disturbance. Find also the period of oscillation of a water-particle.

3. Draw curves to show how the pressure and displacement of the air in a pipe along which a simple uniform train of sound waves is passing vary (*a*) with the time at a particular point, (*b*) with the distance along a pipe at a particular instant.

4. Describe the changing conditions of a mass of air traversed by a succession of sound waves that proceed from a regularly vibrating source. How does the velocity of such waves depend upon the temperature, pressure, and hygrometric condition of the air?

Mention some observation that indicates that the velocity of a sound wave is independent of its length, and of its amplitude of vibration.

5. Two bells are struck regularly and simultaneously 40 times per minute. At first they are equidistant from an observer, but the distance of one of them is gradually increased. Assuming the velocity of sound to be 1120 feet per second, calculate what the difference between the distances of the bells from the observer will be before he again hears them sound simultaneously.

Why would it be difficult to verify the calculation by direct experiment?

6. How does the velocity of sound in a gas depend upon (i) temperature, (ii) pressure, (iii) the nature of the gas?

How has the velocity of sound in water been determined?

7. Describe a method of determining the velocity of sound in the open air. Point out the sources of error, and explain how some of them may be allowed for.

8. A tuning-fork, vibrating in oxygen gas, produces waves, each four feet in length and travelling with a velocity of 1100 feet per second. Determine its frequency of vibration, explaining the process. How would the velocity, wave-length, and frequency be affected if the same fork were sounding in hydrogen at the same pressure and temperature, the density of oxygen being 16 times that of hydrogen at the same pressure and temperature?

9. What is the nature of sound in air? Give reasons depending upon experiment for your answer.

10. How would you show that the prongs of a sounding tuning-fork are in vibration, and that the intensity of the note emitted depends upon the amplitude of the vibration?

Describe carefully some method of determining the number of vibrations that take place per second.

11. Describe a siren and explain how it may be used to find the frequency of a tuning-fork.

If there are 32 holes in the disc which makes 1050 revolutions per minute, what is the frequency of the note emitted by the siren?

12. Describe any method of counting the number of vibrations made by a tuning-fork. In what way would you expect temperature to affect the number and why?

13. What experimental evidence have we that sound is propagated by waves of compression and rarefaction in the air? How does the length of a wave produced by a tuning-fork of given pitch depend (1) upon the pressure of the air, (2) upon its temperature?

14. Describe experiments by which, in the case of a given sound, the frequency and length of the waves can be ascertained.

15. Upon what properties of a solid does the speed of sound in the solid depend? Are all kinds of waves in a solid propagated with the same speed? Why does sound travel faster in steel than in air?

16. The specific gravities of nitrogen and oxygen gases are as 14 to 16. At what temperature will the velocity of propagation of sound through oxygen be the same as that through nitrogen at 15° C.?

17. Explain why the rise of temperature due to compression and the fall of temperature due to rarefaction in a sound wave *both* tend to raise the velocity of propagation of the wave.

18. What are the conditions necessary for two musical notes to produce distinct beats? Show that the number of beats per second is equal to the difference between the two frequencies. Fifty-six tuning-forks are arranged in a series in order so that each fork gives 4 beats per second with the next lower fork. The last fork gives the octave of the first. Find the frequency of the first.

19. Show that the expression

$$y = a \sin \frac{2\pi}{\lambda} (x - vt)$$

represents a train of waves of amplitude a and wave-length λ moving along the x axis with velocity v. Draw curves showing that the variation of the displacement y

(1) varies with the time at a point $x = \frac{3\lambda}{2}$, and

(2) with x at a time $\frac{\lambda}{v}$.

20. If a train of sound waves in a gas of density ρ is represented by $y = a \sin \frac{2\pi}{\lambda} (x - vt)$, where λ is the wave-length, v the velocity of sound, and y the displacement, show that the mean energy per unit volume is proportional to $\frac{a^2 \rho}{\lambda^2}$.

CHAPTER VI.

FORCED VIBRATION. RESONANCE.

76. Free and Forced Vibration.—As the series of conditions which constitutes a wave-system travels through the air, the density at any fixed point increases and diminishes alternately, and the pressure changes proportionally to the changes of density. If there is a solid body stationary in the air, the pressure on its surface rises and falls as the waves pass it, varying harmonically (Art. 8) if the waves are harmonic, or, in any case, rising and falling proportionally to successive ordinates by their wave-form. As all bodies yield more or less to pressure, the body will vibrate.

Such vibrations, caused by continuous changes of external pressure, are called *forced vibrations*, to distinguish them from the *free vibrations* which take place when a body is displaced from a position of equilibrium and then left to itself. In most cases it is not possible to determine by elementary methods the exact character of the forced vibrations which a given wave-system will cause in a given body; we can only say that, as the differences of pressure in different parts of an ordinary sound wave are very small, the vibration produced, if it depends on the changes of pressure actually taking place at the moment, will usually be very small also.

77. Resonant Forced Vibration.—But if the successive waves are all exactly equal and similar, so that the pressure change is exactly periodic, and if the period of this pressure change is related in a certain way, to be explained presently, to one of the periods of free vibration of the

body, the effects of a number of successive changes of pressure may be added together, and so vibratory movements of the body may be produced, which so greatly exceed in amplitude those which would be caused by the pressure changes actually occurring at the moment, that the latter may be neglected in comparison. Such forced vibration, greatly exceeding in extent that which would be due to the pressure changes actually occurring, and resulting chiefly from past pressure changes, is called *resonant forced vibration*. We shall sometimes call it simply *resonant vibration*, though this term is often applied also to free vibration when caused by a previous sound.

78. Experiments on Resonant Vibration.—We have next to determine under what circumstances such resonant forced vibration is produced, and for this purpose we may try some simple experiments.

Exp. 23.—Hang up two heavy weights, A and B, by strings of equal length, and set them swinging ; it is easy by slightly altering the length of one string to get them to swing in almost exactly equal times, so that if started in the same direction they will keep time with each other for some hundreds of consecutive swings. Now stop B, and tie a fine thread to it, and every time A passes the middle point of its swing in one direction, say from right to left, give a very slight momentary pull to the thread, not enough to produce any visible movement of B. After a few seconds, B will swing very perceptibly, and at the end of a minute or two it will swing through quite a large arc. The weight swings to a distance on each side of its mean position much greater than the displacement which would be due to the actual pull of the thread ; often to a greater distance than the thread could pull it aside without breaking.

The same thing happens in many other cases. The regular tramp of soldiers crossing a bridge will break the bridge down if its period of vibration agrees with the interval between the steps ; and the vibrations of the air caused by an organ (it is said even by the voice) will break a pane of glass in a window if the pane is of such a size that it vibrates with the same frequency as the waves reach it.

79. Different Effects of Harmonic and Non-harmonic Pressure Changes.

Exp. 24.—Alter the length of A's string a few inches, so that the times of swing are slightly unequal, and repeat last experiment. No large vibration of B will be produced. Now lengthen A's string till A makes only one vibration to two of B (or, which is simpler, if A swings with the same period as B, pull the thread only at the end of every two double swings of A). Note that the movement of B is gradually increased till it becomes nearly as large as before.

The same occurs if we pull the thread every time A passes its lowest point from right to left, or at any regular intervals which are exact multiples of B's free-vibration period. So that in some cases periodic impulses can cause a resonant vibration whose period is an exact submultiple of their own. But there is a very important exception to this. If the periodic impulses, instead of being forces which begin and stop suddenly, like the pull of the thread, are forces which increase and diminish *harmonically*, they will not set in resonant vibration a body whose natural period is an exact submultiple of their own. This can easily be shown by a slight variation of Exp. 24.

Exp. 25.—Hang up A and B (still by strings of equal length) a long distance, say 20 feet, apart, and tie one end of a piece of very thin elastic, about 2 feet shorter than this distance, to B. Hold the other end of this elastic in your hand, exactly under A, so that it is slightly stretched. Set A swinging, with an amplitude of about a foot, along a line passing through B, and move your hand as A swings, so as to keep the end of the elastic exactly under A. The pull of the elastic on B will then increase and diminish harmonically, or very nearly, and it will be found that B will begin to vibrate, and keep increasing its vibration, if its free-vibration period is the same as that of A. Now lengthen the string of A till it makes only one swing to two of B, and then keep the end of the elastic under A as it swings, no considerable movement of B will be produced. B does move, of course, but the movement does not increase beyond that due to a single pull.

Now, we saw that a slight pull on a thread tied to B, occurring once in every two swings of B, would produce a large vibration. The difference between the two cases is this:—The pull of the thread is a force which, at regular intervals, increases very suddenly from zero to its maximum, and, after lasting a very short time, falls very rapidly

to zero. In other words, the thread exerts on B a force which varies periodically but *not harmonically*, while the pull of the elastic varies harmonically. We conclude that a periodic force which increases and diminishes harmonically does not cause resonant vibrations of periods which are submultiples of its own, though a force which varies non-harmonically may do so.

Exp. 26.—*Test the unison of two forks.* Rest the shanks of two forks on a sounding board, one of them being excited. After a few seconds remove and silence the excited fork. The other should be emitting its note, it having been excited by resonance, if its frequency is identical. Otherwise nothing should be observed.

80. Period and Amplitude of Forced Vibrations.

Exp. 27.—Now adjust the lengths of the strings till A swings nearly, but not exactly, with the same frequency as B. Suppose 30 swings of A require the same time as 31 of B. Repeat Exp. 25 with the elastic, B will still be set vibrating, and its amplitude of vibration will increase for a time, but will soon reach a limit which is much smaller than when the periods agree more exactly. Note also that the vibration of B agrees in period exactly with that of the pulls given to the elastic, and does not depend on its own natural period of vibration.

If the period of A's vibration is 2 seconds, the forced vibrations of B will have a period of 2 seconds also, whatever the natural period of B, but, unless that natural period is about 2 seconds, the vibration of B never becomes very much greater than that which one of the pulls would produce.

Though the vibration produced in a body by a periodic force, which only nearly agrees in period with the free vibrations, is always of smaller amplitude than it would be if the periods agreed exactly, the extent of the difference depends very much on the mass of the vibrating body and the amount of friction resisting its motion. If the mass is large and the friction small, as in the case of a heavy pendulum or tuning-fork, the resonant vibration produced by impulses of *nearly* the natural period of the vibrating body is very small compared with that produced by equal impulses of *exactly* the right period. But if the mass is small and the friction large, as in the case of a boat rolling

in the waves, or air vibrating in a narrow tube, impulses of nearly the right period will produce nearly as strong resonant vibration as impulses of exactly the right period (see Art. 83).

Practically, when we have to do with a body which is not very light, and periodic pressure changes which are not very violent, we may assume that no *perceptible* movement will be caused by a *harmonic* pressure change of any period which is not almost exactly one of the periods in which the body can vibrate freely.* So *harmonic* waves do not cause strong vibration of bodies which they pass, unless the waves arrive with frequencies which are nearly free-vibration frequencies of the bodies.

We have not yet discovered what are the conditions under which *non-harmonic* waves cause resonant vibration, though we have seen that a non-harmonic rise and fall of pressure, such as non-harmonic waves would produce, sometimes causes resonant vibration where harmonic waves, arriving with the same frequency, would fail. To determine what non-harmonic waves will cause resonant vibration of a given body we must resort to a mathematical device, explained in the next chapter.

81. Further Experiments on Forced Vibration.—The impulses instead of being communicated by hand may be communicated through the support.

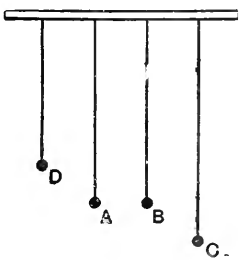


Fig. 65.

Exp. 28.—Suspend four simple pendulums, *A*, *B*, *C*, and *D*, from a light wood lath fixed horizontally with its "width" in a vertical plane, as shown in Fig. 65, in which the plane of vibration of the pendulums is supposed to be at right angles to the plane of the paper. Let *A* be the actuating pendulum with a rather heavy bob, *B* a pendulum of the same length as *A*, *C* a slightly longer, and *D* a slightly shorter pendulum. Set the pendulum

* Under exceptional circumstances this may be untrue; an instance is given in Art. 181.

A in vibration. The lath is set in forced transverse vibration of small amplitude, but of the same period as the pendulum by the periodic force applied to it at the point of attachment of the swinging pendulum. As a result of the vibration of the lath a periodic force of the same period as that of pendulum A is applied at the points of attachment to each of the pendulums B, C, and D. The application of this periodic force tends to set the pendulums in vibration, and it will be found that the pendulum B of the same length, and therefore of the same natural period, as A is quickly set in vibration. This is an instance of the particular case of forced vibration known as *resonance*. The pendulums C and D, however, first get up a slight swing, then come to rest and continue this process for some little time, but ultimately they settle down into steady vibration, with exactly the same period as that of the pendulum A. It will be found on examination that the mode of vibration of C is that shown in Fig. 66, while that of D is shown in Fig. 67.

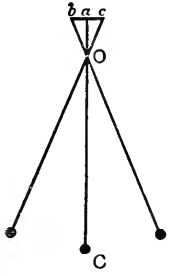


Fig. 66.

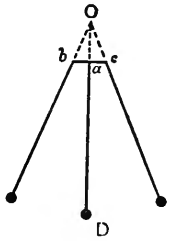


Fig. 67.

In each case it will be seen that as a the point of attachment of the pendulum vibrates in the path bc (which may be *very short*) with a period equal to that of the pendulum A the suspension thread adjusts itself so that each pendulum vibrates as a pendulum of length equal to that of A and with the same period as A. It will be noticed also that the amplitude of vibration produced in C and D is less than in the case of A, and that while the pendulums A, B, and D are in the same phase, C is in the opposite phase or differs from them in phase by half a period.

Exp. 29.—The last experiment may be varied by setting up a pendulum with a heavy bob A and suspending from this bob another pendulum with a light bob B. A may be of lead and B of cork. The suspension may be cotton thread.

(1) Make the length of A greater than the length of B (Fig. 68) so that the natural period of A is greater than the natural period of B.

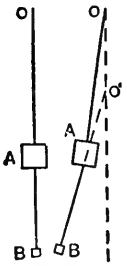


Fig. 68

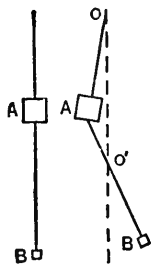


Fig. 69.

Then set A in vibration. The vibration of B is at first irregular, due to a co-existence of free and forced vibrations, but the free vibrations gradually die out and B finally settles down to a vibration of the same period as A and in the same phase as A.

(2) Make the length of A less than the length of B (Fig. 69) so that the natural period of A is less than the natural period of B. Then set A in vibration. The vibration of B is at first irregular, but it finally settles down to a regular vibration of the same period as A, but in opposite phase.

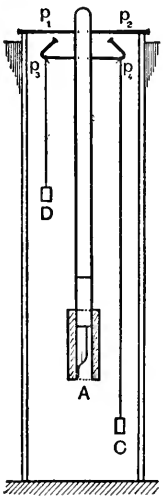


Fig. 70.

Another most instructive experiment which can be performed with about the minimum amount of trouble is as follows :

Exp. 30.—Take a penholder A (Fig. 70), weight it at the bottom by wrapping a piece of lead-foil around it. Stick two pins p_1, p_2 into the other end and rest the pins on the covers of two books so that the holder can swing between them. The arrangement constitutes a pendulum. Remove A and stick two bent pins p_3, p_4 into it just below p_1 and p_2. To p_3 attach a thread pendulum D of about two-thirds the length of A, and to p_4 attach one, C, a little longer than A. The bobs of D and C may be small weights (gram weights or other small pieces of metal). Set A swinging. Observe that the motions of C and D are at first irregular, giving

all the symptoms of "beats," and that finally they swing regularly, D being in agreement, *i.e.* in phase with A, and C being in opposition, *i.e.* out of phase with A. The best method of observation is to look down on the pendulum from above.

82. Mathematical Treatment.—Consider a mass m which vibrates in simple harmonic motion of period T under the action of a controlling force (such as gravity in the case of a pendulum bob, and elastic forces in the case of a mass attached to a spring). When the displacement of m is equal to x the acceleration of m is $-\left(\frac{2\pi}{T}\right)^2 x$, and the outward acting force is $-m\left(\frac{2\pi}{T}\right)^2 x$, the minus sign being affixed because the acceleration is towards the origin and the tendency of the force is to decrease x. Now suppose a harmonically changing force of period T_1 act on m (in Exp. 30 this force acts, say, on D by making the point of suspension p_3 oscillate). This force may be represented by $F \sin \frac{2\pi}{T_1} t$. The period of vibration of m now becomes equal to that of the force, and therefore when the displacement of m is x the acceleration of m is $-\left(\frac{2\pi}{T_1}\right)^2 x$, and the outward acting force on m is $-m\left(\frac{2\pi}{T_1}\right)^2 x$.

This force must be the resultant of the original force and the impressed force, *i.e.*

$$-m\left(\frac{2\pi}{T_1}\right)^2 x = -m\left(\frac{2\pi}{T}\right)^2 x + F \sin \frac{2\pi}{T_1} t$$

$$4\pi^2 m . x \left\{ \frac{1}{T^2} - \frac{1}{T_1^2} \right\} = F \sin \frac{2\pi}{T_1} t,$$

or

$$x = \frac{F \sin \frac{2\pi}{T_1} t}{4\pi^2 . m\left(\frac{1}{T^2} - \frac{1}{T_1^2}\right)}.$$

(1) If the period of the applied force is greater than the natural period, $T_1 > T$, *i.e.* $\left(\frac{1}{T^2} - \frac{1}{T_1^2}\right)$ is positive and x is of the same sign as $\frac{2\pi}{T_1} t$, *i.e.* the forced vibration is in phase with the force. (This is the case of pendulum D in Exp. 30.)

(2) If the applied period is less than the natural period, $T_1 < T$, and the forced vibration is in opposite phase to that of the force. (This is the case of pendulum C in Exp. 30.)

(3) If the applied and natural periods are equal, $T_1 = T$, and $x = \frac{F \sin \frac{2\pi}{T_1} t}{0}$. Mathematically this means that the displacement becomes infinite; practically it means that the displacement becomes larger and larger until the loss of energy from the vibrating body due to air resistance, etc., is equal to the energy given to the vibrating body by the applied force. This case is one of *resonance*.

83. True and False Resonance.—The following non-mathematical explanation covers a great many phenomena. Consider a non-viscous body of a definite natural period. Being non-viscous, it does not dissipate its energy in the form of heat, and therefore if it is set vibrating it will continue to vibrate in its natural period for ever with unchanged amplitude. No body customarily met with is so perfectly free from viscosity; but a tuning-fork of frequency 256 which continues audible for two minutes (or 30,720 periods) after being struck is a sufficiently near approximation.

Let the body be disturbed by a periodic force and let the period of the force differ slightly from the natural period of the body. For instance, let a fork of frequency 258 be acted on by a periodic force of frequency 256, so that the fork would naturally gain two periods on the force in 1 sec., or a half period in $\frac{1}{4}$ sec., *i.e.* after making $64\frac{1}{2}$ complete vibrations. Due to the impulses of the force the fork will be set in vibration. Suppose the amplitude of vibration due to one impulse is a. Since the fork is non-viscous its vibrations were it left to itself do not decrease in magnitude, and therefore the motion of the fork at any instant is the resultant of all the separate motions due to the successive impulses of the force. Now consider the action on the fork of two impulses of the force one of which is given $\frac{1}{4}$ second later than the other. The vibrations produced in the fork by these two impulses will be of equal amplitude a, but their phases

will be exactly opposite, for there will have been $62\frac{1}{2}$ vibrations of the fork due to the first impulse during this $\frac{1}{4}$ second while the force will be acting in the same direction as before. It follows that the oscillations set up by the two impulses considered exactly neutralise, and in general that each oscillation produced by the periodic force is effectively damped by the oscillation produced $\frac{1}{4}$ second later. The total motion produced in the fork is therefore very small, as it can never rise above the motion produced in the first fraction of a second before the damping action commences.

No really considerable disturbance can therefore be given to the fork by periodic force whose period is sensibly different from the natural period. The fork only vibrates considerably when acted on by a harmonic disturbance of period equal to its own, within very narrow limits indeed. This is the condition for *True Resonance.*

Consider next a viscous body. The law obeyed by viscous friction is that the resisting force is proportional to the first power of the velocity.* A vibrating body affected by viscous friction has a period a little longer than if it were non-viscous, but this difference is negligible if the viscosity is small. What is important, however, is that the vibration is gradually destroyed, its amplitude being diminished by equal fractions in equal times. If, for instance, the amplitude is halved in 20 vibrations it will be multiplied by $(\frac{1}{2})^2$ or $\frac{1}{4}$ in 40, and by $(\frac{1}{2})^3$ or $\frac{1}{8}$ in 60. The logarithm of its amplitude diminishes by log 2 for every 20 swings, and therefore decreases at a uniform rate.

[The Logarithmic Decrement is defined as the diminution which takes place in the logarithm (Napierian) of the amplitude per unit time. If the body here considered had frequency 258 as before, the logarithm (to base e) of the amplitude would diminish by $\log_e 2$ or ·693 in a time 20/258, and the logarithmic decrement would be

$$\cdot 693 \times 258 \div 20 = 8 \cdot 94.]$$

Let now this body be acted on by a similar disturbing force of period 256. The effect of one single disturbance is a periodic vibration which starts with amplitude a say. After $\frac{1}{4}$ second the amplitude of this vibration has fallen to less than $a/8$. The disturbance which would come $\frac{1}{4}$ second after the first would produce a vibration of amplitude a in the contrary phase to the above, but more than 8 times as great. Consequently this disturbing effect is not expended in destroying the motion already existing; only an eighth of it is wasted in this way. The disturbing force is therefore enabled to produce much more effect than in the former case. Hence we get a really large forced vibration in a somewhat viscous body when the period of the disturbing force is near, but not identical with, a natural period of the body. This is called *False Resonance.*

It takes place only in vibrating bodies that are quite appreciably viscous; and the antecedent condition is that the disturbing force shall have a period near, but not identical with, the free period.

* *I.e.* resisting force = constant × velocity.

If the free frequency had been 256, identical with that of the disturbing force, then any one disturbance would produce a vibration of amplitude a which would fall to $a/8$ in $\frac{1}{4}$ second as before. But this would now be in the same phase as the disturbance of amplitude a which is being impressed, so that the resultant would be $9a/8$ instead of $7a/8$ as before. Consequently the inequality of frequency, 258 to 256, makes the final amplitude of the whole vibration about 7/9 what it would have been had the frequencies been equal; and the intensity of the sound (proportional to square of amplitude) is about $(7/9)^2$ or about six-tenths what it would have been.

With a still more viscous body the difference of intensity corresponding to such a small difference of period might have been very much less. A typical case is a narrow air column. If it be made to resound accurately to a tuning-fork, its natural period may be changed (by altering its length, Art. 145) often as much as 10 per cent. without making the resonance faint.

Still greater resonance is shown by a sounding box (Fig. 62). It will resound to any tuning-fork, for it has a large number of natural frequencies, and the frequency of the fork is sure to be near one of them, but it resounds to some forks more than others. The body of a violin, however, is so very viscous that it does not seem to resound better for one frequency than another. It shows False Resonance in a predominant degree, and, though it must have natural free periods, there is none of that evident preference for special frequencies which characterises True Resonance.

84. Other Effects of Friction.—Besides Viscous Friction, which is an internal resistance varying as the *first power* of the velocity, the air and other vibrating bodies exhibit internal friction proportional to the *square of the velocity*. The effects of this are very intricate, and some will be alluded to later (Art. 85, on Combination Tones). One is this, that a body can be set in vibration by a disturbance of half its natural period, and this vibration is of the natural period of the body. Thus an air column resounds (but faintly) when excited by a fork of half its period.

85. Combination or Resultant Tones.—An elastic body or system which vibrates subject to viscous friction and is disturbed simultaneously by two harmonically altering forces of frequencies m and n will vibrate in a manner which is the resultant of only two simple harmonic displacements of these frequencies. But if its friction is not purely viscous, that is, is not proportional to the first power only of the velocity but also to the square of the velocity, the vibration is much more complicated. It has harmonic components of which the largest and most

important terms have frequencies m, n, $m + n$, $m \sim n$, $2m$, $2n$, $2m + n$, $m + 2n$, $2m \sim n$, $m \sim 2n$, $2m + 2n$, $2m \sim 2n$. The chief of these are the first four. All except m, n, $2m$, $2n$ are called Combination Tones, for their occurrence depends on both disturbances.

The component of frequency $m \sim n$ is called the Difference Tone, and was discovered by the violinist Tartini. It was supposed to be a sound produced by the beats, for the frequency of the beats would be $m \sim n$ [Art. 58]. This explanation had to be abandoned, however, when the Summation Tone $m + n$ was discovered, since no beats occur of this frequency.

Part of the mechanism of the ear has great internal friction (Arts. 98, 110), and consequently combination tones can be heard very distinctly. They probably cannot be produced in such a non-frictional system as a tuning-fork. But if a fork of frequency $m + n$ be in air disturbed by frequencies m and n, the combination tone $m + n$ is formed in the air and sets the fork into very feeble vibration that can only be detected by optical methods of the very greatest delicacy (Art. 110, 3).

EXAMPLES VI.

1. Explain *forced vibration* and *resonance*. Describe experiments to exhibit them.

2. Set up a simple pendulum with a leaden ball (or bullet) and a cotton thread (say 50 cms. long). Tie on the bullet another pendulum composed of a rounded cork and a cotton thread. Swing the combined arrangement and study what happens when length of upper pendulum is (1) greater than, (2) equal to, (3) less than, the length of the lower pendulum.

3. Show that the forced vibrations of a body are in or out of phase with the periodic force according as the period of the force is greater or less than the natural period of vibration of he body.

4. Distinguish between True and False Resonance. Which are the resonance boxes fitted to tuning-forks an instance of?

5. What are combination tones?

6. Show how a vibration may be maintained by a succession of properly timed impulses, and point out how they either may or may not affect the periodic time according to the way in which they are given. Illustrate your answer by considering (1) a clock pendulum, (2) an electrically maintained tuning-fork.

7. (Difficult.) If a body execute a simple harmonic motion in time T_1 under one straining force and in time T_2 under another, what will be its time under both forces together? Two tuning-forks with straight prongs, the lengths of which are $\frac{3}{8}$ of the length of the second's pendulum, are in exact unision and have each a frequency of 64 when held upright. If one is inverted show that when sounded together there will be about one beat a minute. (Given that the length of the simple pendulum which is equivalent to a rod suspended at one end $= \frac{2}{3}$ length of rod.)

CHAPTER VII.

FOURIER'S THEOREM.

86. Components.—We saw in Chapter I. how the rise and fall of pressure at any point can be represented by a curve. Suppose that the dotted line *ACBDZ* (Fig. 71 or 72) represents the periodic rise and fall of pressure at some point *P*, on a scale of, say, an inch to a second, the ordinate *A'A* representing the amount by which the pressure is below the average at a given moment *T*; an ordinate $\frac{1}{8}$ inch

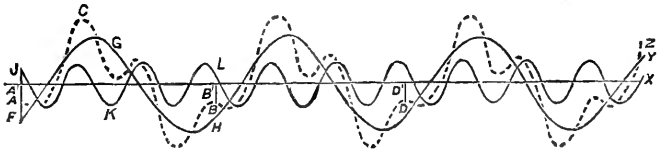


Fig. 71.

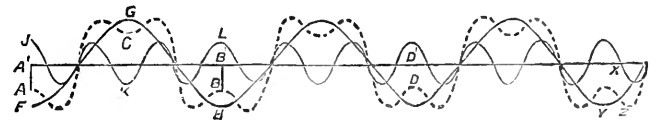


Fig. 72.

to the right of *A'A* representing the difference from the average pressure $\frac{1}{8}$ second later, and so on. Now, there is no physical difference between a force *f* and two forces, acting together, whose sum is equal to *f*. So that, instead of saying that the pressure-difference * at the moment *T* is

* The term "pressure-difference" will be used to mean the difference between the actual pressure of the air at a given moment and its average pressure.

$A'A$, we can say that it is the sum of pressure-differences represented by $A'F$ and $A'J$. So that, if $FGHY$ and $JKLX$ are any two curves whose ordinates everywhere add up to those of $ACBDZ$, there is no difference between a rise and fall of pressure represented by $ACBDZ$ and two pressure-changes, happening at the same time, represented by $FGHY$ and $JKLX$.

Pressure-changes such as those represented by $FGHY$ and $JKLX$, which add up to a given pressure-change such as that represented by $ACBZ$, are called **components** of the pressure-change represented by $ACBZ$.

In Figs. 71, 72, $FGHY$ and $JKLX$ are drawn as harmonic curves, and so represent harmonic pressure-changes, but the change $ACBZ$ could obviously be resolved into two or more component curves of different shape in an infinite number of ways. Whatever form we give to $FGHY$, we could find a form for $JKLX$ which would make $ACBZ$ the resultant. And in all these cases the effect of the pressure-change $ACBZ$ would be the algebraic sum of the effects of the two components.

Consequently we have a very wide choice of the kind of resolution which we will adopt. But in practice harmonic curves are always taken, for the following reasons. A harmonic pressure-change produces on an elastic body a vibration which is also harmonic and of its own period (Art. 82). If this period be a natural period of the body there is resonance (Art. 79), and the vibration persists, remaining harmonic of same period as its frequency dies away. Thus the harmonic components of a pressure-change produce harmonic components of the total vibration produced. Had we taken components of any other type, they would not individually produce vibrations of their own type, and the effect of each separate component could only be found with great difficulty. If some component, not harmonic, had a period equal to the free period of the body it might, or might not, cause resonant vibration; and it might cause resonance of other periods than its own.

Another advantage of harmonic components is that they seem to act specially on the sense of hearing. Each

harmonic component sets into vibration one, and only one, fibre in the inner ear. If this be not exactly the case, it is still certain that it affects this one fibre very much more than others (Art. 98). Hence we can regard the harmonic components as so many separate sensations; and by analysing the pressure-change into harmonic components we are best able to calculate its effect not only on vibrating bodies generally but specially on our sensory organs.

A consequence of these properties of harmonic vibration has already been found by mathematical analysis (Art. 82). It is that if the period of a force component be equal to that of a possible free period of the body the motion produced is of great amplitude, *i.e.* resonance occurs. The motion produced by other force components whose periods are unequal to free periods of the body are usually so very much smaller that they can be neglected in comparison.

87. Fourier's Theorem.—Figs. 26, 71, and 72 show that there are *some* periodic non-harmonic curves whose ordinates are the sums of the ordinates of the harmonic curves, so that if a rise and fall of pressure corresponds to one of these non-harmonic curves, its effect in producing resonant vibration can be easily calculated by finding the effects of pressure-changes corresponding to the harmonic curves. A very important theorem, due to Fourier,* shows that this can be done not only in some cases, but in all.

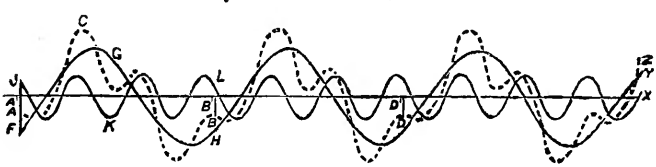


Fig. 71.

For our purpose we may state Fourier's Theorem in this way. Let there be any given periodic curve, such as the dotted line *ACB* in Fig. 71 or 72, which does not overhang

* Fourier, *Analytical Theory of Heat*, 1822.

anywhere, so that no ordinate cuts it more than once, and let A, B be two points on it. Then it is always possible to find sets of *harmonic* curves whose ordinates will add up to those of the curve ACB. And (which is the important point for our purpose), *if A and B are similarly situated points on the curve whose ordinates are equal*, there is, among the sets of harmonic curves whose ordinates add up to those of the curve ACB, *one* set (and only one) which consists entirely of harmonic curves whose lengths (the length of a double bend measured along the axis) are contained an exact number of times between the ordinates at A and B.

Figs. 71, 72 illustrate this. The ordinates at A and B are equal, and the ordinates at every point of ACB are the sums of the ordinates of the two harmonic curves FGH and JKL, whose lengths are each contained an exact number of times in the length $A'B'$. We could find other sets of harmonic curves which would add up to ACB, but there is only this one set of harmonic curves *whose lengths are contained an exact number of times between A' and B'* which will add up to ACB.

To simplify the diagram, we have illustrated a case in which this set of harmonic curves consists of only two. Usually there are more than two, and the number may be infinite.

88. Application to Sound.—Fourier's Theorem is purely a mathematical one. Its connection with sound arises from the fact that, as the pressure of the air cannot have more than one value at any one point at the same moment, any possible rise and fall of pressure can be represented by a curve which does not overhang anywhere, and that if this rise and fall of pressure is *periodic*, or exactly repeats itself at equal intervals, this curve may be divided into lengths which are all alike, so that the first and last ordinate of each length are equal.

Let us suppose, for example, that the dotted line $ACBZ$ (Fig. 71 or 72) represents a periodic rise and fall of pressure, and that ACB represents one complete cycle of changes of pressure, so that the rest of the curve consists

of repetitions of *ACB*. Let *FGH* and *JKL* be the set of harmonic curves whose lengths are contained an exact number of times between *A'* and *B'*, and which add up to *ACB*. Then, if *FGH*, *JKL* are continued to the

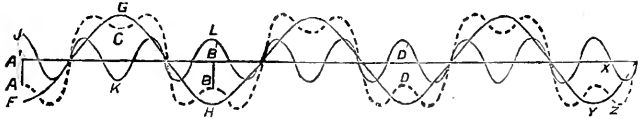


Fig. 72.

right, they repeat themselves in the next period *B'D'*, and these portions add up to the same curve as before —that is, to the form *BD*. So that, however far we continue *ACBDZ*, the harmonic curves *FGHX*, *JKLY*, also continued, will everywhere add up to the curve *ACBDZ*.

As the harmonic curves *FGH*, *JKL* repeat themselves exactly an exact number of times in the length *A'B'*, they represent harmonic pressure changes which repeat themselves an exact number of times in the period represented by *A'B'*, that is, they are harmonic pressure changes whose frequencies are exact multiples of the frequency of the non-harmonic pressure change represented by *ACBZ*. In the figure, *FGH* represents a harmonic rise and fall of pressure occurring once, and *JKL* a harmonic rise and fall of pressure occurring three times in one complete period of the non-harmonic pressure change represented by the dotted line *ACB*.

89. Harmonic Components.—If there is a non-harmonic pressure change repeating itself with frequency n, then Fourier's Theorem shows us that we can always find a series of *harmonic* pressure changes, of frequencies which are exact multiples of n (n itself being included), such that, occurring together, they are the given non-harmonic pressure change. These harmonic pressure changes are called the *harmonic components* of the non-harmonic pressure change.

It should be clearly understood that the statement that a non-harmonic pressure change of frequency n is always the sum of harmonic pressure changes of frequencies which are multiples of n does not at all imply that any actions are taking place which could cause pressure changes of these frequencies, or that these pressure changes have any separate existence. It is only true in the same sense that anything 5 feet long is the sum of two parts, one 3 feet long and the other 2 feet long; it does not mean that there is any physical distinction between the parts.

For instance, if a man rows a boat, making (we will suppose, for convenience, that it is possible) one double stroke per second, he exerts on the oar a push-and-pull whose frequency is 1 per second. This push-and-pull is not harmonic; instead of gradually rising to a maximum and then gradually diminishing, it is nearly uniform till near the end of the stroke. Fourier's Theorem shows us that it is the sum of a number of harmonic pushes and pulls whose frequencies are 1, 2, 3, 4, 5, etc., per second, happening together. This does not mean that the man is really giving pushes and pulls at these rates; he pulls continuously for more than half-a-second at a time. But if we replaced the man by a number of men, of whom one pushed and pulled harmonically once a second, one twice a second, one three times a second, and so on, then, if the intensities of these harmonic pushes and pulls, and the moments of beginning them, were rightly chosen, the push-and-pull on the oar would be the same as the non-harmonic push-and-pull exerted by the actual rower. And this is all that is meant by calling these harmonic pushes-and-pulls the components of his non-harmonic push-and-pull.

The proposition of Art. 86 shows that the effect of a non-harmonic change of pressure in producing vibration is the sum of the effects which its harmonic components (or any set of components) would produce separately. And we saw in the last chapter that a harmonic rise and fall of pressure produces *resonant* vibration in a body if the frequency of the pressure change is one of the

free-vibration frequencies of the body. Putting these facts together, we see that a non-harmonic rise and fall of pressure on a body will set it in resonant vibration if, and not unless, one of the harmonic components of the non-harmonic pressure change has a frequency which agrees with one of the free-vibration frequencies of the body.

Of course, *any* periodic pressure change on a body sets it in vibration. The statement just given relates only to *resonant* vibration. The changes of pressure, as sound-waves pass, are so small that, unless the effects of a number of changes are added by resonance, we may generally neglect the movements produced.

We now see why, in the experiment described in Art. 78, a momentary pull every 4 seconds sets a pendulum, whose free-vibration frequency is 2 seconds, in resonant vibration. The pull of the string on the pendulum rises and falls non-harmonically once in 4 seconds, and one of the harmonic components of this non-harmonic variation of force is a pull rising and falling harmonically twice in 4 seconds. We do not actually pull twice in 4 seconds, but a harmonic pull whose frequency is twice in 4 seconds, occurring at the same time with other harmonic pulls whose periods are contained exactly in 4 seconds, would be the non-harmonic pull (increasing and diminishing again very rapidly once every 4 seconds and then remaining zero the rest of the period) which the string really gives to the pendulum.

90. The Fundamental and its Harmonics.—If a non-harmonic pressure change has a frequency n, then a harmonic pressure change of frequency n is called the *fundamental* pressure change of the non-harmonic one, and harmonic pressure changes whose frequencies are $2n$, $3n$, $4n$, etc., are called the 2nd, 3rd, 4th, etc., *harmonics* of that fundamental. The harmonic components of a non-harmonic pressure change are all included among its fundamental and the harmonics of that fundamental. If the fundamental is one of the harmonic components, it is called the fundamental component.

The same method and the same terms may be applied to anything which varies with the time, and which can therefore be represented by a curve. For instance, a non-harmonic vibration of a point of frequency n is always the sum (Art. 11) of harmonic vibrations of frequencies which are multiples of n, and these harmonic vibrations are called the harmonic components of the non-harmonic vibration.

91. Harmonic Components of a Wave-System.—We may also apply the same theorem to the wave-system which causes the rise and fall of pressure. Instead of representing a pressure change, suppose now that the curve $ACBZ$ (Fig. 71 or 72) is the wave-form of any non-harmonic wave-system, so that ACB is the wave-form of one wave-length. Then Fourier's Theorem shows us that we can always find a set of imaginary harmonic wave-systems whose wave-forms, $FGHY$ and $JKLX$, add up to the wave-form of the non-harmonic system, and whose wave-lengths are contained an exact number of times in one wave-length of the non-harmonic system. These imaginary harmonic wave-systems are called the harmonic components of the real non-harmonic wave-system. Each wave-system would produce, as it passed any point, a rise and fall of pressure represented by a curve which is the same as the wave-form of the system, so that the effect of any non-harmonic wave-system is the sum of the effects which its harmonic components, if they were real wave-systems, would produce separately.

A non-harmonic wave-system, therefore, sets a body in resonant vibration if any one of the harmonic components of the non-harmonic system is one whose waves, if they were real, would arrive with a frequency corresponding to one of the free-vibration frequencies of the body.

A given non-harmonic wave-system is often said to "consist of" its harmonic components. This is rather misleading, because it suggests the idea that this particular set of components has a real existence in some different sense to other sets of components, harmonic or other, whose wave-forms would also add to that of the actual

wave-system. This is not the case; we could always find sets of non-harmonic waves, and even sets of non-harmonic waves all of the same kind, whose wave-forms would add up to the wave-form of any given wave-system, and the real waves "consist of" any of these sets of non-harmonic components exactly in the same sense as they "consist of" the harmonic ones. All "components," harmonic or other, are imaginary, and the only reason why we do not usually refer to any other set except the harmonic ones is that the effect of a succession of similar harmonic waves in producing *resonant* vibration is so easy to predict.

It is only when we want to know what *resonant* vibrations a wave-system can produce (and in one other case which will be considered later) that there is any advantage in Fourier's device of adding the effects which the imaginary harmonic components would produce separately, to find the effect of the real waves.

We shall see later that, owing to the nature of the mechanism of hearing, a separate *sensation* corresponds to each of the *harmonic* components of a system of non-harmonic waves. This of course does not in any way indicate that the harmonic components have an external physical existence, any more than the six distinct colour sensations which we receive from the spectrum indicate that there are six physically different kinds of light. It only shows that the mechanism of hearing is such that each distinct sensation depends on the *resonant* vibration of a distinct body, so that the sensations which a wave-system can cause depend on the bodies it can set in *resonant* vibration. We shall see in the next chapter how this mechanism works.

92. Harmonic Analysis.—As the harmonic components of a wave-system, and the harmonic components of the rise and fall of pressure produced by it, are represented by the same harmonic curves, to find the harmonic components of the pressure-change is equivalent to finding the harmonic components of the wave-system. If we knew the character of the rise and fall of pressure exactly, so that we could draw the curve representing it, we could find

by geometrical methods the harmonic curves which add up to that curve, and these represent the harmonic components of the pressure change. But it is not practically possible to determine either the wave-form of a series of waves or the rise and fall of pressure (represented by the same curve) which the waves produce. It is much easier to determine the harmonic components of the pressure change by finding experimentally what bodies the pressure change will set in resonant vibration. It is not necessary to have bodies of every possible free-vibration period for this. The harmonic components of a pressure change are all pressure changes whose frequencies are exact multiples of the frequency of the actual rise and fall of pressure.

If, for instance, the complete cycle of changes of pressure is repeated 100 times per second, the harmonic components of the pressure change are all pressure changes whose frequencies are multiples of 100. So that, if we expose to this rise and fall of pressure bodies whose free-vibration frequencies are 100, 200, 300, 400, etc., some of these will be set in resonant vibration, and the frequencies of these are the frequencies of the harmonic components of the actual rise and fall of pressure.

For this experiment it is evidently desirable that the bodies used should be bodies which have only one free-vibration frequency. There are very few such bodies; the best for the purpose is the air contained in a globular or cylindrical vessel which has an opening much narrower than the greatest diameter of the vessel. Such a vessel is called a "resonator" (Art. 97).

The determination of the harmonic components of a pressure change, vibration, wave-system, etc., is called the *harmonic analysis* of it (see Arts. 97 and 103).

The harmonic analysis of curves may be effected mathematically by means of infinite trigonometrical series* or by mechanical rules deduced from these.†

* See Byerly's *Fourier's Series and Spherical Harmonics* (Ginn and Co.), Ch. II., and Gibson's *Treatise on Graphs* (Macmillan), Ch. VII.

† See Watson's *Text-book of Practical Physics* (Longmans and Co.), § 11, and Gibson's *Introduction to the Calculus* (Macmillan), Ch. XI., for an account of Runge's method.

93. Relation of Harmonic Components to Source of Waves.—In some cases, as shown in Art. 54, a non-harmonic wave-system is produced by two or more sources, each of which alone would produce a harmonic system. In this case, as we saw, if the vibrations are of small amplitude, the wave-form of the actual waves is the sum of the harmonic wave-forms of the wave-systems which the sources would produce separately. These harmonic wave-systems are therefore the harmonic components of the actual wave-system.

Even where there are not any sources which really vibrate harmonically, the harmonic components of a wave-system, though quite imaginary, are often similar to real harmonic wave-systems which might be produced by the actual source. It oftens happens that a body vibrates so that every part of it moves harmonically in the same period; in this case it usually produces nearly harmonic waves. If there are several different ways in which the body could vibrate harmonically, with different periods, there are several harmonic wave-systems of different wave-lengths which it could produce. If such a body vibrates in any other way, it is usually one in which its movement is the *sum* (Art. 11) of two or more of the harmonic vibrations which it could perform; in this case the waves from it have a wave-form which is the sum of the wave-forms of the harmonic waves which these vibrations would produce, so that these harmonic waves are the harmonic components of the actual wave-system.

In these cases the harmonic components of the waves produced are similar to real waves which might be produced by the same sources. It must not be supposed that this is always the case, still less that the possibility of finding imaginary harmonic systems whose wave-forms would add up to that of the waves from a given source depends at all on whether the source of sound could vibrate so as to produce such harmonic systems. In many cases there is no relation between the harmonic components of a wave-system and any real harmonic waves which the source could possibly produce. For instance, if air is blown from a pipe against a revolving disc having 100 holes

drilled in a circle at the same distance from the axis as the end of the pipe, a puff of air will come through each hole as it passes—100 puffs per second if the disc rotates once in a second. At the disc, every process which occurs at all is repeated 100 times a second, and not oftener; there is no operation being repeated 200 or 300 times a second. A hundred similar waves, each about 11 feet long, start from the disc in each second. These waves are far from harmonic in character, the pressure of the air on the side of the disc away from the pipe rising very suddenly as a hole comes opposite the pipe and falling very suddenly when the hole has passed, instead of the gradual harmonic rise and fall. But the wave-form of these waves, as Fourier's theorem shows, might be made by adding the ordinates of the wave-forms of harmonic waves 11 feet, $\frac{11}{2}$ feet, $\frac{11}{3}$ feet, etc., in length. So that harmonic waves of these lengths are harmonic components of the waves from the disc, although to produce real harmonic waves $\frac{11}{3}$ feet long would require a movement repeating itself exactly 300 times in each second, and nothing that happens at the source is repeated with this frequency,

94. Change of Components.—As stated in Art. 51, intense waves change their form a little as they travel. At some distance from the source their form cannot be made by adding the same harmonic curves which add up to their form near the source. In other words, the harmonic components change as the waves travel; the components at a distance are not the same as near the source. But if the fundamental is one of the components at one place it always is so at any other place.

95. Different Wave-systems with same Components.—As shown in Figs. 71, 72, the same harmonic curves, in different relative positions, add up to quite different forms. So that two wave-systems, of quite different wave-forms, may have the same harmonic components, in different relative positions. Two such wave-systems set the same bodies in resonant vibration, and therefore cannot be distinguished from each other by merely observing which of a series of resonators they excite.

In this chapter we have considered chiefly the rise and fall of pressure on a body as waves pass it, because it is to changes of pressure that the resonant vibration of the body is directly due, and the chief use of Fourier's device is to determine what bodies will be set in resonant vibration. But the vibration of the particles of the air as the waves pass may also be represented by a curve, and may therefore be considered as the sum of a number of harmonic vibrations, of frequencies which are multiples of the frequencies of the actual vibration. The curve which represents the movement of a particle of air (the displacement curve, Art. 4) is not the same as the curve which represents the rise and fall of pressure in that particle, which is the same as its velocity curve (Art. 18). The two curves, however, can always be made by adding the same harmonic curves in different relative positions. So the harmonic components of the vibration have the same frequencies as the harmonic components of the change of pressure.

The *constitution* of a non-harmonic pressure change means the frequencies and amplitudes of its harmonic components without reference to their relative phases, and the same term is applied to vibrations, wave-systems, etc. So that the vibrations, pressure changes, or wave-systems represented by the dotted curves in Figs. 71 and 72 are said to have the same constitution, though not the same character.

By the aid of Fourier's Theorem we can often simplify problems in interference. Thus in the first experiment of Art. 55, instead of saying that absolute silence can only be produced if the waves are symmetrical, we can say that it can only be produced if the difference between the paths *ABD* and *ACECD* is an odd number of half wave-lengths for each component of the actual waves, which obviously can only occur if the components are all odd harmonics of the fundamental. By comparing Figs. 71, 72, 26, we see that waves with only odd harmonic components are symmetrical, the wave-form of the rarefaction being that of the condensation inverted, but that this is not the case if an even component is present.

96. Wave-form.—No accurate method seems to be known of determining the wave-form of an undulation. The phonautograph described in Art. 68 gives traces which are different for waves of different character, and these traces are sometimes loosely spoken of as the wave-forms, but neither this method nor any method which depends on the yielding of a membrane can give the wave-form as defined in Art. 18. If a membrane could be massless and perfectly loose, it would move exactly as the air moved, and might be made to trace a displacement curve of the movement of the air. If a membrane could be stretched infinitely tightly, its displacement at each instant would be proportional to the pressure on it at that instant, and it might be made to trace a true pressure-curve or wave-form, though the amplitude of this would be indefinitely small. The movement of any real membrane is a compromise between them, modified considerably by the mass of the membrane and its liability to be thrown into resonant vibration if any of its own very numerous modes of free vibration agree in frequency with any of the harmonic components of the waves reaching it.

The phonograph (Art. 69) reproduces quality very nearly.

The wave-forms of undulations then cannot at present be determined with any great accuracy, and are, in fact, unknown, except in the regions close to large vibrating surfaces, where they cannot differ much from the velocity curves of the surfaces themselves. We can, however, determine, by the method explained in Art. 91, the frequencies of the harmonic waves whose wave-forms would add up to that of the actual waves. That is not at all the same as knowing the actual wave-form, because we cannot find either the relative amplitudes or the relative phases of these harmonic waves at all accurately, and harmonic waves of the same frequencies, but of different amplitudes and relative phases, may add up to quite different wave-forms. Still, for many purposes, to know the frequencies of the harmonic waves whose wave-forms would add up to the actual wave-form is as useful as to know the wave-form itself, and this can be ascertained by means of resonators.

97. Resonators.—A resonator is a globular or cylindrical box (Fig. 73) with a wide hole at one end, and a small one, fitted with a tube, at the other. Its frequency of free

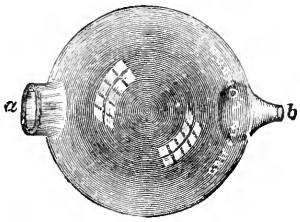


Fig. 73.

vibration is found by blowing across the wide hole, and determining the frequency of the sound produced, by the siren, phonautograph, or other method. This frequency can be altered by slightly altering the size of the larger hole, for a tube whose diameter is diminished at an open end has a longer vibration period than an ordinary open tube (see Exp. 61). In the case of cylindrical resonators, which are made in two pieces to slide one inside the other like the joints of a telescope, the frequency can also be adjusted by altering the length. A set of such resonators is prepared whose free-vibration frequencies are exact multiples of the frequency of the waves to be analysed, and, if the tubes from the smaller openings of these resonators are placed in turn in the ear, we hear a loud sound from those whose free-vibration frequencies correspond to those of the harmonic components of the arriving waves.

As the components can only be detected by this method one at a time, it is only suitable for the analysis of sounds which can be produced continuously. For those of short duration we must be able to determine which resonators are vibrating at a given moment. For this purpose the smaller opening of each resonator is fitted with a manometric capsule and the flames from all the capsules are arranged close one above another in front of a rotating

mirror, as in Fig 74. The appearance of the flames in the mirror shows which resonators are vibrating. In this way the analysis can be shown to a large audience.

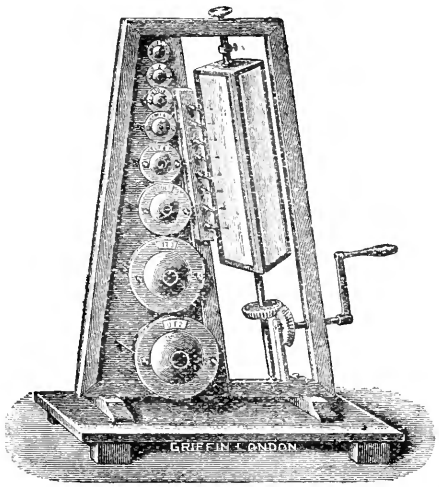


Fig. 74.

This method is not suitable for the detection of feeble harmonic components in a loud sound, as, on account of the small mass of the air in a resonator, a strong component of a wrong frequency will produce more vibration than a feeble component of the right frequency, so that the presence or absence of the latter cannot be ascertained. Tuning-forks are much better detectors of weak components, since the forced vibration produced in a fork by even a very strong component of different frequency cannot be detected. The resonant forced vibration produced by a weak harmonic component of the same frequency as the fork is small, but it can be detected by a device which calls in the aid of the Interference of Light (see Art. 110, 3).

EXAMPLES VII.

1. Explain carefully what is meant by the harmonic components of a non-harmonic wave-system.

2. Could the wave-form of a non-harmonic system be built up out of components similar to one another which are not harmonic? If so, state generally what is the advantage of supposing it built up of harmonic components.

3. What is the condition necessary in order that a body may be thrown into resonant forced vibration by a non-harmonic wave-system?

State clearly the difference between the effect of a force which varies harmonically, and one that does not.

4. State Fourier's theorem, and apply it to analyse a compound curve. Draw the curves.

CHAPTER VIII.

THE EAR AND HEARING.

98. Structure of the Ear.—The rise and fall of pressure which occur as sound waves arrive at the ear, produce, by their effect on its structures, the sensation of sound, and the nature of this sensation depends on the nature of the rise and fall of pressure; in other words, on the character of the sound waves. We may, therefore, learn much about sound waves from the sensations they produce. But, to understand what physical facts about the waves are indicated by different sensations, we must know something about the structure of the ear.

This apparatus is a very complex one, but it is constructed on quite simple principles. The external ear, or *pinna*, has, in ourselves, no connection with hearing, which is not affected by its removal; but in many animals it is funnel-shaped, and it then serves to increase the intensity of the vibration, in a way explained in the next chapter. A funnel-shaped pinna is also useful for ascertaining the direction from which the waves come. From the pinna a tube, called the *meatus*, in man about $1\frac{1}{4}$ in. long, leads directly inwards, and is closed at the inner end by a stretched membrane, the *membrana tympani* or *drum-skin of the ear*. This membrane is connected by a chain of three small bones, the *malleus*, *incus*, and *stapes*, or the *hammer*, *anvil*, and *stirrup*, to another membrane, the *fenestra ovalis*, which forms part of the wall of the hearing chamber, or *cochlea*.

The principle of this hearing chamber may be understood if we imagine a long, low, narrow building of two storeys (a ground-floor and one above it), having at one

end two doors, one into the ground-floor, the other into the upper storey. The floor which separates the storeys is not complete; it does not extend quite to the further end of the building from the doors, and along one side of the building it does not reach, as a solid floor, up to the wall; along this side, however, but not at the end, it is completed by two sheets of membrane, the lower one completing the ceiling of the lower room, the upper one the floor of the upper one; between these membranes there is a space, which is entirely shut off from the rooms, since the membranes meet and join at the end furthest from the doors. The lower and upper rooms are in free communication at the end furthest from the doors, where the floor between them is entirely wanting. The doors are thin and flexible. The whole space inside the building is filled with fluid.

The upper membrane is thin and simple in nature. The lower one broader at one end than at the other, and consists of a series of tightly stretched parallel fibres embedded in tissue. The membrane is but loosely stretched parallel to its length so that these fibres are free to vibrate almost independently of their neighbours.

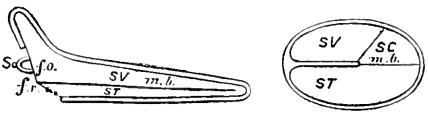
In the space between the two membranes there are several (four to six) thousand rods of different lengths; these stand on the lower membrane, in two parallel rows running its whole length; they are fastened at their lower ends to the membrane on which they stand. The rods in each row lean towards the other row, and at their upper ends each touches, and is joined to, its fellow in the other row; each such pair is free except where the lower ends are attached to the lower membrane. The whole series forms "a sort of gable roof."

If in such a building the thin material of the door of the upper storey was pushed in, a wave, very similar to an ordinary water wave on a canal, would travel all along the upper room to the further end, depressing all portions of the membranous part of the floor in turn as it passed along; as the liquid is incompressible, the membranous ceiling of the lower room would be forced down at the same time, and the flexible door of the lower room would be forced out. Every movement of the door of the upper room

produces a movement of the membranous part of the ceiling of the lower room and of the rods supported by it, and if the door is made to vibrate regularly, any fibre in this membrane whose natural vibration period corresponds will be set in resonant vibration.

In the actual ear, a chamber constructed on a principle similar to the building above described is coiled into a spiral so close that one side of the building, where the solid floor is complete, becomes a mere central pillar. The chamber resembles a snail-shell in form; hence the name *cochlea*. The upper door is the *fenestra ovalis*, already mentioned; the other is called the *fenestra rotunda*. The rods are called the *fibres or rods of Corti*. The membrane to which they are attached is called the *membrana basilaris*. A nerve-filament from the brain is connected to each pair of rods.

If the *cochlea* was uncoiled and straightened out, a longitudinal section of it would be something like Fig. 75 (*a*), and a transverse section something like Fig. 75 (*b*); but these figures merely indicate the general arrangement, not the proportions of the parts.



(*a*) Longitudinal section. (*b*) Transverse section.

Fig. 75.

SV, *ST*, *Scala vestibuli*, and *Scala tympani*, the two storeys; *f.o.*, *fenestra ovalis*, the oval window; *S*, *stapes*, the last of the chain of bones connecting the *fenestra ovalis* to the *tympanum*; *f.r.*, *fenestra rotunda*, the round window; *SC*, *Scala cochleae*, the space between the membranes; *m.b.*, *membrana basilaris*, to which the rods are attached.

When sound waves reach the *tympanum*, it vibrates, and its movements cause corresponding, but smaller, movements of the *fenestra ovalis*, with which it is connected by the linkage of bones. These movements are transmitted to the liquid which fills the *cochlea*, and thus to fibres of

the *membrana basilaris*. The sensation of sound depends on the vibration of these fibres; usually on their resonant vibration, since ordinary sound waves do not produce sufficiently great vibration unless the effects of a number of waves are added by resonance. There is no way of ascertaining whether the resonant vibration of each fibre produces a separate sensation, because we cannot make one fibre vibrate alone. As the fibres are attached to a membrane they move with considerable friction, and the resonant vibration caused by the movements of the liquid is not much stronger in the fibre whose free-vibration period exactly corresponds than in a number of others whose periods are not very different (Art. 80). Harmonic waves, therefore, set in resonant vibration not one fibre, but a group of consecutive fibres. The resulting sensation is, however, always a simple one; it is only when two groups of fibres vibrate, while intermediate ones do not, that we have the sensation of hearing two sounds at once, and this cannot be caused by harmonic waves.

It is not, however, universally admitted that the ear acts by resonance. The *telephonic theory* is that each wave, when it reaches the basilar membrane and its associated structures, sends a nervous impulse to the brain, and that the sensation depends on the frequency with which these impulses are received. It will be seen that this view differs radically from the resonant theory. On the resonant theory periodicity extends only as far as the basilar membrane; the nervous impulse sent to the brain is not periodic, and the pitch of the sensation depends on which nerve-fibre is stimulated, not on how it is stimulated, so that if the nerve-fibre whose natural frequency is 200 could be stimulated in any other way, for instance by a non-periodic stimulus such as an electric current, we should hear the note which is usually produced by 200 waves per second, though nothing would really be repeated with that frequency or repeated periodically at all. On the telephonic theory the nervous impulses transmitted to the brain have the same frequency as the waves, and the same sensation depends on the frequency of these nervous impulses, so that no non-periodic stimulation could produce a sensation of definite

pitch. As the telephonic theory does not profess to explain any of the observed relations between waves and sensations of sound, it hardly admits of experimental proof or disproof, but the resonant theory is more in accordance with analogy, since in other cases the stimulation of a particular nerve produces a particular kind of sensation, irrespective of the nature of the stimulus; a blow on the eye produces the sensation of light.

In the following pages we speak of the sensation as due to the resonant vibration of the fibres of the basilar membrane, without meaning to imply that this theory is the correct one.

A study of the structure and action of the drum-skin of the ear shows that it is well adapted to receive and respond to longitudinal wave motion, and that it probably can, to a limited extent, be adjusted in sensitiveness for response to waves of different types. It should be noticed that the drum-skin responds, not to the displacement of the air particles in the incident wave, but to the changes of pressure which attend the cycle of compressions and rarefactions which constitute a longitudinal wave. When the air adjacent to the outer surface of the drum-skin is compressed the *increase of pressure* forces the membrane inwards, and when the air is rarefied the *decrease of pressure* allows the membrane to move outwards. The amplitude of motion of the drum-skin is however very small, probably not more than a tenth of a millimetre at most, and as small as a millionth or a ten-millionth of a millimetre for very faint but audible sounds.

99. Characteristics of a musical sound.—Musical sounds differ from each other in three essential particulars. These are, *loudness*, *pitch*, and *quality*. The meaning of loudness and pitch as applied to musical sounds is generally understood.

Loudness is characteristic of all sounds, whether musical sounds or noises, and is applicable, in the general sense, to sounds of all degrees of loudness from the slightest and softest to the loudest. It relates to the magnitude of the sensation produced.

Pitch is applicable only to musical sounds, and refers to the characteristic which differentiates the notes of a musical scale. As we go up the scale the pitch of successive notes rises, and as we go down the pitch of the notes falls. Low bass notes are said to be of low pitch and high treble notes of high pitch. When a note is sharpened its pitch rises slightly, and when it is flattened its pitch falls slightly.

Quality, in its technical sense, is the characteristic of a musical sound which, quite apart from difference in pitch or loudness, enables us to distinguish, by the general character of the sound, between sounds of different instruments. Thus two notes, one produced by a tuning-fork and the other by the string of a violin, may be of exactly the same pitch and loudness, but they are of very different quality or *timbre*.

100. Pitch.—The sensation differs according to the frequency of the vibration. This difference is called a difference of *pitch*, and the sensations produced by harmonic waves of greater and smaller frequency are said to be sensations of *higher* and *lower* pitch respectively. There seems no relation between the ordinary meanings of the words "high" and "low" and the sensations produced by waves of different frequencies arriving at the ear; probably the names are derived from the sensations felt in the vocal organs in producing such waves.

Strictly, the term pitch is applied only to the sensation, not to the vibration which produces it, but the term is often loosely used as equivalent to frequency.

To show that pitch depends on frequency the following experiments may be performed.

Exp. 31.—Stretch several strips of corded silk (ribbons) of different "grain" on a thin piece of wood (the cover of this book is a corded cloth). Place a finger-nail lightly in contact with the silk and run it quickly along the strip. Observe that the more rapidly the nail moves the higher the note heard; also if it is passed with about the same speed over different strips, then the note is of higher pitch when the cording is fine than when it is coarse.

By moving the nail over the cording a quick succession of taps is produced, and the observations show that the more rapid the taps the higher the pitch.

Exp. 32.—Set a "cardboard" siren and a Savart wheel in action and arrange so that the speed of revolution can be easily varied. It will be found both with the wheel and the siren that when the frequency changes the pitch changes. When the frequency is increased the pitch rises, when it is decreased the pitch falls, and when it is constant the pitch is constant.

The relation between pitch and frequency is very satisfactorily illustrated by the apparatus* shown in Fig. 76.

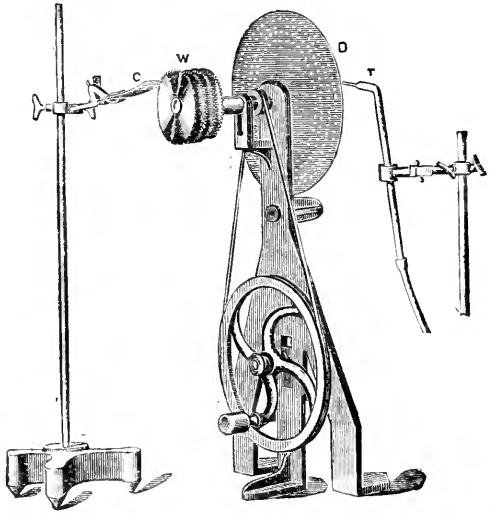


Fig. 76.

Four Savart wheels, *W*, and a siren plate, *D*, pierced with four concentric rings of equidistant holes are fitted on the same spindle and geared so as to be rotated at any required speed by means of the driving wheel shown in the figure. The number of teeth on the wheels and the number of holes in the rings of the siren plate are in the ratio 4 : 5 : 6 : 8, the number of teeth on any one wheel being the

* See Poynting and Thomson's *Sound*, p. 9.

same as the number of holes in the corresponding ring of holes in the plate. A card or thin piece of vulcanite *C* is arranged so that it can be pressed lightly to any one of the four wheels, and the tube *T*, through which a jet of air can be directed on to the siren plate, is fixed so that it can be adjusted to any ring of holes in the plate.

Exp. 33.—With this apparatus adjust the card to *any one* of the toothed wheels and the jet of air to the corresponding ring of holes. It will then be found that at whatever speed the apparatus may be revolved the pitch of the two notes produced is always the same, although the quality of the notes is very different. It will also be noted, as in Experiment 32, that as the speed of revolution is varied the pitch for each note rises and falls in exactly the same way. That is, so long as the frequency is the same for two notes, however differently they may be produced, the pitch is the same.

Exp. 34.—With the same apparatus, while the speed of revolution is practically constant, touch each of the four wheels in rapid succession with the card, beginning with the one with the smallest number of teeth. The succession of notes produced will be at once recognised as the common chord for the note of lowest pitch. In the same way, if the jet of air is directed in rapid succession against each of the four rings of holes, the same sequence of notes will be produced.

The pitch of a musical note is thus determined by the frequency of the source. When the frequency is constant the pitch is fixed and definite, and when the frequency changes the pitch changes. Pitch is therefore the essential characteristic of a musical sound, for, being determined by the frequency, it is directly associated with the periodicity of the note. A noise, being non-periodic in character and of no definite frequency, cannot be said to have pitch.

It is found, however, that the sensation of pitch is produced by a very slight persistence of periodicity. The incidence of two or three complete wave lengths in direct succession on the ear is sufficient to establish the sensation of pitch. Hence, if a vibrating body executes only two or three complete vibrations, the sound heard may be a note of well-defined pitch.

Exp. 35.—Construct a cardboard siren plate and mark it for five concentric rings of holes with the same number of holes for each ring. Cut out (with a cork borer) two consecutive holes in the innermost ring, four in the next, six in the next, and ten in the outermost ring. Mount the card for rotation, and as it rotates let the jet of air be directed on each of the incomplete rings of holes and notice the pitch of the notes produced. It will be found that, at any given speed, the pitch is the same for each ring, and that, while the pitch is more definite the greater the number of holes, it is fairly clearly defined even for the innermost ring, where the periodicity of the note is determined by the sequence of only two complete vibrations.

There are two ways of stating the pitch. The physicist states it in terms of the frequency, the musician in terms of the place of the note in a musical scale. See Art. 107.

101. Limits to Audible Sound.—There is an upper and a lower limit in pitch for audibility. Methods of obtaining the lower limit of audition are open to the objection that, unless we are certain that the very low note produced is pure, we may be hearing its harmonics; hence the low numbers given by some writers are questionable. Savart, using a rotating rod to send pulses through the air, found the frequency of the lower limit was about 8. Helmholtz disputed this, using the reason above; he himself used a double siren and also an adjustable tuning-fork, and found that the frequency of the lower limit is about 27. (This is the frequency of the lowest note on an ordinary piano.)* Below that the vibrations are heard separately.

Exp. 36.—Take a large tuning-fork constructed so that it can be adjusted by sliding weights on the prongs to a frequency varying from 25 to 35 per second. It will be found that when the fork is excited the note is quite inaudible at the lower limit of its range, but distinctly audible at the higher limit. It is somewhat difficult, however, to determine the exact frequency at which audibility occurs.

The upper limit has been found by Appunn and Preyer, who used small tuning-forks, and by Rayleigh, who used

* It is sometimes stated that the waves, when very slow, are heard as separate shocks, but there seems no evidence of this, at any rate in the case of harmonic waves.

artificial bird-calls (see Exp. 56). It varies within wide limits for different individuals, extreme frequencies of 16,000 and 40,000 being found with an average of about 20,000. Above a frequency of 4,000 the note can hardly be considered musical. The frequency of the highest note on an ordinary piano is about 3,500.

Galton's whistle (Fig. 77) is of great service in determining the upper limit of audibility. It is a miniature organ pipe fitted with a screw piston. By turning the screw around the volume of air in the whistle can be altered and the pitch of the note given out changed. The frequency for any given position is read from graduations on the screw.

Fig. 77.

Mr. Galton also investigated the upper limits of audition in animals by observing whether they noticed a high note when produced near them. In this way he found that the limit for many animals is higher than for mankind, and that the cat tribe has especially high hearing powers.

The ear is very sensitive to differences of frequency between sounds of frequencies such as are commonly produced by the voice; a change of $\frac{1}{4}$ per cent. in the frequency of such sounds is easily detected. Outside the limits of the voice, the sensitiveness of the ear to changes of frequency is much smaller.

102. Loudness.—Differences of intensity in the waves reaching the ear produce different loudness, or quantity of sensation; but no simple relation between the intensity and the loudness can be stated, all sensations, indeed, not admitting of quantitative measurement. Of two wave-systems of the same frequency and wave-form, the one of greater intensity sounds the louder, but this is not necessarily the case if the frequencies or wave-forms are different.

Waves of very low and very high frequency (near the limits of audible frequency) sound not nearly as loud as waves of the same intensity which are nearer the middle of the audible range.

The intensity depends upon the amplitude of vibration, the frequency, and the density of the medium; being proportional to the squares of the amplitude and frequency and to the density of the medium (Art. 27).

Exp. 37.—Set a tuning-fork or a violin string in vibration, and note that the greater the amplitude of vibration the louder the sound heard by an observer stationed at a fixed point.

Exp. 38.—Suspend a small bell in a large flask fitted so that it can be filled, as required, with air, water vapour, coal gas, hydrogen, carbon dioxide, or other gas. When the bell is shaken it will be found that the loudness of the sound heard varies with the density of the gas, being greater the greater the density.

The loudness of the sound emitted by a vibrating body may in some cases be greatly increased by the action of a *sounding board.** When a tuning-fork, for example, is sounding, the loudness of the sound emitted may be very greatly increased by resting the end of the fork handle on a piece of wood, such as a light table top or box cover. This is explained by the fact that the vibration of the fork sets up, as explained in Chap. VI., forced vibration in the wood particles, and the surface over which energy is communicated from the fork to the medium being thus greatly increased, the intensity of the wave motion at any point in the medium is proportionately increased and the sound greatly intensified. The duration of the sound is, however, proportionately decreased, for the initial energy of the vibrating fork is approximately constant and the rate of radiation of energy is greatly increased.

No accurate method is known of measuring the amplitude of the vibrations of gases; such estimates as those in Art. 27 are arrived at by assuming that all the energy observed to be expended in keeping the source in vibration is converted into sound, and determining the amplitude of vibration necessary at any given distance from the source

* The use of a sounding *box* involves also the principle of resonance. A *board* involves false resonance. See Arts. 80, 83.

in order that the air at that distance may transmit energy at the same rate as it leaves the source. The true amplitude, of course, cannot be greater than this, but it is on theoretical grounds that it is believed to be not much less.

It has been calculated from experimental data by Lord Rayleigh that for a sound that is just audible the amplitude of vibration of the air particles is about a ten millionth of a centimetre (10^{-7} cm.), or a millionth of a millimetre; or ten tenth-metres (Angström units). (See p. 62.)

103. Quality.—Non-harmonic waves produce in the ear the same effect as on any other system of resonators; that is, they set in resonant vibration all the fibres whose frequencies of free vibration correspond with the frequencies of any of their harmonic components. The resulting sensation differs very much in different persons, and even in the same person, according to the amount of attention he gives. A person who has been musically trained, and listens attentively, will generally be conscious of hearing, at the same time, the different sounds which real harmonic waves, similar to the harmonic components, would produce separately. With less training or attention, only a single sensation is perceived, which usually seems to be of rather higher pitch than would be produced by harmonic waves of the same frequency, but is not exactly like the sensation produced by harmonic waves of any frequency. There is then a difference between sensations produced by waves of different forms, even if they are of the same frequency, and this is called a difference in the *quality* of the sound. The terms *character*, *timbre*, *clang-tint*, are used by different writers as equivalent to quality.

It is this difference in the quality of the sensations which enables us to distinguish between continuous notes of the same frequency produced by different instruments, a flute and a harmonium for instance, though we may be quite unable to say which is of higher pitch. But where the sound is not continuous we are aided in our judgment about its origin by the way in which the sound begins and ends.

Two wave-systems which are exactly similar in frequency, amplitude, and wave-form must give rise to the same sensation, for there is no respect in which the two can differ. But it does not follow that two wave-systems of different wave-form will produce different sensations.

Quite different curves may be produced (Art. 95) by adding the ordinates of the same harmonic curves in different relative positions, and waves of these different forms set the same resonators in vibration. If the fibres of the basilar membrane were *independent* resonators, it is probable that two wave-systems of different wave-forms would produce exactly the same sensation, if their wave-forms were such as might be formed by adding the same harmonic curves in different relative positions. If this was the case, we could say that the quality of the sensation produced by non-harmonic waves depended on the frequencies and amplitudes, but *not* on the relative phases, of their harmonic components.

It was Helmholtz's view that this actually was the case, and he designed the following experiment to prove it. He had 13 tuning-forks, whose frequencies were in the ratios 1 : 2 : 3 : 4 : etc., mounted each in front of a resonator (Art. 97) of corresponding frequency; the openings of these resonators could be closed by sliding doors. All the forks could be kept in continuous vibration by the same momentary electric currents passing at regular intervals round electro-magnets placed close to the prongs; the interval was an exact multiple of the period of the slowest, and therefore of every other, fork; and, as the force varied non-harmonically, it produced resonant vibration in them all (Art. 79). The sound of any fork was hardly audible, except when the door of its resonator was open. Helmholtz first determined, by means of the resonators described in Art. 97, the harmonic components of some sound of marked and peculiar quality (say that of a trumpet) which was of the same frequency as the slowest of the forks. He then set all the forks in vibration, and found that the sound from the open resonators vibrating together was similar in quality to that of the trumpet. Now in this experiment the harmonic waves from the resonators would

not, unless by accident, be in the same relative positions as the harmonic components of the waves from the trumpet; there was, therefore, usually no resemblance between the wave-forms of the two wave-systems. The fact that the sensations were similar seemed to prove that waves of different form produced the same sensation if they could be made of the same harmonics in different relative positions.

This experiment is often described as the synthesis of a given non-harmonic sound. It is, however, only the sensation, not the wave-system, which is reproduced, so that the experiment is purely a physiological (and psychological) one. There is no physical resemblance whatever (except in length) between the original waves and Helmholtz's copy of them, and if we *saw* sound waves instead of hearing them, it would be obvious that they were totally unlike. The apparent resemblance is due simply to the very imperfect way in which the ear distinguishes between waves of different form, just as the apparent resemblance between the white lights produced by adding different pairs of complementary colours is due to the imperfect way in which the eye distinguishes between non-harmonic light waves.

Though Helmholtz failed to detect any difference in the sensations produced by the trumpet and by the forks, it is not impossible that resonant bodies which, like the fibres of the basilar membrane, are all connected to the same membrane may vibrate differently according to the relative phases of the different harmonic components, and though Helmholtz has shown that this difference will be small, there is some experimental evidence to show that waves having the same harmonic components in different relative positions are sometimes distinguishable by the ear. It is therefore best to state that the quality of the sensation depends on the wave-form of the waves which produce it, but that waves of different wave-form produce *nearly* the same sensation if there is no difference except in relative position (phase) between the harmonic curves of which the different wave-forms can be built up. The sensation produced by harmonic waves is by some called a

tone, that due to non-harmonic but periodic waves a note. A note may be regarded as a number of tones heard at the same time.

The property of a wave which determines the quality of the sound is most clearly exhibited by means of the displacement or strain curve for the wave.

Fig. 78.

Figures 78, 79 represent the displacement curves for the waves of two musical sounds of the same pitch but of quite different quality. The wave length is the same for the two curves, but the character of the sequence of states which make up a wave length, *as shown by the* form *of the curve,* is different. The *quality* of a sound is thus indicated by the *form of the displacement curve* or strain curve for the wave, and, for this reason, by a transference of terms, the quality of a sound is sometimes said to depend upon the *form* of the wave.

Fig. 79.

The curve in Fig. 78 represents approximately the sound wave produced by the vibration of a tuning-fork. It differs little from the simple sine curve representative of a simple harmonic wave. The curve in Fig. 79 represents the sound wave produced by the vibration of a violin string. The curve is strictly periodic, but differs widely from the simple harmonic form.

These two curves differ essentially in *form,* and the two sounds they represent differ essentially in *quality.* The first, represented by a simple harmonic curve, is said to be a tone; the second, represented by a curve which is really compounded of a series of simple harmonic curves, is said to be a note formed by the blending together of a series of simple tones.

104. Koenig's Wave Siren.—By substituting for a disc with holes a disc with its edge cut in harmonic waves, the siren may be made to give more nearly harmonic sounds. In this case the air is blown from a single radial slit, which is entirely or only partly covered by the disc, according to the position of the latter. The amount of air issuing from the slit varies nearly harmonically if the teeth of the disc are of harmonic form, so that the waves as they start are approximately harmonic. They become less so, however, as they proceed, so that the difficulty of beats between the harmonic components is not entirely overcome.

By cutting the edge of such a disc into waves or shallow teeth of different forms, waves of different wave-form can be produced, and M. Koenig showed that discs whose edges are cut into different curves which have the same harmonic components (Art. 95) may produce sounds of different quality, and he considered that this disproved Helmholtz's view (Art. 103) that the quality of the sound due to wave-systems having different forms but the same components is the same. M. Koenig's experiment, however, in no way disproves Helmholtz's view unless we assume that the harmonic components of the waves produced are the same as the harmonic components of the form of the teeth of the disc, and there is no reason to believe that this is the case. There are, however, more satisfactory arguments against Helmholtz's view.

105. Noise.—All the sounds we hear are, however, not musical sounds. Sounds other than musical sounds vary greatly in character, but may all be included under the general term *noise*. The essential characteristic of a *noise*, as distinguished from a musical sound, is irregularity or *want of periodicity*. Any source in *non-periodic* motion giving rise to an irregular wave disturbance with sufficiently rapid variations of pressure may produce a noise of a character determined by the nature of the disturbance which reaches the ear. The wave disturbance associated with a noise will be of no definite wave length, the sequence of states of condensation and rarefaction which constitute the disturbance will follow no regular law, and, as there is no definite wave length, there will be no recurrence of a definite cycle of states, wave length after wave length, as in the case of the periodic wave associated with a musical sound.

Exp. 39.—Press a card against a revolving wheel notched at the edge with irregular teeth of different sizes. Note the difference between the sound given out in this experiment and in **Exp. 32.**

Some noises, however, are on the borderland of musical sounds, for they give rise to a short note of definite pitch. This pitch is appreciated more if the noises are graduated to give the musical sounds, say, of the common chord or the full diatonic scale. Thus if pieces of wood or glass rod cut to certain dimensions (Exp. 91) are thrown in succession on a stone or hard table the musical sound is very evident. Other examples are the sounds made by the drops from dripping taps, and the withdrawing of corks from bottles.

It should be noticed that the originating disturbances in the case of most noises are not the large and sometimes violent air displacements which accompany the production of the sound, but very rapid movements of very small amplitude or rapid changes of pressure produced somewhere at the source of the sound. Thus, when a piece of wood is struck with a hammer, the source of sound is not the air displacement produced by the moving hammer, but the rapid irregular vibration of the surface particles of the wood, caused by the stroke of the hammer. The sound produced by clapping the hands is caused, not by the motion of the hands in the air, but by the sudden expulsion of the air from between the hands at the instant of contact. The noise of an explosion is produced, not by the violent displacement of the air at the centre of explosion (the " wind " of the explosion), but by the compressions and rarefactions communicated to the surrounding air by the sudden changes of volume in the explosion region. The sound of thunder is caused by the sudden expansion and the subsequent sudden contraction of the intensely heated air along the track of the lightning flash.

All these sources of disturbances, however irregular they may be in action, set up wave disturbances of irregular non-periodic type in the air, and the incidence of these disturbances on the drum of the ear gives rise to sound sensations or noises characteristic of the disturbances.

The difference between a musical sound and a noise is well exhibited by means of the displacement or strain curves of the wave disturbance associated with these sounds. Figs. 80, 81 evidently represent the displacement curves for musical sound waves. The

curves are periodic in character, and, although the waves they represent evidently differ in wave length and in type (as indicated by the form of the curve), they are both characteristic of musical sounds.

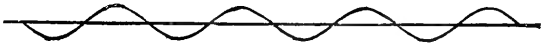


Fig. 80.

Fig. 81.

Fig. 82, on the other hand, represents a wave disturbance such as would be associated with a particular kind of noise. If the strain or pressure curve for this displacement curve be drawn as explained

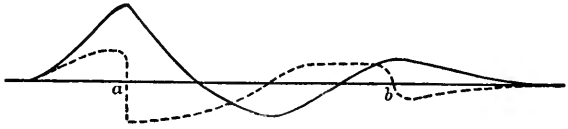


Fig. 82.

in Art. 17, it will be found, as shown by the dotted curve in the figure, to be of a very irregular character. The sudden changes of pressure indicated at a and b produce the characteristic sensations of the noise.

106. Discord.—When two sources sound together which would, separately, produce harmonic waves of nearly equal length, the actual wave-system consists of alternate groups of more and less intense waves (Art. 57). As these reach the ear, the sound heard keeps increasing and diminishing in loudness; these variations are called *beats*. These beats may be considered a consequence of the fact, explained above, that there is not much difference between the intensity of the resonant vibration of the fibres of the basilar membrane caused by waves of exactly their own free-vibration period and that caused by waves of nearly their own period. So that, if two sources of sound have nearly equal vibration frequencies, there are fibres which

either of the sources would set in resonant vibration, the frequency of this vibration being always that of the source, not the free-vibration frequency of the fibre (Art. 80). The movements which these fibres would be executing if one of the sources was vibrating alone are sometimes in the same direction, sometimes in the opposite direction to those which they would be executing if the other source was vibrating alone, and the actual movement of these fibres keeps increasing and diminishing.

From this it will be seen that no beats will be heard if the frequencies of the sources are so different that they do not cause resonant vibration of the same fibres. This is found to be the case when the difference of frequencies is more than $\frac{1}{5}$ of the smaller frequency.

These constant variations in the intensity of the sound, like the variations in the light of a flickering or "bobbing" flame, are very unpleasant within certain limits of frequency. These limits are different for sounds of different pitch. With very low or very high notes, the beats are hardly noticed. For notes of medium pitch (within the range of the voice) they are hardly distinguished if less frequent than 2 per second. From 2 to 10 per second, each beat is mentioned separately, but the effect is not very unpleasant. Above 10 per second the beats are no longer heard separately, but produce the peculiar jarring sensation known as *discord*. The unpleasantness of this increases with the frequency of the beats up to a certain point, about 30 beats per second for the middle of the voice-range. The effect then becomes less unpleasant, with increasing frequency, and becomes imperceptible when the beats are more than 1 to every 4 vibrations, or when they are more than about 80 per second, whichever is first reached.

As each fibre is set in vibration by harmonic components nearly agreeing with itself in frequency, just as if these components were the whole sound, we have the sensation of beats or discord when two non-harmonic wave-systems arrive together, if any two among their harmonic components have frequencies whose differences are within the limits just given.

In consequence of this, non-harmonic wave-systems are often discordant when harmonic systems of the same frequencies would be concordant. A tuning-fork vibrating gently 200 times a second sounds quite harmonious with one of any frequency above 240, since they do not affect the same fibres, and tuning-forks are very free from harmonics. But a note of frequency 200 and one of frequency 280 on a piano would be discordant, since the third harmonic, 600, in the sound from the first would be discordant with the second harmonic, 560, in the sound from the second. Indeed, if we assume that all the possible harmonics are present in each note, we shall find that there will be discord between almost any two notes of moderate frequency, and even between the different harmonics of the same note if it is low enough. But if only the three or four lowest harmonics of each note are intense enough to affect the ear, we shall find that a combination of two notes of moderate frequencies is usually concordant if the frequencies are in a simple ratio, and discordant if not. Thus 200 and 280 are discordant if the harmonics up to the third are intense enough to affect the ear, but 200 and 250 have no discordant harmonics below 800 and 750, so that there is no discord unless the fourth harmonic of the lower note is fairly strong, while 200 and 300 or 200 and 400 have no discordant harmonics at all.

Though the concord or discord of two notes thus depends chiefly on their frequency-ratio and the harmonics present, it also depends to some extent (owing to the fact that beats more frequent than about 80 per second do not produce noticeable discord) on absolute frequency. Thus 200 and 300 are concordant whatever harmonics are present, but 100 and 150, the same ratio, are discordant if the first four harmonics of each are present, and in general the lower the notes the simpler must be the ratio, or the fewer the harmonics present, to avoid discord.

When the frequency of one note is double that of another, the first is said to be an *octave* above the second, for a reason which will appear shortly, and all notes whose frequencies lie between any number n and $2n$ (inclusive) are said to be "in the same octave."

When one note is an octave above another, all the harmonic components of the first are among the possible harmonic components of the second; if the two notes contain many harmonics, there is a strong similarity between the sensations produced, so that from the point of view of sensation the first is often considered to be not a different note from the second, but "the same note an octave higher." This may be shown by the following experiment.

Exp. 40.—Fix on a revolving axle three toothed wheels, having respectively say 60, 120, and 130 teeth. Hold a card in quick succession against the first and second, the first and third, and the second and third. A resemblance will be noted in the sounds produced by the first and second wheels, which is quite absent in the other cases, although in the last case the notes are much nearer in pitch than those produced by the first and second.

For this reason two such notes are called in music by the same name and are for most purposes regarded as the same note.

Any notes of the same name can be sounded together without discord; indeed, this simply amounts to strengthening some of the harmonic components of the lowest, so that the result is a note of the same pitch as the lowest, but of slightly different quality. But any other combination of notes within the same octave has harmonic components whose frequencies are not multiples of the lowest, and is different in its effect from a note of any quality.

107. Musical Scales.—A combination of two or more notes sounded together is called a *chord*. If it produces the jarring effect described above it is called a discord, if not, a *concord*. Though a discord by itself, or if long continued, is unpleasant, a series of discords following each other according to certain rules may be pleasant, and modern musical compositions usually consist largely of discords. But it is essential that a series of discords should lead up to a concord; concords form an essential part of all musical compositions in which more than one note is sounded at once. The *scale*, or series of notes used

in a musical composition, must therefore be so chosen that concords can be formed from it. All musical scales consist of notes related to each other, directly or indirectly, by simple frequency-ratios. This is not entirely due to the difficulty of combining other notes without discord, for before harmony, or the pleasing combination of notes, was attempted it had been found that melody, or the pleasing succession of notes, required the choice of notes which are now known to be related by simple frequency-ratios. This, of course, cannot depend on beats, and is probably due to the fact that when the notes used are related in this way, the same frequencies often recur, in various combinations, among the harmonics. Both for melody and for harmony, then, we must have simple relations between the frequencies of the notes used, but the conditions for the two are by no means identical, and a scale may be quite suitable for melody, and not for harmony. As we saw above, it is also a condition for harmony (though not for melody) that we should be able to produce the notes used without intense harmonics higher than the third; notes without any harmonics higher than the fundamental, however, are wanting in expression.

The simplest ratio of frequencies other than identity is 1 : 2. Tones so related are said to be an **octave** apart. They are always concordant. The next simplest ratio is 2 : 3 or 4 : 6; tones so related are said to be a **fifth** apart. They, too, are always concordant for notes within the range of an ordinary musical instrument, except for very low ones. If the lower note be raised an octave, its name is unaltered, but we get the ratio 3 : 4, which is called a **fourth.** The next simplest ratios are 4 : 5 and 5 : 6, called **thirds**; since $5/4 > 6/5$, the first is called a **major** and the second a **minor** third. These intervals are concordant between high notes, but discordant between low notes.

All musical concords are made up of notes of frequencies in ratios 4 : 5 : 6 combined with the octave 1 : 2. Four notes of frequencies in ratios 4 : 5 : 6 : 8 form a **Common Chord** Three notes in ratio 4 : 5 : 6 form a **Major Chord.** Taking the 6 as basis of another major chord, its notes would be represented by 6, $7\frac{1}{2}$, 9; and taking 9 as basis we

should have 9, $11\frac{1}{4}$, $13\frac{1}{2}$. Reducing to a common denominator by multiplying by 4, this gives

$$16,\ 20,\ 24,\ 30,\ 36,\ 45,\ 54.$$

Each of these frequencies may be doubled or halved without altering the name of the note. Performing this process on 16, 20, and 54, and altering the order, we get

$$24,\ 27,\ 30,\ 32,\ 36,\ 40,\ 45.$$

These seven notes form the ordinary or **Major Scale**. The frequencies given are only relative frequencies. The fundamental or **Tonic** (represented by 24) may have any actual frequency. If it be the middle C of a piano tuned to concert pitch, its actual frequency is about 261, the next note D would be $\frac{27}{24} \times 261$ or 293, and so on.

But whatever be the actual frequency of the note represented by 24, it is called the **Tonic**. The note in the fifth place from it is represented by 36, and the frequency ratio is 3 : 2; this is called the **Dominant**. The octave of the tonic is 48; it is itself the dominant of $\frac{2}{3} \times 48$ or 32, hence this is called the **Subdominant**, but it is convenient to notice that the subdominant is the note below the dominant. The interval from the first note to the third note ($30/24 = 5/4$) is called a major third; from the 30 note to the 36 ($36/30 = 6/5$) a minor third, etc. In music, the interval from 27 to 32 is also called a third because it extends over three notes, and is a minor third because 32/27 is more nearly equal to 6/5 than to 5/4.

The numbers given above,

$$24,\ 27,\ 30,\ 32,\ 36,\ 40,\ 45\ (48),$$

are in ratio

$$1,\ \tfrac{9}{8},\ \tfrac{5}{4},\ \tfrac{4}{3},\ \tfrac{3}{2},\ \tfrac{5}{3},\ \tfrac{15}{8}\ (2),$$

the last of these being the lowest note of a similar series having the same ratios, and so on, both upwards and downwards. This scale is called the **Major Diatonic** scale.

The successive intervals between the notes of this scale are

$$\tfrac{9}{8},\ \tfrac{10}{9},\ \tfrac{16}{15},\ \tfrac{9}{8},\ \tfrac{10}{9},\ \tfrac{9}{8},\ \tfrac{16}{15}.$$

The intervals $\frac{9}{8}$, $\frac{10}{9}$, $\frac{16}{15}$ are called a **major tone**, a **minor tone**, and a **semitone** respectively, so that the sequence of intervals in the major diatonic scale is

Major tone, minor tone, semitone, major tone, minor tone, major tone, semitone,

or, neglecting the difference between major and minor tones,

tone, tone, semitone, tone, tone, tone, semitone.

On the system called the Tonic Sol-Fa system these notes are called (whatever their actual frequencies) by the names

Doh, Ray, Me, Fah, Soh, Lah, Te (Doh),

the names being repeated in the octaves above and below. When it is necessary to distinguish between notes of the same name in different octaves, this is done by dashes above or below the name; thus Me″ means a note two octaves above the note indicated by Me.

If we chose Lah (40) as lowest note of the scale, we should have, doubling all numbers below 40,

40, 45, 48, 54, 60, 64, 72, 80,

or 1, $\frac{9}{8}$, $\frac{6}{5}$, $\frac{27}{20}$, $\frac{3}{2}$, $\frac{8}{5}$, $\frac{9}{5}$, 2.

All scales, ancient and modern, include those whose frequencies are $\frac{3}{2}$ and $\frac{4}{3}$ of the tonic, *i.e.* the dominant and subdominant. If we replace $\frac{27}{20}$ by $\frac{4}{3}$, which is nearly equal to it, we get a satisfactory scale with ratios

Lah	Te	Doh	Ray	Me	Fah	Soh	Lah
1	$\frac{9}{8}$	$\frac{6}{5}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{9}{5}$	2.

This is the original form of the *Minor Diatonic scale.* In modern music the $\frac{9}{5}$ is usually changed to $\frac{15}{8}$, and the $\frac{8}{5}$ sometimes changed to $\frac{5}{3}$, making these notes the same as in the major scale.

The successive intervals between the notes of the original form of the Minor Diatonic scale are

$$\frac{9}{8},\ \frac{16}{15},\ \frac{10}{9},\ \frac{9}{8},\ \frac{16}{15},\ \frac{9}{8},\ \frac{10}{9},$$

so that the sequence of intervals in this scale is

Major tone, semitone, minor tone, major tone, semitone, major tone, minor tone,

or, neglecting the difference between major and minor tones,

tone, semitone, tone, tone, semitone, tone, tone.

The **interval** between any two notes of the diatonic scale is named according to the number of notes of that scale from one to the other, both inclusive; thus the interval between Me and Lah is called a fourth. When two different intervals have the same name, the larger is called *major* and the smaller *minor*; thus the interval between Doh and Me (ratio 5 : 4) is called a major third, while that between Me and Soh (ratio 6 : 5) is called a minor third. Where one or two nearly equal intervals is much simpler than the other, the simpler is distinguished as *perfect*; thus the interval between Doh and Soh (ratio 3 : 2) is called a perfect fifth, while the interval between Te and Fah (ratio 64 : 45) is a diminished fifth. The diatonic scale with its notes in the exact ratios given above, and the intervals of that scale, are called *just* to distinguish them from tempered scales and intervals such as those described in the next section.

If there are any three notes, x, y, z, in ascending order, the interval between x and z is called the *sum* of the interval between x and y and the interval between y and z. If N_x, N_y, N_z are the frequencies of the three notes, $\frac{N_z}{N_x} = \frac{N_y}{N_x} \times \frac{N_z}{N_y}$, or the ratio corresponding to the sum of two intervals is the *product* of the ratios corresponding to the intervals themselves. Thus the sum of a perfect fifth and a perfect fourth is an octave, since $\frac{3}{2} \times \frac{4}{3} = \frac{2}{1}$. For many purposes it is more convenient to take the *logarithm* of the ratio of the frequencies as the numerical measure of the interval. Then the logarithmic measure of the sum of two intervals is the sum of the logarithmic measures of the intervals themselves. Thus a perfect fourth added to a perfect fifth makes an octave, since

$$\log \tfrac{4}{3} + \log \tfrac{3}{2} = \log 2,$$

and a major tone added to a minor tone makes a fourth, since

$$\log \tfrac{9}{8} + \log \tfrac{10}{9} = \log \tfrac{5}{4}.$$

The only combinations of more than two notes within the same octave which involve no dissonance, if the first three harmonics of each note are present, are in the ratios 4 : 5 : 6 and 10 : 12 : 15. The former is called the *major triad* or *major chord*, the latter the *minor triad*. The diatonic series includes (in each octave) three major triads, viz.:—Doh, Me, Soh; Fah, Lah, Doh′; Soh, Te, Ray′; and three minor triads, Lah, Doh′, Me′; Ray, Fah, Lah; Me, Soh, Te; the second of these being very slightly inaccurate unless the Ray is modified. In either the major or minor scale the tonic, subdominant, and dominant are each of the lowest note of the triad, major or minor; these triads are called the tonic, subdominant, and dominant triads respectively. In either scale every note forms part of at least one of these triads. In the major scale these triads are all major; in the minor scale the tonic triad is minor, and the others may be so.

Each triad consists of a major and a minor third, but in the major triad the major third is below the minor, and *vice versa*.

Of the many other scales which were used before the development of harmony led to the disuse of scales which did not allow a sufficient variety of concords, the most important was the **Pythagorean**, whose notes have to the key-note the ratios

$$1 \quad \frac{9}{8} \quad \frac{81}{64} \quad \frac{4}{3} \quad \frac{3}{2} \quad \frac{27}{16} \quad \frac{243}{128} \quad 2,$$

every note of which is derived from the key-note by taking octaves and fifths upwards and downwards. An approximation to this scale is often used by soloists using instruments without fixed notes, like the violin.

The musical effect of a composition depends chiefly on the order and duration of the chords and intervals which succeed each other, *i.e.* on the *relative*, not the absolute, frequencies of the notes, at least to the extent that the composition is recognisably the same when the frequencies of all the notes are raised or lowered in the same ratio. Thus any series of notes of proper durations, whose frequencies have the ratios of $1, \frac{15}{16}, \frac{5}{6}, \frac{15}{16}, 1, \frac{9}{8}, \frac{5}{4}, 1, \frac{4}{3}, \frac{5}{4}, \frac{9}{8}, 1, \frac{15}{16}, \frac{5}{6}, \frac{15}{16}, \frac{3}{4}$, etc., is recognised as the air "Men of

Harlech." All these are diatonic intervals, so that if we had an instrument whose notes formed a diatonic scale (having, for instance, notes of the frequencies 240, 270, 300, 320, 360, 400, 450, with the doubles and halves of each of these, and so on), we could produce on it the air "Men of Harlech" and many other simple compositions.

But an instrument with only these notes would be very restricted in its use. We could produce on it the air "Men of Harlech" by sounding notes of frequencies 240, 225, 200, 225, etc., or, if we preferred it higher, we might use notes of twice these frequencies, but we cannot get notes, having the right ratios, of any intermediate frequencies. The pleasurable effect of a musical composition depends largely on the appropriateness of the absolute pitch of the notes used, and absolute pitch is of still greater importance when the instrument is to accompany the voice, since a small change of absolute pitch may make some of the notes too high or too low for the singer to produce. It is therefore very probable that neither the lower nor the higher series of notes given above would be satisfactory, though either would give the required air.

To be satisfactory in this respect an instrument would require to be able to produce the notes of any one of at least six diatonic scales. Besides this, for any but the simplest musical effects, it is necessary to be able to pass from one scale to another whose key-note is related to that of the first, and so on, and in most music notes are used occasionally which do not belong to the diatonic scale.

To fulfil all these conditions, and yet retain the exact ratios of the notes in each scale, we should require several additional notes in each octave. For example, the major diatonic scale has frequencies as shown in the top line of the adjoining table. The scale on the second note is got by multiplying the top line throughout by $\frac{9}{8}$ and shifting the resulting figures on one to the right. This makes the old 24 the new 27 and preserves the original sequence of intervals.

The result is given on the second line, the new notes, $30\frac{3}{3}$, $33\frac{3}{4}$, $40\frac{1}{2}$, $50\frac{3}{8}$, being underlined. The last is replaced by $25\frac{5}{16}$ to bring it within the original scale. The scale

on the third line is got by multiplying the first scale by $\frac{5}{4}$, and gives the new notes, $37\frac{1}{2}$, 50, and $56\frac{1}{4}$, or, taking the last two within the original scale, 25, $28\frac{1}{8}$, and $37\frac{1}{2}$. The fourth line yields new notes, $42\frac{2}{3}$ and $53\frac{1}{3}$ (or $26\frac{2}{3}$).

	Doh	Ray	Me	Fah	Soh	Lah	Te	Doh′	Ray′	Me′	Fah′	Soh′	Lah′
Doh	24	27	30	32	36	40	45						
Ray	$25\frac{5}{16}$	27	$30\frac{3}{8}$	$33\frac{3}{4}$	36	$40\frac{1}{2}$	45	$50\frac{5}{8}$					
Me	25	$28\frac{1}{8}$	30	$33\frac{3}{4}$	$37\frac{1}{2}$	40	45	50	$56\frac{1}{4}$				
Fah	24	$26\frac{2}{3}$	30	32	36	40	$42\frac{2}{3}$	48	$53\frac{1}{3}$	60			
Soh	24	27	30	32	36	$40\frac{1}{2}$	45	48	54	60	64		
Lah	25	$26\frac{2}{3}$	30	$33\frac{1}{3}$	$37\frac{1}{2}$	40	45	50	$53\frac{1}{3}$	60	$66\frac{2}{3}$	75	
Te	$25\frac{5}{16}$	$28\frac{1}{8}$	30	$33\frac{3}{4}$	$37\frac{1}{2}$	$42\frac{3}{16}$	45	$50\frac{5}{8}$	$56\frac{1}{4}$	60	$67\frac{1}{2}$	75	$84\frac{3}{8}$

The fifth line yields no extra notes. The sixth line gives $66\frac{2}{3}$ (or $33\frac{1}{3}$) and the seventh $84\frac{3}{8}$ (or $42\frac{3}{16}$). Thus, summarising the table, we get the complete scale,

Doh Ray Me Fah

$24, \left\{\begin{matrix}25\\25\frac{5}{16}\end{matrix}\right. . \left.26\frac{2}{3}\right\}, 27, \{28\frac{1}{8}\}, 30, \{30\frac{3}{8}\}, 32,$

Soh Lah Te Doh′

$\left\{\begin{matrix}33\frac{1}{3}\\33\frac{3}{4}\end{matrix}\right\}, 36, \{37\frac{1}{2}\}, 40, \left\{\begin{matrix}40\frac{1}{2}.42\frac{3}{16}\\42\frac{2}{3}\end{matrix}\right\}, 45, 48,$

which includes eleven notes in addition to the diatonic scale, or nineteen notes in all including the octave. This number is not practicable in instruments with fixed scales. For such instruments, therefore, the scale must be altered.

One way of doing this is as follows: The intervals 25 : 24, $30\frac{3}{8}$: 30, $40\frac{1}{2}$: 40 are so small that the new notes 25, $30\frac{3}{8}$, $40\frac{1}{2}$ may be neglected, and we are thus left with eight new notes. Each of the remaining pairs of new notes is then replaced by a single intermediate note, and thus we get five new notes, called in order De, Re, Fe, Se, and Ta, or twelve notes in all, excluding the octave. All the intervals are then usually modified further by tempering.

108. Tempered Scales.—A tempered scale is one in which the frequencies of the notes are not exactly in simple ratios; by sacrificing this advantage it is possible to make a sufficient variety of scales out of only twelve notes to the octave. The temperament most in use is the *equal* temperament. On the equal tempered scale the octave is divided into twelve equal intervals called semitones. Since these twelve equal intervals added together make up the octave, it follows that if x denote the ratio that measures the interval $x^{\frac{1}{12}} = 2$ or $x = 2^{\frac{1}{12}} = 1{\cdot}059463$. From this it readily follows that the equal temperament scale consists of a series of notes having the ratios

Doh	De	Ray	Re	Me	Fah	Fe
$2^{\frac{0}{12}}$	$2^{\frac{1}{12}}$	$2^{\frac{2}{12}}$	$2^{\frac{3}{12}}$	$2^{\frac{4}{12}}$	$2^{\frac{5}{12}}$	$2^{\frac{6}{12}}$
1	1·059	1·122	1·189	1·260	1·335	1·414

Soh	Se	Lah	Ta	Te	Doh'
$2^{\frac{7}{12}}$	$2^{\frac{8}{12}}$	$2^{\frac{9}{12}}$	$2^{\frac{10}{12}}$	$2^{\frac{11}{12}}$	2
1·498	1·587	1·682	1·782	1·888	2

The ratio of the chief notes are

Doh	Ray	Me	Fah	Soh	Lah	Te	Doh'
1	1·122	1·260	1·335	1·498	1·682	1·888	2

so that the tempered diatonic scale is made up of the sequence

tone, tone, semitone, tone, tone, tone, semitone,

as in the just diatonic scale, but the tones are all equal and exactly twice the semitone.

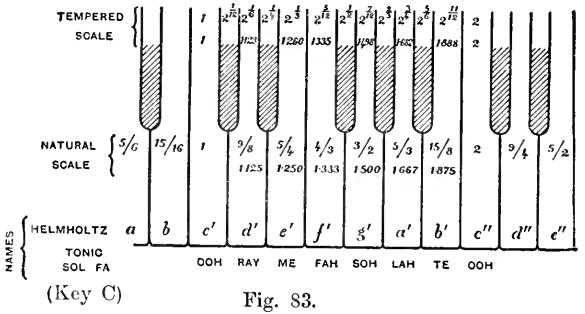
If we compare the above ratios of the tempered scale to the just diatonic ratios

1 1·125 1·250 1·333 1·500 1·667 1·875 2

we see that none of them differ by as much as 1 per cent., so that when two notes of the tempered scale which correspond to a perfectly harmonious combination on the just scale are sounded together the beats are not rapid enough to be disagreeable. It is this very remarkable approximation of the powers of $1\frac{2}{3}/2$ to simple fractions which makes an equally tempered scale possible. The most important intervals, the fourth and the fifth, are, by a curious coincidence, the nearest to those of the just scale, being within about $\frac{1}{5}$ per cent.

In key-board instruments the series of notes provided is on the tempered scale; any one of the notes may be taken as the key-note of either a major or a minor scale. So that from this series of notes we can make twelve major and twelve minor scales, whose key-notes differ in frequency, each from the next, by less than six per cent., and we can take any note of any scale which we are using as the key-note of a fresh scale, which may be either major or minor.

On a piano or harmonium Doh, Ray, Me, Fah, Soh, Lah, Te, Doh are given by the white keys, and De, Re, Fe, Se, Ta



(Key C)

Fig. 83.

by the black keys (Fig. 80). The succession of octaves may be written

d_1 r_1 m_1 f_1 s_1 l_1 t_1 d r m f s l t d′ r′ m′ f′ s′, etc.,

d being the "middle" doh of the piano, and d′ the octave above it.

In the above treatment we have used the Tonic Solfa notation to identify the notes, but Helmholtz's notation is more usual amongst musicians. Helmholtz represented the notes of the scale by the letters

c, d, e, f, g, a, b, c.

Different type is used for different octaves, so that on his notation the succession of octaves reads

C D E F G A B c d e f g a b c′ d′ e′ f′ g′ a′ b′ c″ d″ e″, etc.

c′ is what is usually called the middle c of the piano, c″ and c being the next upper and lower c's.

In Helmholtz's notation the remaining notes of the scale do not receive special letters, but are denoted by the letter of the note below with the sign ♯ (sharp), or by the note above with the sign ♭ (flat). Thus on Helmholtz's notation a full octave of the scale is written

$$\mathrm{C}.\left\{\begin{matrix}\mathrm{C}\sharp\\ \mathrm{D}\flat\end{matrix}\right\}.\mathrm{D}.\left\{\begin{matrix}\mathrm{D}\sharp\\ \mathrm{E}\flat\end{matrix}\right\}.\mathrm{E}.\mathrm{F}.\left\{\begin{matrix}\mathrm{F}\sharp\\ \mathrm{G}\flat\end{matrix}\right\}.\mathrm{G}.\left\{\begin{matrix}\mathrm{G}\sharp\\ \mathrm{A}\flat\end{matrix}\right\}.\mathrm{A}.\left\{\begin{matrix}\mathrm{A}\sharp\\ \mathrm{B}\flat\end{matrix}\right\}.\mathrm{B}.\mathrm{c}.$$

On just diatonic scales the sharp and flat between any two notes are not identical, but there is no distinction between them on the tempered scale. In making a semitone change from a note it is usual to keep to the letter of the first note.

Endeavours have been made recently to increase the number of notes in the octave in order to permit keyed instruments to give a truer expression of the just diatonic scale, and no doubt with increased perfection in mechanism such attempts will ultimately prove successful.

109. Standards of Pitch.—The standard of absolute pitch assigned to the upper C of the piano, *i.e.* c″, varies slightly, but in England *concert pitch* requires a frequency of 546 for this note. The old *Philharmonic pitch* gave 540 as the frequency for c″, and the new *Philharmonic* or French *Standard pitch* adopted in 1899 requires c″ and a′ tuning-forks to have frequencies of 522 and 439 respectively at 68° F.

The equal tempered scale from c′ (261) to c″ (522) in accordance with this standard pitch is therefore

c′	c′♯	d′	d′♯	e′	f′	f′♯
261,	276·6,	293,	310·4,	328·9,	348·4,	369·2,

g′	g′♯	a′	a′♯	b	c″
391·1,	414·4,	439,	465·1,	492·7,	522.

The Tonic Solfa c″ was fixed at 507, this being the pitch of Handel's fork. Tonic Solfa pitch is thus about a semitone below concert pitch.

For scientific purposes the c″ is taken as 512, this number being chosen because, as it is a power of 2, all the lower c's will be integers; a′ is therefore 427 and g′ 384.

110. Beats and Combination Tones.—*The controversy between Koenig and Helmholtz.*

1. Beats.—As pointed out in Art. 106 on discord beats will be only produced between two notes if these are near enough for the two patches of fibres of the basilar membrane which they excite to overlap. This was **Helmholtz's** theory, and according to him the interval must be less than a *minor third.*

If two notes which are first in unison gradually separate from each other the sensation of discord increases up to a certain point, then decreases, and finally disappears when the interval becomes as great as a major third. Hence it is evident that the disagreeableness of discord depends on the number of beats, and in the middle part of the scale it appears to reach a maximum when the number of beats is about thirty-three a second. The sensation, however, depends not only on the number of beats, but also on the position of the notes in the scale, thus very low down two notes of 33 and 66 vibrations a second would give the perfect concord of an octave and yet would give 33 beats a second.

Koenig has shown that when two notes at first in unison gradually separate the beats become more rapid, till in general they cannot be distinguished. When, however, the interval approaches an octave beats begin to be heard

again, which become slower as the octave is approached. Their number is the same as if the octave of the lower note was beating with the upper note. As the upper note passes the octave the beats again die out, but appear again when the twelfth, whose frequency is three times that of the fundamental, is approached, and so on, beats being heard whenever the upper note is nearly an exact multiple of the lower note.

Koenig's experimental facts thus seem to disagree with Helmholtz's theory. The difficulty might be got over by recalling the fact that the notes produced by musical instruments are very seldom pure. In the best musical note not only is the fundamental present, but other secondary tones accompany it the frequencies of which are multiples of the frequencies of the fundamentals, the whole series forming a compound tone. If then Koenig's beats are produced when the lower note is accompanied by its octave as an upper partial, the beats might be produced by this octave beating with the higher note which is itself nearly the octave of the other. The beats would then be consistent with Helmholtz's law.

This theory may account for the facts when wind instruments are used, as these are usually accompanied by harmonic upper partials, but unfortunately it fails to account for it when tuning-forks are used, as the notes given by forks are remarkably clear of harmonic partials, and Koenig on carefully testing his large forks declares that no harmonics are present.

Lord Rayleigh has recently suggested a way out of the difficulty. If a stretched string is set in vibration near another string whose fundamental frequency is half its own, then, since the first harmonic of the lower note is equal to the note given by the vibrating string, the stationary string will be set in motion by notes whose frequencies are twice and similarly three and four times its own. Lord Rayleigh assumes that the strings in the membrana basilaris are in like manner set in vibration by their octaves, twelfths, etc., and that beats are thus produced which are consistent with Helmholtz's law.

Another explanation is given below.

2. Combination Tones.—It was shown in Art. 85 that if there are pressure changes affecting the same body with frequencies m and n, the pressure change in the air will have components of frequencies $m + n$ and $m - n$, besides multiples of m and n, and these will of course produce the corresponding sensations. These sensations are called *resultant* or *combination tones*, that corresponding to $m - n$ being called the *difference tone*, and the other, which is less intense, being called the *summation tone*. The ear membrane and fibres have great internal friction, and therefore are particularly affected by this cause. This was Helmholtz's theory of combination tones. Koenig and others advanced the theory that the difference tone was produced by the beats, *i.e.* that variations of intensity when they became too rapid to be heard separately produced the same sensation of tone as variations of pressure of the same frequency This of course left the summation tone unexplained, and Koenig and others went so far as to deny its existence. Rucker and Edser, however, performed experiments conclusively proving the objective existence of the difference and summation tones by getting a fork of the right frequency to respond to them.

These tones explain the beats heard between two notes nearly an octave apart. If m be nearly $= 2n$, $m - n$ nearly $= n$, and beats can be heard between the difference tone and the original tone n. Thus if we have forks 256 and 510 the difference tone is $510 - 256$ or 254, and there are 2 beats per second between the 254 and 256. If we have forks $m = 256$, $n = 760$, so that they are nearly a twelfth apart, $n - 2m = 248$, and this gives 8 beats with 256.

3. Rucker and Edser's Experiment.—The acoustical part of the apparatus consisted of a Helmholtz double siren (Fig. 84 and *S* Fig. 85). This siren consists of two ordinary sirens mounted, facing each other on the same axis. Each siren is provided with four circles of holes, and one or more of these can be put in use at will by means of the stops fitting into the side of the instrument. Both sirens should be blown from the same air-chest. The difference between the frequencies of the two notes given was always adjusted to be exactly 64. The siren was placed in front of a large Koenig resonator *R* (Fig. 85) adjusted to respond to a note of

frequency 64. The sound waves entered a large cone of wood C and were concentrated towards the apex. The apex of the cone was cut off and the opening nearly blocked by a thin slip of wood W carried by a prong of a good tuning-fork F. The frequency of the fork was 64, and it responded to a note of exactly 64, but to nothing else. The shank of the fork was embedded in a lead block D

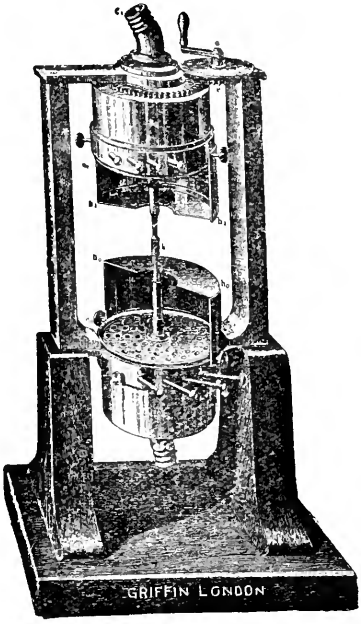


Fig. 84.

The optical part of the apparatus was a Michelson and Morley Interferometer. This consists of a parallel-sided plate of glass M_1 which partly transmits and partly reflects a beam of monochromatic light from a lantern L incident on it at an angle of 45°. The two parts are reflected back along their own courses by silvered mirrors, one M_2 fixed, the other M, of equal mass to W, fastened to the other prong of the tuning-fork. When these reflected beams reached the plate of unsilvered glass again, part of the beam which was originally transmitted is reflected, and part of the beam which was reflected is transmitted, and these parts coincide and can be received in the

same telescope *B*. The apparatus is arranged so that the light reaching the centre of the screen by the two routes has travelled the same distance when the fork is still; the centre of the field of view is bright, but the parts of the field of view to which the two paths differ by an odd number of half wave-lengths of light are, by the principle of interference, dark; there is a system of approximately hyperbolic dark and bright bands. A movement of the prong of the fork *F* to the extent of a quarter wave-length, *i.e.* to $\frac{1}{160000}$ of an inch, makes the dark parts bright and *vice-versa*; a trembling of the prong to this or any greater extent interchanges dark and light so rapidly that no bands are visible. This effect is produced by an extremely feeble sound of exactly the frequency of the fork, but not by very loud sounds of other frequencies.

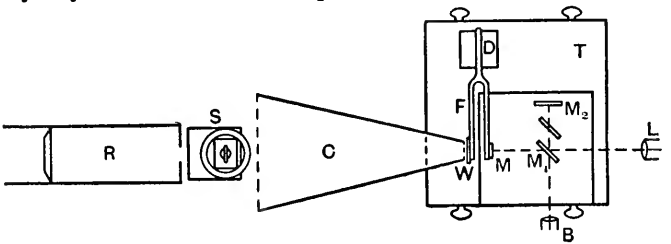


Fig. 85.

To ensure the steadiness of the apparatus, the interferometer and the block supporting the fork were mounted on a large stone slab *T*, so supported as to be little affected by external vibrations.

When the interval between the two notes of the siren is small the frequency of the difference tone is equal to the frequency of the beats, or the beat-tone as Koenig called it, both being equal to the difference of the frequencies of the sounding notes. If, however, the interval is large the frequency of the beats or "beat-tone" is equal to the difference between the frequencies of the upper note and the octave of the lower note, while the frequency of the difference tone is still the difference between the frequencies of the two notes. In all the experiments the frequency of the difference tone was made 64, while the frequency of the "beat-tone" was varied from 64 to 12·8. In all cases the objectivity of the difference tone was affirmed, but experiments made to determine whether the beat-tone was objective gave negative results.

The siren was then tuned to give a summation tone of 64, and in each case the interference bands were disturbed. Helmholtz was therefore correct in stating that the siren produces two objective tones of frequencies equal to the difference and sum of the frequencies of the notes sounded, and the theory of Koenig is wrong.

4. Physiological Difference Tones.—Since the physical resultant tones depend on the amplitude of vibration being too large for the principle of superposition to be applied, they must be of extremely small intensity except when the same air is strongly agitated by both the sources of sound. In fact the tuning-fork method, described above, failed to detect them in any case in which the sources were really independent, even though near together, though they were easily detected when a double siren was used or when two strings vibrated on the same sounding-box. The tuning-fork can detect sounds far too faint to affect the ear, yet, as will be shown, the ear can often hear resultant tones which the fork does not show.

Whatever the explanation of difference tones may be there can be no doubt that when two notes are sounded loudly a note whose frequency is equal to the difference in their frequencies is heard; thus if we sound together two tuning-forks whose frequencies are 2048 and 2304 their difference tone of 256 vibrations per second can be plainly heard.

The difficulty of explaining this note as a difference tone is that tuning-forks are notoriously difficult to produce difference tones with, and that as the forks are sounded in the open air without any effort to produce a great disturbance at any point in the air it is probable that the difference tone in this case is *produced within the ear itself*. Helmholtz has suggested how this can be done, and these notes in some cases are probably ear-made notes in the sense that they are produced in the mechanism of the ear.

Thus it can be shown that if a pressure change, whose harmonic components have frequencies m and n, occurs at the surface of a membrane which is very subject to friction, or is more easily displaced one way than the other, the forced vibrations of the membrane will have harmonic components of frequencies $m - n$ and $m + n$ as well as m and n. The drum-skin is such a membrane. The new harmonic components introduced by it are called *physiological combination tones*. It seems to be proved by the experiments just mentioned that the summation and difference tones heard when two independent sources vibrate are almost entirely ear-made.

Both physical and physiological combination tones increase in intensity in a much greater ratio than the sounds due to the sources separately, so that they are much more easily detected when the sounds themselves are loud.

Owing to combination tones, it often happens that two notes are discordant when sounded together, which on the principles explained in Art. 106 we should expect to be concordant. Thus closed cylindrical organ pipes (which produce only the odd harmonics) of frequencies 400 and 780 have no harmonics which can clash, but the difference tone 380 is discordant with 400, and the summation tone 1180 with the odd harmonic 1200. Discord depending on combination tones differs from discord depending on harmonics in being hardly noticeable unless the sound is loud.

The summation tone may be audible when both notes are below the limits of audible frequency, and the difference tone when both notes are above the limit.

The difference tone is sometimes used in music when a lower note is required than the instrument can produce. Thus organ pipes of frequencies 80 and 120, sounded together, produce a note of frequency 40. This method can only be applied with loud sounds.

Exp. 41.—Obtain a double whistle, such as is used by football referees and policemen. It consists of two adjoining stopped pipes. When blown separately each gives a high whistle note, the pitches differing by about a tone. Blow hard through both pipes at once; a loud difference tone is generated. So loud is this tone that the primary tones are practically extinguished.

Find the frequency of the three notes given out by such a whistle by comparison with a piano, and so confirm the law that the frequency of the difference tone is equal to the difference between the frequencies of the primary tones.

One such whistle gave primary notes of frequencies 1160 and 1450, and a difference tone of frequency 290. A police whistle gave primary notes of frequencies 2230 and 1950 and a difference tone of frequency 280.

111. The statement that beats produce a tone is still sometimes made, but it is difficult to attach a distinct meaning to it. If we produce beats of frequency n by sounding together two sources whose frequencies differ by n, we necessarily produce a difference tone of that frequency while there is no other known mode of making the intensity of a sound vary n times a second in which the

pressure change produced has not a harmonic component of frequency n. For instance, if the sound from a tuning-fork of frequency 1000 is allowed to pass through a hole which is opened and partly closed ten times a second, the sound on the other side of the hole becomes louder and fainter ten times a second, so that beats of that frequency are heard. The pressure change in the air beyond the hole repeats itself completely only ten times a second, since though the pressure actually rises and falls 1000 times a second all these pressure changes are not equal. The actual pressure change, repeated completely ten times per second, has a harmonic component of that frequency (Art. 89). If the hole is opened and closed 100 times a second, the beats are too rapid to be heard separately, and a note of frequency 100 is heard together with the note of the fork, but as there is now a harmonic component of frequency 100 present it is not clear that the note should be attributed to the beats. The only clear meaning of the statement that beats produce a tone is that when two sources vibrate together with frequencies whose difference is n, n being within the limits of audible frequency for vibrations, a note is heard, corresponding in pitch to n vibrations, which does not increase in loudness, when the sounds are increased, more rapidly than the sounds, as the difference tone does. But there seems no evidence of this.

If, instead of adding two harmonic curves where bends are of equal heights, as in Fig. 50, we add two harmonic curves one of which has higher bends than the other, the resultant curve is like the third line in Fig 50, but the crests are not equidistant, but are closer in the more intense parts than in the less intense parts, or *vice versa*. It has sometimes been supposed from this that there would be a difference of pitch, as well as of loudness, between the more intense and the less intense parts of the waves from two unequal sources of different frequencies. On the resonant theory of the ear, there should be no difference due to this cause, since the less and more intense parts have the same harmonic components, and the effect depends on the harmonic components of the pressure change, not on how many times a second the actual pressure rises and falls. The difference seems to be actually heard, but another possible explanation is that the resonant bodies of the ear (like stretched strings) have not the same free frequency for large as for small vibrations, so that the fibre which resounds most strongly to a loud sound is not the fibre which resounds most strongly to a faint sound of the same frequency.

112. Variation of Pitch with Motion, Doppler's Principle.—The velocity with which a pulse travels *relatively to the air*, when it has once been produced, is the same whether the vibrating body which produced it was at rest in the air or moving through it, and the same whether the air is moving relatively to the earth or not. Suppose that

a source of sound, say a whistle, produces n condensations per second, and let a be the velocity with which it is moving, relatively to the air, towards an observer. (If it is not moving directly towards the observer, a is to mean the component, in the direction towards the observer, of the velocity, so that if it is moving directly away from the observer, a will be the speed with a negative sign.) Let V centimetres per second be the velocity with which the waves travel relatively to the air. When the whistle begins to sound, a condensation starts away from it in every direction with a velocity V; at the end of $1/n$ second, when the whistle is just producing the next condensation, the first has travelled V/n centimetres while the whistle has travelled a/n towards the observer, and the part of the first which travels towards the observer has therefore a start of $(V - a)/n$ centimetres in front of the part of the second which travels the same way. This is therefore the length of the waves travelling towards the observer.

Let b be the velocity with which the observer is moving relatively to the air, away from the source, reckoned in the same way as a. Then a total length $V - b$ of successive waves will pass the observer in each second; as the length of each wave is $(V - a)/n$, the number of waves which pass the observer per second is $(V - b) \div \{(V - a)/n\}$ or

$$n \frac{V - b}{V - a}.$$

From this expression we easily deduce :—

(1) If the source and the observer move the same way with the same velocity, the frequency of the waves received is the same as that of the waves sent out.

(2) If the source and observer approach (*i.e.* if $a > b$) the frequency of the waves received is greater than that of the waves sent out, and *vice versâ*, but in this case the frequency of arrival depends on the velocities of both the source and observer relatively to the air and not simply on the rate of approach $a - b$. The change of frequency produced by a certain velocity of the source relatively to the air is not the same as that produced by an equal velocity of the observer relatively to the air.

If a and b are very small compared with V,

$$(V - b)/(V - a) = 1 + (a - b)/V$$

nearly, so that the frequency of arrival is in this case nearly the same whether it is the source or the observer which is moving relatively to the air. The reason of the alteration of frequency is, however, quite different in the two cases; in one the length of the waves is altered, while in the other nothing is altered but the rate at which they pass the observer.

(3) The ratio of the frequencies of the waves received by an observer before and after passing a source at rest relatively to the air is the same as the ratio before and after a source, moving with the same speed, passes an observer at rest, though the actual frequencies are different.

Now the frequency with which waves arrive at the ear determines the pitch of the sound heard, greater frequency corresponding to higher pitch, and the ratio of two frequencies determines the musical interval between the sounds. So that if our distance from a source of sound is increasing, we hear a sound of lower pitch, and *vice versâ*. The change in the pitch of the sound heard when a whistling locomotive, which has been approaching us, passes and begins to recede is a familiar instance of this. From (3) we see that the musical interval between the notes heard is the same for the same speed, whether it is the whistle or the observer that is moving relatively to the air, though the notes themselves are different.

The altered pitch due to movement of the observer relatively to the air begins and ends with the movement, but that due to movement of the source does not begin till the waves, which left the source as its motion began, have reached the observer.

Movement of the air relatively to the earth makes no difference to the formulae given above provided that a and b are measured relatively to the air, but if we wish to state the frequency of arrival of the waves in terms of the velocities of source and observer relatively to the earth, the formulae are different. Let a', b', w, be the velocities of source,

observer, and wind, *relatively to the earth*, in the direction from source to observer (or their components in this direction), while V is the velocity of sound *relatively to the air*. Then $a' - w$, $b' - w$, are the velocities relatively to the air, and must be substituted for a and b. Hence

$$\text{frequency of arrival} = n\left(\frac{V - b' + w}{V - a' + w}\right).$$

From this we see that wind, in the same direction as the sound, lessens the change of pitch due to a given speed of source or observer, and *vice versâ*. It comes to the same thing as a change in the speed of the sound.

The fact that the frequency of the arrival of waves at a point is affected by movements of the point, or of the source of the waves, is called *Doppler's Principle*.

Buijs Ballot and Scott Russell put Doppler's principle to test by examining the pitches of whistles and trumpets carried on railway engines as the engines approached and passed them. The principle was confirmed. Another simple way of verifying the principle is for two cyclists to ride swiftly past one another, one of them ringing his bell. If the speed is about twelve miles an hour the pitch of the bell will appear to the other cyclist to drop a semitone as the bell passes him.

Exp. 42.—Fit securely a whistle or small organ pipe of high frequency into about two yards of stout rubber tubing. One experimenter must place the free end of the tubing in his mouth and whirl the whistle around his head, meanwhile blowing steadily through the tube. Another experimenter standing some distance away will observe the pitch of the whistle gradually rise and fall. The maximum change of pitch will occur when the whistle is moving directly towards or away from the observer. To calculate the number of revolutions that must be made in order that the pitch must alter by a semitone use the formula given above. The frequency of the note heard when the whistle approaches the observer is $nV/V - a$, and the frequency heard on recession is $nV/V + a$. Therefore taking the interval of a semitone as $\frac{18}{17}$ we get $\frac{V + a}{V - a} = \frac{18}{17}$, *i.e.* $35a = V$. Taking V as 34000 cms. per second $a = \frac{34000}{35}$ cms. per second approximately. If N = number of revolutions per second and R = the effective radius of the circle, $a = 2NR$.

Now R will be about 150 cms. $\therefore N = \frac{7}{44} \times \frac{1}{150} \times \frac{34000}{35} = \frac{34}{33}$, or just over one revolution a second.

If two whistles of the same pitch are fastened to the ends of a rotating rod and then blown, an observer at a distance will observe beats. Beats can also be observed if a tuning-fork mounted on a resonance box is sounded and moved quickly to or from a wall.

This principle is only one of curiosity when applied to sound, but Doppler, who announced it in 1842, first applied it to stellar motions. If a star approaches the earth the apparent frequency of any particular set of waves in its spectrum will be increased so that its spectrum lines will be displaced towards the violet end of its spectrum. By throwing a spectrum from a stationary source containing the same elements alongside the stellar spectrum the shift of the lines can be easily measured and the change in wave-length deduced. From such observations it has been found that the bright star Sirius (α Canis Majoris) was receding from the earth with a relative velocity of 29 miles per second, and that the velocity of Saturn's rings was greater near the planet than further away, thus confirming Maxwell's prediction on mathematical grounds that these rings were composed of satellites.

113. Perception of Sound Direction.—The question as to how far the ears alone are capable of determining the direction of a source of sound has been attacked by Lord Rayleigh. When the alternative is between left and right the discrimination can be made with certainty without moving the head no matter what notes are sounding, but when directly in front or behind, no pronouncement can be made with pure tones, but with sounds of other characters front and back can often be distinguished. An object the size of the head forms an effective screen for small waves, so that with high notes the observer can decide whether a sound is produced on his right or left by mentally comparing the intensities of the sound at the two ears. When, however, the wave-length is much longer than the diameter of the head, the head does not cast an appreciable shadow, and the intensities at the two ears are practically identical. In this case Lord Rayleigh has shown that the observer decides between right or left by means of the phase difference between the parts of the waves meeting the two ears at the same instant. When the phase at the right ear is in advance of the phase at the

left ear, the observer decides that the sound is on his right hand, and when the phase is earlier at the left ear the observer estimates the sound to be on his left hand.

EXAMPLES VIII.

1. Describe the structure of the ear. What is the special function of the fibres of the basilar membrane?

2. What is the essential feature of a musical sound? How does it differ from a noise?

3. What are the three characteristics of musical sounds? How is the movement of the air-particles affected by (*a*) change of pitch, (*b*) change of intensity?

4. A circular disc, pierced with a concentric ring of equidistant holes, rotates about an axle which passes through its centre. A stream of air from a small fixed jet is blown against the holes. On what do the *intensity* and *pitch* of the note produced depend?

5. Explain clearly upon what properties (1) the loudness, (2) the musical pitch, of a note depend.

6. Why is it more difficult to determine the lower limit of frequency of audible sound than the upper?

7. The note given by a vibrating string is readily distinguished from one of the same pitch given by a flute. Explain in what respects they differ from each other.

8. Two notes of frequencies 200 and 300 are sounded. What is the *interval* between them?

9. What physical fact is implied when we say that one note is a fifth above another?

10. What is the interval between a note and its octave? Check your answer by adding the intervals between the 8 notes of the octave of the just diatonic scale.

11. What is the interval of a major third, a fourth, and a semitone? What is the difference between the intervals of a major and minor tone?

12. What is the *common chord* of the scale?

13. If a certain C be produced by 256 vibrations per sec., find the frequency of G, upper G, and lower G.

14. If G be produced by 320·0 vibrations per sec., find the frequency of C in the same octave.

15. Describe a musical interval so that, for example, three-fifths shall be as large as one-fifth. How is it supposed that our ears are able to recognise simple musical intervals?

16. If we used the natural scale, how many notes should we need per octave to provide the correct intervals in the keys of the seven principal notes?

17. What do you understand by temperament? Describe the scale of equal temperament.

18. Show that on the scale of equal temperament three major thirds make an octave.

19. What is understood by Summation and Difference Tones? Discuss the question of their objectivity, stating the explanations given by Von Helmholtz and Koenig. Describe recent experiments that have been made with reference to this subject and state the results.

20. If two vibrating sources whose frequencies are 400 and 802 are sounded together, beats are heard. Why? What is the frequency of the beats?

21. Mention any fact which goes to show that it is the frequency with which the waves arrive at the ear, and not simply the length of the waves emitted, that determines the pitch of the note.

22. Explain why the note of a bicycle bell seems different in pitch when the cycle is approaching you than when it is receding from you. Deduce a formula connecting the variation of pitch with the velocity of the bicycle.

23. A whistle sends out 200 waves a second. What number of waves will a person receive per second (*a*) if the whistle comes towards him with the velocity of sound; (*b*) if the whistle goes away from him with the velocity of sound; (*c*) if he goes towards the whistle with the velocity of sound; (*d*) if he goes away from the whistle with the velocity of sound?

24. An engine in a cutting between two bridges is travelling at the speed of 56 feet per sec. Compare the frequencies of the echoes heard by an observer on the edge of the cutting.

CHAPTER IX.

REFLECTION AND REFRACTION OF SOUND.

114. Reflection with Change of Sign.—In Art. 15 we described the transmission of a condensed or rarefied condition along a row of elastic bodies such as the carriages of a train. We will now consider what will happen when the pulse reaches the last carriage, which we will call Z. As in the latter part of Art. 15, we will suppose Z in contact with a fixed obstacle, such as the end of a siding, and that all the carriages are to some degree compressed between the engine and this obstacle. We will also suppose ourselves looking from a position such that the direction from A to Z is from right to left, as in Fig. 27.

When a condensation, produced in A by a push of the engine, reaches Z, Z moves in its turn to the left, compressing the buffers between it and the end of the siding. When Z comes to rest, these are more compressed than those between Y and Z, and therefore Z begins to move back again, and stops only when it has transferred its condensed condition to Y. Then Y moves in the same direction, and in this way the condensation travels back again to A, each carriage moving in turn a short distance towards the engine.

In the same way, if a rarefaction travelled from A to Z, Z would move in its turn towards A, but, as the end of the siding is fixed, this leaves the pressure between Z and the end of the siding less than that between Z and Y, and so Z moves back, Y follows, and a rarefaction travels back to A, each carriage moving a short distance away from the engine.

In these cases it will be noticed that the condition that travels back is of the *same* kind as that which travelled to Z from A, but the *direction* in which the carriages themselves move is *reversed* when the wave travels back again. When this is the case, the wave is said to be reflected *with change of sign*. It should, however, be noticed that, as the direction of movement of a wave is determined when the direction of movement of the substance and the kind of pulse (condensation or rarefaction) are given, the wave can only be reflected by reversing the sign of one of these. So that "reflection with change of sign" is really reflection in which it is the velocity and not the condensation of the undulating substance which is reversed in sign.

Reflection with change of sign takes place not only at a fixed obstacle, but at any point where the movement of a smaller mass is transferred to a larger one. If there is a row of trucks of which, say, those from A to F are empty, and the rest loaded, then, when a condensation produced in A reaches F, a smaller condensation travels on along the rest of the train, while another condensation travels back from F to A. The energy in these two condensations is equal to that in the original condensation.

In exactly the same way condensations or rarefaction travelling through the air (or any substance) are reflected when they reach a substance of greater density. Thus let MN (Fig. 86) represent a line of particles in the

Fig. 86.

medium at right angles to the reflecting surface AB, and let b be the last particle in the line at N, and a the first particle in the reflecting surface. When a longitudinal wave travelling along MN reaches the particle b this particle attempts to transmit the forward displacement to a. The particle a, however, is fixed or only slightly displaceable, according as AB is a rigid surface or the surface of

a dense medium, and b, in attempting to displace a *forwards*, is itself, by reaction, displaced *backwards*, and so originates the reflected wave back along NM.

Hence, if the upper curve along MN in Fig. 87 is the curve of displacement of the incident wave at the instant the disturbance reaches the reflecting surface, then the

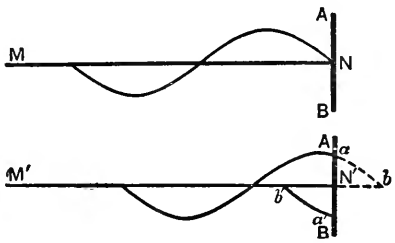


Fig. 87.

lower curves along $M'N'$ may represent the curves of displacement for the incident and reflected waves at an interval rather less than a quarter of a period later. In this interval the portion of wave represented by ab has been reflected at AB, and the displacement curve for this reflected portion is given by $a'b'$. The successive displacements in the incident wave are here shown reversed in the reflected wave.

The resultant displacement in the air near the surface AB is obtained by adding the curve $a'b'$ to the remaining portion of the incident wave. The resultant curve runs into the surface AB at N'.

The kind of strain in the wave is not, however, changed by reflection under these conditions. Thus in the curves along $N'M'$ in Fig. 87 the states of gradually decreasing compression found from b to a in the incident wave are found unchanged in the reflected wave from b' to a'. That is, each kind of strain is reflected unchanged at the surface AB, compression being reflected as compression and rarefaction as rarefaction.

Exp. 43.—Fix one end of a wave machine spiral (see Exp. 11) and originate a compression at the other end. It will be found that when the compression reaches the fixed end it is *reflected back as a compression.* Similarly, if a rarefaction is transmitted along the spiral to the fixed end, it is there *reflected back as a rarefaction.*

It is obvious that if AB is a smooth and practically rigid surface the incident wave will be almost totally reflected. A small portion of it will probably be diffused or scattered at the surface, and a very small portion may be transmitted on into the substance behind the reflecting surface. If, however, AB is the surface of separation between the medium of the motion and a denser medium, then only a portion of the energy of the incident wave is reflected, and a considerable portion may be transmitted on into the denser medium. The greater and more sudden the change of density the larger the proportion of the energy which is reflected.

115. Reflection without Change of Sign.—When the movement of one portion of an undulating substance is communicated to another of smaller mass, a different kind of reflection takes place. Suppose that there is a series of trucks, A, B, C, D, E, F, G, ... of which those from A to F are loaded, while the rest are unloaded, and that a push is given to A which compresses it. Each truck from A to E moves in turn to the left, and is brought to rest by the increasing pressure between it and the next. When F moves on in its turn, and the pressure between F and G begins to increase, G, being lighter, moves on more quickly than the heavier trucks did, so that the pressure between F and G does not increase so fast as that between E and F did; F therefore moves further than the other loaded trucks before it comes to rest, and G moves the same distance as F This evidently leaves F in an expanded or rarefied condition, and also leaves a greater space between F and E than between E and D. E therefore begins to move towards F, expanding its buffers as it does so, and then D towards E, and so on, and in this way a rarefied condition travels back from F to A, while a condensed condition travels forward from G along the rest of the

train. As before, these two contain together the energy of the original condensation.

In this case it is to be noticed that the reflected pulse is of the opposite kind to the original one, rarefaction instead of condensation, but that the movement of the trucks when the reflected pulse reaches them is in the *same* direction as it was when the original condensation reached them. The condensation is reversed in sign, the velocity is not. This is called reflection *without change of sign.* A rarefaction arriving at F starts a condensation back in the same way.

Reflection of this kind happens when a wave which has been travelling in a dense medium reaches a rarer one, for instance, when sound waves produced under water reach the surface, or when waves travelling in air reach the surface of a gas flame. Every condensation that arrives starts a rarefaction back, and *vice versâ.* If AB in Fig. 87 represents the surface of separation in this case, the particle b in attempting to displace the particle a forwards meets with little resistance, and consequently undergoes at any instant a greater *forward* displacement than that communicated to it directly by the incident wave. The additional forward displacements thus impressed on b are transmitted back along NM, and constitute the reflected wave.

Exp. 44.—It can be seen by experimenting with a wave spiral that if both ends of the spiral are free a *compression* (or rarefaction) originated at one end is reflected at the other *free end* as a *rarefaction* (or compression).

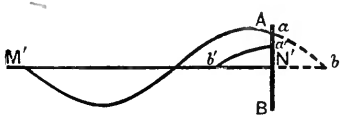


Fig. 88.

The curves along $M'N'$ in Fig. 88 represent, as explained with reference to Fig. 87, the curves of displacement for the incident and reflected waves for reflection at the surface

of a rare medium at an instant rather less than a quarter of a period after the first incidence of the wave on AB. In this interval of time the portion of the wave represented by ab has been reflected at AB, and the displacement curve for this portion when reflected is given by $a'b'$. The successive displacements in the incident wave are not reversed in the reflected portion, but they are diminished in magnitude as, under the conditions considered, the reflection can only be partial. The state of strain is, however, reversed by the reflection, for the phases of gradually decreasing compression found from b to a in the incident wave become the phases of gradually decreasing rarefaction found from b' to a' in the reflected wave.

As before, the resultant displacement in the air near the surface AB is obtained by adding the curve $a'b'$ to the remaining portion of the incident wave.

In this case of reflection at the surface of a rare medium it is evident that only a small portion of the energy of the incident wave will be reflected. The main portion of it is transmitted on into the rare medium.

Exp. 45.—Take two wave machine spirals (of the same diameter) one of which is made of thicker wire than the other, and suspend them so that they hang in the same horizontal line, end to end. Tie the turns at the adjacent ends of the spiral firmly together with thread, so as to make a secure junction between the two spirals. This junction represents the surface of separation of two media represented by the two spirals, the denser medium being represented by the spiral made with the thicker wire.

With this compound spiral the reflection of longitudinal wave motion at the surface of a denser or rarer medium can be easily studied. A *compression* (or rarefaction) originated at the free end of the light spiral travels along it up to the junction, where it is in part reflected back as a *compression* (or rarefaction) and in part transmitted on along the heavier spiral. Similarly, a *compression* (or rarefaction) originated at the free end of the heavier spiral is in part reflected at the junction as a *rarefaction* (or compression) and in part travels on along the lighter spiral.

In either kind of reflection, each pulse which arrives at the second medium sends a pulse forward into the second medium as well as one back again through the first,

though the forward pulse may be of very small intensity if the difference of density is great. This forward pulse in the second medium is of the same kind as the original pulse in the first medium, neither the kind of pulse nor the direction of movement of the substance being reversed in it.

Reflection with change of sign is also called reflection *without change of type*, and *vice versâ*.

116. Reflection in Tubes.—As might be expected, the first kind of reflection occurs when sound waves travelling along a tube reach its closed end; each pulse as it arrives starts one of the same kind back. The second kind of reflection occurs when sound waves passing along a tube reach its *open* end.

It is difficult at first to see why this should happen, since the air beyond the end of the tube is not less dense than that in the tube, but the reason is somewhat as follows:— In the case described in Art. 115 the reason why a condensation reaching F started a rarefaction back again was that when F moved on, the pressure in front did not increase so fast as it had done with the other heavy trucks, so that F moved further than they did. The same occurs when a pulse comes to the open end of the tube. Let AB (Fig. 89) be a tube of 1 sq. cm. area of cross section, and let CD be a thin layer of air in the tube. Suppose CD displaced 1 mm. to the right in $\frac{1}{33000}$ sec. At the end of this interval the effect will not have spread beyond 1 cm. of the tube on each side of CD, so that an extra hundred cubic millimetres of air have been forced into the cubic centimetre of space immediately to the right of CD, and the pressure there increased accordingly. But if a layer EF at the end of the tube moved a millimetre to the right in $\frac{1}{33000}$ sec., at the end of this interval the condensation would affect all the air within a radius of 1 cm. from the end of the tube, and only the same volume of

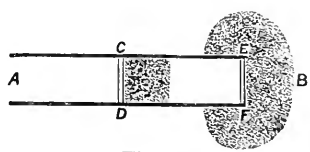


Fig. 89.

extra air, 100 cubic mm., has been added to this much larger space.

The rise of pressure to the right of *EF* is therefore not nearly so great as it was in the case of *CD*. So that when a wave of condensation passes along *AB*, and each layer of air passes on its energy to the next and comes to rest itself, *EF* will not have expended all the energy passed on to it by the previous layer when it has moved the same distance as the other layers moved. *EF* will therefore move further than the other layers did, and so, while it transmits a condensation to the air in front, it will leave a rarefaction behind it, which will travel back along the tube just as in the case of the railway trucks. In the same way, a rarefaction reaching the end of the tube spreads a rarefaction through the air beyond the end, but starts a condensation back along the tube.

117. Experimental Illustrations.—Echoes form the commonest example of the reflection of sound waves at the surface of a denser medium. They will be further treated in Art. 122.

Exp. 46.—Arrange two tubes. each about 3 feet long and 2 inches in diameter, at an angle as shown in Fig. 91 (tubes of thick paper are sufficient). Adjust a flat surface of a book cover, a slate, or a square of wood or cardboard, as shown at *ab*, so that the reflection from this surface of the ticking of a watch at *A* is most clearly heard at *B*. A felt screen should be placed between *A* and *B* to cut off the direct sound from *A*. When this adjustment is made it will be found that the reflecting surface is equally inclined to both tubes, and that the axes of the two tubes and the normal to the surface are in the same plane. That is, sound waves incident along the tube from *A* are reflected at the surface *ab*, so that the angle of reflection is equal to the angle of incidence, and the directions of incidence and reflection are in the same plane as the normal to the surface.*

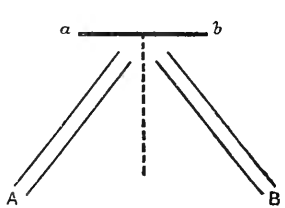


Fig. 90.

* Cf. *Text-book of Light*, Art. 23

Exp. 47.—Set up a large concave mirror, and place a watch on the axis of the mirror between the principal focus and the centre of curvature of the mirror. Attach a length of rubber tubing to the stem of a small funnel, and, using the funnel as a sound collector, find the point on the axis of the mirror at which the reflection of the ticking of the watch is most clearly heard. This is the point at which the sound of the ticking is brought to a focus after reflection at the surface of the mirror, and it will be found that the relation between the distance of this point from the mirror and the distance of the watch from the mirror is in agreement with that required by the theory of the reflection of wave motion at a concave spherical surface.*

This experiment may be modified by using two large mirrors facing each other, the watch being placed at the principal focus of one mirror and the funnel at the principal focus of the other mirror. The ticking of the watch is reflected from the first mirror to the second one, and there again reflected to the focus, where it can be distinctly heard by means of the funnel and tube collector.

Exp. 48.—Repeat Exp. 46, using a flat gas flame as the reflecting surface. It will be found that good reflection takes place at the surface of separation of the air from the hot gas of the flame. Reflection will also be found to take place at the surface of the hot current of air and gases ascending from the flame.

The experiments on reflection and refraction work much better if very short sound waves are used. A bird-call or shrill whistle is a very convenient source of such waves, and a sensitive flame forms a very good detector. These instruments are described in Art. 140.

118. Relation between the Wave Length and the Effective Transmission of Sound Waves through Apertures.—In the cases of wave-propagation which we have so far considered, the *wave-fronts*, or continuous surfaces drawn through those points where the air is in the same phase of its vibration, have been either complete spheres or have been bounded at the edges by the walls of a tube; in either case the wave-front can only travel at right angles to itself; expansion in its own plane is impossible. But when wave-fronts, expanding through the air, reach a fixed obstacle, only the pieces of the wave-fronts which are stopped by the obstacle are reflected. We require therefore to know how pieces of wave-front travel through the

* Cf. *Text-book of Light*, Art. 40.

air when they are not confined at the edges by the walls of a tube.

Suppose we have a screen or wall *EF* (Fig. 91) of some material which does not transmit sound, and that in this screen there is a hole *G*. If waves or shells of condensation

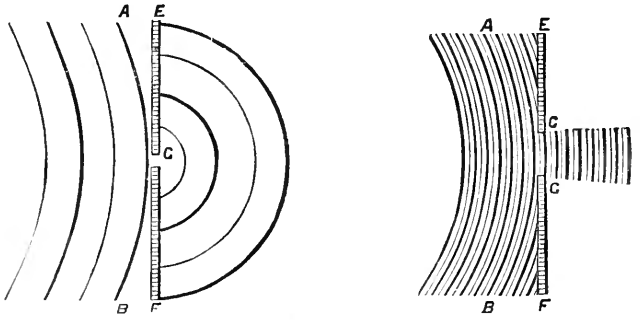


Fig. 91.

and rarefaction, such as *AB*, arrive at this screen from a source to the left of it, a piece of each wave-front passes through the hole, and spreads through the air to the right of the screen. How it spreads depends on the relation between the diameter of the hole and the distance between successive shells of maximum condensation (wave-length).

As each shell of condensation arrives at the hole, the air in the hole becomes condensed and moves to the right, exactly as it would do if a solid piston fitting the hole moved to the right, and, if the distance between a wave-front of maximum condensation and one of maximum rarefaction is larger than the diameter of the hole, a shell of condensation spreads spherically in every direction through the air to the right of the screen, exactly like the waves produced by the vibrating piston in Fig. 89; rarefactions spread similarly in their turn. This is illustrated in Fig. 91 (*a*); the thick circles are wave-fronts of maximum condensation, the thin ones those of maximum rarefaction. In this case the sound is audible at any point.

But, if the wave-length is much smaller than the diameter of the hole, the piece of each shell which passes through hardly spreads at all after reaching the other side, but travels at right angles to its own wave-fronts* as shown in Fig. 91 (*b*). In this case no sound is audible except at points from which straight lines can be drawn to the source of the sound; we have in fact on the further side of the hole a *beam* of sound in a definite direction from the source.

This may be considered as an instance of "superposition" (Chapter IV.). Let H (Fig. 92) be any point from which the source of sound would not be visible, and let waves be arriving from the left whose wave-length is much smaller than the diameter of the hole. Then the distances of H from different parts of the hole differ by more than half a wave-length. We may suppose the hole divided by cross lines into small holes, and we can pair off each of these small holes with another whose distance from H is half a wave-length greater or less. Let J, K be such a pair. Then, if all the opening except J was blocked up, waves would spread from J in all directions to the right, and a condensation would reach H at the same moment that a rarefaction would reach H if all the opening except K was blocked up, so that, if J and K are open together and the rest of the opening blocked, there is no vibration at H. As this applies to every pair of small holes, there is no vibration at H when the whole opening is free. (It is not usual that they can all be paired off in this way, but the unpaired excess is generally a small fraction of the area of the hole; and the sound intensity due to it is very small.)

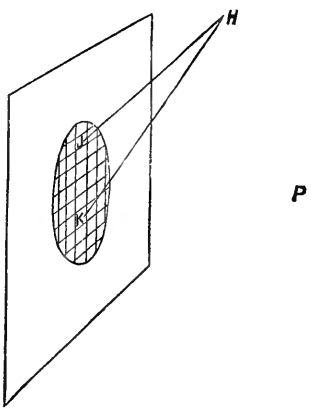


Fig. 92.

* This would also be the case with the waves produced by a vibrating solid surface if they were short compared to the surface, but practically solid surfaces cannot be made to vibrate fast enough for this, so that the waves spread from them in all directions.

It is evident that this argument does not apply to a point such as P, from which the source would be visible through the opening, for the different parts of the opening are all practically at the same distance from P. So that there are waves from the opening in the direction of P, but not in the direction of H. Also the argument would not apply if the wave-length was greater than twice the diameter of the hole, for then we could not find two parts of the opening whose distances from H differed by half a wave-length. In that case there would be vibration at H, wherever H was taken, that is, the waves would spread in all directions from the opening, but the intensity would be less intense as the direction was more oblique. There is not a definite line between the two cases; the smaller the wave-length compared to the opening, the less the pieces of wave-surface which come through the hole spread sideways, and the more nearly the vibration is confined to the regions from which the source would be visible.

The part played by the screen is simply to limit the size of the piece of each wave-front. It does not matter what limits the size; any limited piece of a travelling shell of condensation or rarefaction, if small compared with the distance between two successive shells, spreads in every direction in front of it, but a piece of a travelling shell which is large compared with the distance between two shells advances at right angles to its own surface, and does not spread. As each piece of shell is accompanied by a nearly constant quantity of energy, the intensity of the sound diminishes very slowly with distance when the pieces of shells do not increase in size.

The different behaviour of waves of different lengths is easily shown experimentally.

Exp. 49.—Take a sheet of roofing felt, say 3 feet square, and cut a round hole 6 inches in diameter. Place a large humming top on one side of the felt and 2 feet from the hole. You will find that the sound of the top is quite audible at any point on the other side, and is not perceptibly louder at points from which the top is visible through the hole than at points in quite a different direction from the hole; the sound is, however, nowhere nearly as loud as if the felt was removed. Now we substitute for the top, which gives waves perhaps 4 feet long, a source of sound, such as a watch, which gives waves from 2 inches to half an inch in length. You will find that at any point from which the watch is visible through the hole the sound is nearly as loud as if no screen was interposed, while at points from which the watch is not visible it is hardly audible at all.

119. Relation between the Wave-length and the Effect of Obstacles.—There is a similar difference between the behaviour of waves of different lengths when an obstacle is interposed between a source of sound and the ear. If the interval between successive shells of condensation is greater, or even not much smaller, than the diameter of the obstacle, the waves close in round the edge into the space behind the obstacle, and the sound is heard at any point. But waves which are very close compared with the diameter of the obstacle do not close in much, but advance at right angles to their own surfaces, each shell advancing with a hole in it where part has been stopped by the obstacle. The obstacle thus casts a **sound shadow** whenever the wave-length of the sound is much less than the dimensions of the obstacle. This, like the corresponding case of the opening in a screen, may be shown to be an instance of interference.

Exp. 50.—Hold a quarto magazine between the ear and a watch; the sound is almost cut off. Similarly hold it between the ear and a clock; the sound is not cut off. This is more strikingly shown by placing the watch between the clock and the ear, so that both are heard at once, but in such a position that the watch sounds much the louder. Place a thick quarto magazine in the line from the ear to the two sources of sound; the clock is heard much louder than the watch, even if the watch is audible at all. Now interpose a large screen (such as a pile of open newspapers), the sound from both is cut off; now interpose a packet of post-cards, neither sound is cut off.

120. Huyghen's Construction for Reflected Waves.—When a wave of condensation reaches a plane solid or liquid surface, a wave of condensation begins to travel away from each point of the surface as soon as the original shell of condensation reaches that point. Thus let AA' (Fig. 93) represent a condensation of a plane wave incident on the surface AB. As this condensation impinges on AB the particles A, C, D, etc., in turn become secondary sources of wave emission. By the time the portion A' of the original shell has reached B, the first wave from A has travelled a distance AB' from A where $AB' = BA'$, and if BB' be drawn tangential to the sphere

of radius AB' it will touch all the extreme spheres of disturbance emanating from the points between A' and B'. Therefore BB' gives the direction of the reflected shell of

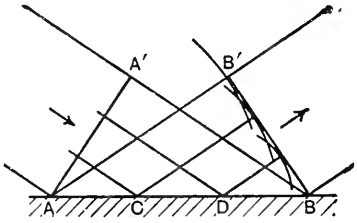


Fig. 93.

condensation and the sound now proceeds in a direction perpendicular to it. The figure is symmetrical about a normal so that the angle of incidence is equal to the angle of reflection, *i.e.* the reflected shell of condensation makes

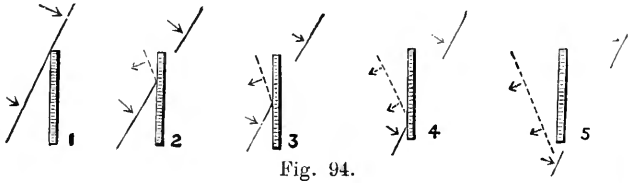


Fig. 94.

the same angle with the plane surface as the original shell. Fig. 94 shows more clearly some successive stages in the formation of such a reflected shell, the reflected part being shown by a broken line.

If the piece of each shell which is reflected is large compared with the distance between two condensed shells, the reflected pieces advance at right angles to their own surfaces without spreading, and the effect of the reflected sound is confined to a reflected beam, which follows the same law as the reflection of light. If the condensed waves are farther apart than the diameter of the reflecting surface, the reflected waves spread in every direction from the reflecting surface, and diminish so rapidly in intensity

that they cannot be detected at a short distance. The same is true when the reflection is "without change of sign," as from the surface of a gas flame; hence in the Exps. 46, 47, 48 the source of sound must produce very short waves.

121. Reflection of Spherical Waves at Spherical Surfaces.

Exp. 51.—Let a tap drip gently into water placed in an elliptical vessel such as a "tin bath." Spherical waves diverge from the point where the drops fall, and if this point is situated at one of the foci of the ellipse, the waves are reflected at the walls of the vessel and shrink in upon the other focus.

When a shell of condensation or rarefaction arrives at a concave solid surface, and is turned back, the reflected

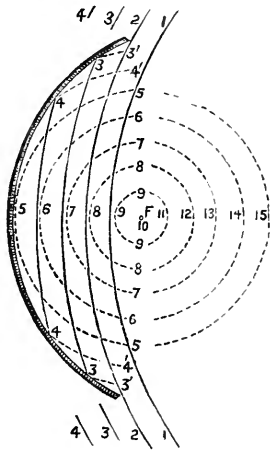


Fig. 95.

shell is concave to the direction in which it is going, if the convexity of the original shell was not too great. Some stages of such reflection are shown in Fig. 95, where the lines marked 1, 2, 3, 4, etc., are successive positions of a shell of (say) condensation which originally started from

a source *C*. As each part of the shell reaches the concave surface, a condensed condition starts back, so that when one part of the shell of condensation has reached the position 33, other parts have already started back and reached the position 33′, so that 3′333′ is a continuous shell of condensation, of which the part shown by the complete line is still travelling towards the mirror, while the broken line parts are travelling away. If these reflected portions are large compared with the distance between a condensed and a rarefied shell, they will advance at right angles to their own wave-fronts without spreading, assuming the successive positions 4′444′, 555, etc., so that the reflected wave may (if the reflecting surface is rightly shaped) converge towards a point *F*. At this point the intensity of the sound is very great, for the diminishing pieces of shells have always the same energy; the sound is brought to a focus just as light is. After reaching *F* the wave diverges again, on the other side, assuming successively the positions 11, 12, etc.

All this is only true if the mirror, and therefore the reflected pieces of the shells, are large compared to a wave-length. If this is not the case, the reflected wave spreads in every direction, instead of converging. In fact, in that case it makes no practical difference what the form of the mirror is. This seems to be lost sight of by some makers of ear trumpets, who attempt, by parabolic reflectors, to converge the sound waves into the ear. No reflector, whatever its shape, can do this unless it is many times larger than the wave-length of the sound waves, and the waves produced in conversation are not often less than 18 inches long. A funnel-shaped tube concentrates sound to some extent, but it acts not by making the wave-fronts concave, but by communicating the movement and energy of each layer of air to a smaller one, so that the amplitude continually increases.

In Fig. 95 the reflected condensed shell is represented converging till it becomes a mere point. This does not really happen, because before it gets so small it ceases to be large compared with the distance between it and the shells of rarefaction in front and behind, and then it no longer advances only at right angles to itself, but spreads at the edges. Fig. 96 shows, roughly, some successive stages in the advance of a concave wave-surface which is no longer large compared to a wave-length.

If *F* is the centre of the reflected shells as long as they remain spherical, it is evident that the total distance from *C* to any point of the mirror, and thence to *F*, must be constant, for every part of

the same reflected wave-front (999, for instance) started at the same time from C, has been travelling with the same velocity ever since, and is now at the same distance from F. It can easily be shown that this is equivalent to saying that lines from C and F to any given point of the mirror make equal angles with the reflecting surface; *i.e.* F and C are conjugate foci for light (*Text-book of Light*, Art. 40), so that the point where the sound is most intense is the point where an image of the source is formed if the mirror is polished. In fact, light waves are reflected just as sound waves are when the sound waves are short in comparison with the linear dimensions of the reflecting surface.

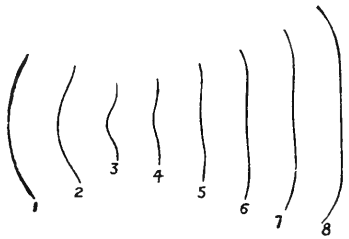


Fig. 96.

If the concave surface is a yard across, a watch placed at C is heard as loudly by an ear at F as if the watch was an inch instead of perhaps several feet away, but there is no increase of loudness due to the mirror at a point a few inches from F. If a large tuning-fork is used instead of a watch, the sound will not be perceptibly louder at F than at other points.

As before, we may consider the intensity of the sound at F, and its absence at points near F, to be due to superposition; to any point except F there are routes from C (*viâ* the reflector) of different lengths, so that condensation would arrive by one route at the same time as rarefaction by another. As long as all routes from C to F *viâ* the reflector are too nearly equal for this to happen, it makes little difference to the intensity at F whether they are *exactly* equal or not. So that roughnesses or irregularities of the mirror are unimportant if they are much smaller than a wave-length; the mirror need not be polished, and may be of gutta-percha or sheet lead.

Cases of reflection which arise when (1) the waves are spherical and the surface is plane, (2) the waves are plane and the surface is spherical, should be dealt with in detail by the student himself.

122. Echoes.—The reflection of sounds from any suitably placed reflecting surface produces what is known as an *echo*. A cliff, a hill-side, the walls of a building, the edge of a forest, clouds, all serve well to reflect sound, and it is by direct reflection at such surfaces that echoes are usually produced.

An echo, to be audible, must be formed by reflection against a surface whose dimensions are very large compared with a wave-length of the sound, for otherwise the sound energy intercepted by the surface is dispersed in all directions, and soon becomes inaudible. The sound of a gun requires a cliff or high wall to form a good echo, but a much smaller surface will give a clear echo of a shrill whistle.

It has been found that when a sound is heard the sensation *persists* about one-tenth of a second after the exciting stimulus has ceased. As a result of this it evidently follows that an echo of any sound can be fully and clearly heard only if the reflected wave returns to the ear of the observer one-tenth of a second *after* the direct wave left it. Hence, if the sound is of practically no duration, as in the case of a pistol-shot or the blow of a hammer on an anvil, an echo will be heard distinctly if the sound takes one-tenth of a second to reach the reflecting surface and return to the observer. Under ordinary conditions the velocity of sound is about 1120 feet per second, so that it travels 112 feet in one-tenth of a second. The echo of a sound of very short duration can, therefore, be heard at a point distant about 56 feet from the reflecting surface.

If, however, the exciting cause of a sound sensation lasts for one-tenth of a second the sensation itself will last for about one-fifth of a second, and a complete echo will be heard only if the distance of the reflecting surface from the observer is about 112 feet. With this distance between the observer and the reflecting surface the echo reaches the ear just as the first sensation ends. Similarly, if the direct wave acts on the ear for, say, ·4 second the sensation will last for about ·5 second, and a complete echo will be heard if the distance of the reflecting surface from the observer is about 280 feet. If, in this case, the distance were 112 feet instead of 280 feet the echo would reach the ear in

about ·2 second, and would, therefore, for the next ·3 second for which the first sensation lasts be confused with, and indistinguishable from, this sensation. For the further ·1 second for which the echo lasts it would be clearly heard for about ·2 second as an echo of the last quarter of the initial sound.

In the case of words uttered by the human voice it is found that the most rapid rate at which short syllables can be uttered and *heard as distinct sounds* is about five per second. That is, the sensation for any short, sharply uttered syllable lasts for about one-fifth of a second, and a complete echo of it can, therefore, be heard only if the distance of the reflecting surface is at least 112 feet. If a number of syllables are rapidly spoken, then, unless the reflecting surface is at a sufficiently great distance, the echo of the first syllables will be confused with the last directly spoken syllables, and only the echo of these last syllables will be clearly heard. Thus, if the distance of the reflecting surface from the observer is about 112 feet, then only the echo of the last syllable uttered will be clearly heard, for the echo of this syllable will reach the ear just as the sound of the syllable itself ceases to be heard. If the distance is 224 feet, then, evidently, the echo of the last two syllables can be heard; and if the distance is n times 112 feet, then the echo of the last n syllables can be heard. Echoes which repeat two or more of the syllables uttered are sometimes called *polysyllabic echoes.*

The data of these calculations, the period of persistence of a sound sensation, and the number of distinct syllables which can be uttered and heard in a second, are, of course, only approximately known and vary within narrow limits for different individuals. It must be remembered, too, that the distance of 112 feet given above in connection with polysyllabic echoes assumes that the conditions are such that the velocity of sound is 1120 feet per second. If the velocity were 1100 feet per second, then 110 feet would be the distance travelled in one-tenth of a second.

When a sound is reflected from a number of suitably placed reflecting surfaces a number of echoes may be

heard, and if the surfaces are at different distances the succession of echoes may last for some time. The rumbling and rolling of thunder, for example, is probably the echoing of the peal of thunder from a number of reflecting surfaces such as surfaces of separation between atmospheric currents, cloud surfaces, rocks, mountain sides, and cliffs. With two suitably placed surfaces, too, it is possible for a sound to suffer successive reflection from one surface to another, and a very prolonged echo may thereby be produced.

Echoes afford a rough method of finding the velocity of sound. If we shout, and observe the interval before the echo is heard, this interval is the time taken by sound to travel twice the distance from us to the cliff and back. A more accurate method is to use a metronome,* beating loudly but slowly. Move the metronome away from a reflecting surface until the ticks coincide with the echoes of the immediately preceding ticks. Then twice the distance of the instrument from the wall divided by the interval between successive ticks is equal to the velocity of sound.

123. Musical Notes produced by Reflection.—If a short, sharp sound, such as may be made by striking two stones together, is subjected to repeated reflection from the walls of a room or passage, or from a flight of stairs, the successive echoes may reach the ear in sufficiently rapid and regular succession to produce the sensation of musical note. The musical ring which sometimes accompanies the sound of a footstep in a bare hall or corridor is an instance of this effect.

Similarly, if a short, sharp sound is produced near a paling or a railing the sound may, if its wave disturbance is sufficiently short, be reflected from each individual pale or rail, and the successive reflections will reach an observer in a regular succession which may be rapid enough to produce the sensation of a musical note. This explains

* A metronome consists of a weighted pendulum whose frequency of vibration can be altered by altering the position of the weight. It is largely used by music teachers to give the "time."

the musical sound which is heard when a sound, such as may be produced by striking two keys together, is produced near an iron railing.

Fig. 97 shows how the reflected waves are distributed. *O* is the source of sound and also the observer, *A*, *B*, *C*, *D*, . . . , are the pales. The figure represents the stage when the

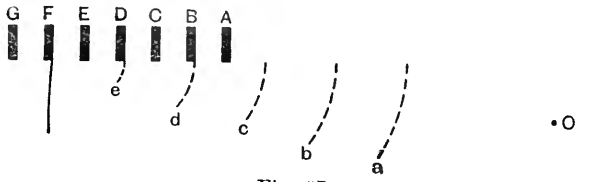


Fig. 97.

shell of condensation has reached *F*. The portions reflected by *A*, *B*, *C*, *D*, *E*, have spread to *a*, *b*, *c*, *d*, *e*; thus the wave-length of the note heard by the observer is approximately equal to twice the distance between the pales.

124. Other Illustrations of Reflection of Sound.—If a room is of such a size and so constructed that the echoes from the walls interfere with the obscure words and sounds almost as soon as they are produced, the room is said to be bad acoustically for speaking or singing. To remedy this defect curtains and screens are sometimes put up to prevent the reflection which gives rise to the echoes.

The action of a speaking tube is a good illustration of the reflection of sound. The wave energy which, without the action of the tube, would spread out into a rapidly increasing volume of air is directed along the tube and, by repeated reflection from the inner surface, confined to the air in the tube. For this reason the intensity of the sound does not increase according to the law of inverse squares, as it should do if the disturbance spread out rapidly from the point of production, but remains practically constant, subject only to loss by the dissipation of energy which accompanies reflection.

From Fig. 98 it can be seen that the wave which would, in the absence of the tube, spread out radially from *O* within the solid angle represented by *AOB* is confined by reflection from the wall of the tube to the air within the

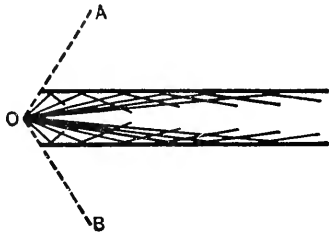


Fig. 98.

tube, along which it travels with very little diminution of intensity with distance.

The ear trumpets used by deaf persons also furnish illustration of the reflection of sound. The wave energy which enters the wide end is directed by repeated reflections through the trumpet to the ear, and as the volume of the air by which the energy is carried decreases as the ear is approached, the intensity of the wave motion is increased.

Another illustration of the reflection of sound is found in the case of a *whispering gallery*. In the circular gallery in the dome of St. Paul's Cathedral a whisper is reflected round the wall of the gallery so as to be audible at any point of its circumference. The action of the gallery is practically that of a circular speaking tube. If sound is uttered at *O* (Fig. 99) the wave radiates in all directions from *O*, but the portion included between the two lines *OA* and *OB* is evidently reflected round and round the gallery in the air between the wall and the cylindrical surface to which the lines *OA* and *OB* are tangential.

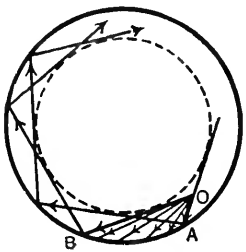


Fig. 99.

In this way a whisper at O can be heard anywhere near the wall of the gallery.

Those who have travelled on a steamboat in a narrow river or canal will have observed that the waves striking the banks are not reflected, but seem to cling to the banks and advance with the boat. Very likely something of the same nature occurs in whispering galleries, for the sound is produced so close to the walls that the ordinary laws of refraction can hardly be applied.

125. Huyghen's Construction for Refracted Rays.—As before (Art. 120), let AA' (Fig. 100) represent a narrow shell of condensation of a plane wave advancing in a medium M on a plane surface AB separating the medium M from the medium N. Each point of AB becomes in succession a centre of disturbance, and by the time the whole shell AA' has reached AB the refracted shell will have advanced to BB', where

$$AB' = \frac{\text{velocity of sound in } N}{\text{velocity of sound in } M} \times A'B.$$

Thus, if the velocity of sound in N is less than the velocity of sound in M, the sound is bent towards the normal, while if the velocity is greater in N than in M the deviation is away from the normal. This change of front, due to the difference of velocity in the media, is characteristic of refraction when the incident wave-front makes an angle with the surface of separation. When, however, the incident wave-front is parallel to the surface of separation, there is no change of front, for the refracted wave-front is parallel to the incident wave-front, but travels on in the new medium with the velocity of wave motion in that medium.

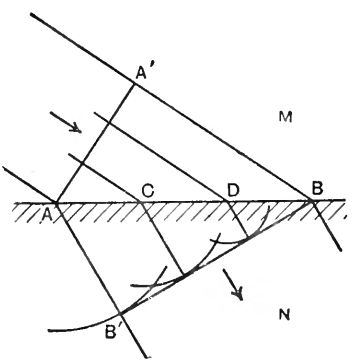


Fig. 100.

126. Refraction of a Spherical Wave at a Plane Surface of Separation between Two Media.—In order to follow the process of refraction of a spherical wave at a plane surface of separation between two media it is convenient to divide the surface up into small elements, such as AB in Fig. 100, and to follow the refraction and subsequent transmission of each element as it reaches the surface of separation.

In Fig. 101 the incident spherical wave diverges from O in the medium M. As the wave advances on to the separating surface, the elements of wave-front 1, 2, 3, 4 are refracted in succession as they reach the surface, and each new element of the refracted wave-front as it is formed advances into the medium N in a direction normal to itself with the velocity of the disturbance in that medium. In this way, as the elements marked 1, 2, 3, 4 are refracted in succession, the refractive wave-fronts indicated by the dotted curves 1, 2, 3, 4 are formed.

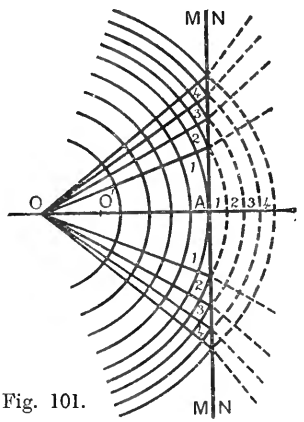


Fig. 101.

The refracted wave-fronts obtained in this way are not spherical. If, however, we consider the refraction to take place only at a small area of the separating surface, taken round A, the foot of the perpendicular from O on to the surface, it can be shown that the refracted wave is approximately spherical and diverges from a point O' on the line OA, such that

$$\frac{OA}{OA'} = \frac{\text{velocity of sound in medium } N}{\text{velocity of sound in medium } M}.$$

Exp. 52.—We may make use of the fact that water waves travel faster in deep water than in shallow water to illustrate wave refraction. Using the apparatus of Exp. 51 place a disc of wood or other solid in the water so that it rests just below the surface. Originate a series of waves in the deep water near the disc. Observe how the curvature of the waves alters as they travel over the disc.

127. Partial Reflection and Refraction.—From what has been said above on the subjects of reflection and refraction of wave motion it will be understood that whenever wave motion is incident on a surface of separation between two media both reflection and refraction take place, and the incident wave gives rise to a reflected wave travelling back into the first medium and a refracted wave travelling on into the second medium. The sum of the energy of the reflected and refracted waves is equal to the energy of the incident wave, provided the loss by other processes at the surface of separation is negligibly small.

128. Experimental Illustration of the Refraction of Sound.—Sound, like light, may be refracted by lenses and prisms of suitable material, and the geometrical relations which apply in the refraction of light at plane and spherical surfaces apply also for sound.

A lens for the refraction of sound waves must be of gas, not of any denser substance, or nearly all the energy will

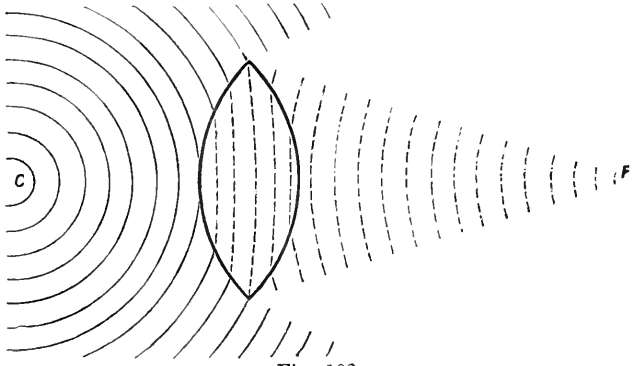


Fig. 102.

be reflected at its first surface. If two convex circular sheets of collodion film are cut from a large collodion balloon and attached by their edges to a metal hoop, and the space between them filled with carbon dioxide gas, we have a lens which will answer the purpose. Sound travels

more slowly in carbon dioxide than in air, so that when each shell reaches the lens it advances more slowly at its centre than at the edges, as shown in Fig. 102, so that when it comes out at the other side it is concave instead of convex towards the direction in which it is advancing. If these concave pieces of shells are large compared with the distance between successive waves, they will converge towards a point *F*, though, as explained above, they do not really become points. As in the case of the concave mirror, it is only sound whose wave-length is short compared with the diameter of the lens which can thus be converged; waves of greater length spread in all directions after passing through the lens, as if the lens was a mere hole, and do not converge.

A similar effect is produced by a concave lens filled with a lighter gas than air; coal-gas is best. Hydrogen does not answer well, the difference of density between air and hydrogen being so great that a large part of the energy is reflected on reaching the lens (Art. 116).

Prisms of collodion film filled with carbon dioxide have also been made, and give a deviated beam of sound if the wave-length is short compared to the prism, but there is no dispersion of sound of different wave-lengths, as in the case of light, since all sound waves travel with the same velocity. (The spectrum analysis of sound has, however, been performed with a gigantic diffraction grating.)

For the use of a bird-call and sensitive flame in experiments on refraction see Art. 140.

129. Alteration of Pitch by Reflection and Refraction.—The statements made above relate specially to harmonic waves. If the waves are non-harmonic, we may conveniently apply Fourier's device, and calculate the result of the reflected or refracted waves as if the "harmonic components" had a separate physical existence, and were reflected or refracted each according to its wave-length. The different harmonic components are of course differently reflected and refracted; thus a concave mirror converges the shorter harmonic components nearly to one point, while it scatters the longer ones in every direction.

The character of non-harmonic waves is therefore often entirely altered by reflection or refraction, or by simply passing an obstacle, having at one place a larger proportion and at another a smaller proportion of harmonic components of very short wave-length. A simple experiment shows this very well.

Exp. 53.—Hold a watch at arm's length, and interpose a large sheet of card between it and the ear. The sound is practically cut off. Now try with a post-card instead of the large sheet. The sound is not much fainter when the card is interposed, but it is a much lower note, or rather, as a trained ear will recognise, all the higher components, which give the peculiar sharp click, are cut off, and only the lowest of the notes produced is heard. The longer harmonic components come round the card to the ear; the shorter ones are absent behind the card (Art. 119).

Exp. 54.—Obtain an echo from a clump of trees or the edge of a wood. Note that the pitch of the echo is often an octave above the pitch of the primary note. The explanation has been given by Lord Rayleigh, who has shown that at reflection from a surface of variable density and compressibility the intensity of the secondary waves must vary inversely as the fourth power of the wave-length. The primary note given out by the observer is thus analysed into fundamental and octave by reflectors of the above type, and the octave strengthened sixteenfold relatively to the fundamental in the reflected sound.

130. Effect of Wind on the Propagation of Sound.—When there is a wind blowing, the air close to the ground travels more slowly than that higher up, so that on the side of the source towards which the wind blows the waves advance more rapidly above than below. As the wave-fronts, being large compared with a wave-length, advance at right angles to themselves, an obstacle such as A, Fig. 103, will not cut off the sound coming from a source C, from an observer at J, for the higher parts of the waves, which have passed over A, soon begin to advance in a slightly downward direction, and so reach the earth at a point beyond. It is for this reason that sound is much better heard on the side of the source towards which the wind blows, especially if there are obstacles between the source and the listener. In the other direction the waves advance less rapidly above than below, and soon leave the earth entirely, as shown in the figure, where the wind is

supposed blowing from right to left. (The difference of velocity above and below is very greatly exaggerated in the figure.)

Even if there are no obstacles, sound is better heard if there is a wind from the source to the observer than if there is not. For the parts of the waves which are close to the ground are constantly losing their energy by the

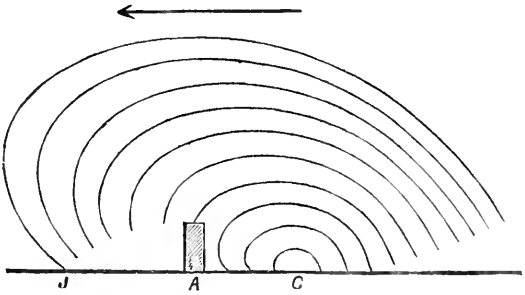


Fig. 103.

process described in Art. 26. As the waves we are now considering are very large compared with a wave-length, each part of a wave may be considered as transmitting its energy only at right angles to itself. If there is no wind, the waves are hemispheres, and the energy travels radially from the source, so that each part of the air near the ground gets its energy from a part also near the ground, *i.e.* from a part already exhausted. But if there is a wind, the waves on the leeward side of the source lean over as shown between *C* and *A*, and as the energy is transmitted at right angles to these, each part of the air near the ground derives its energy from a part of the air at a greater distance from the ground, which has not lost its energy.

It will be seen that there is really more energy lost on the leeward side than on the windward side of the source. If the observer is high enough above the ground to be in the waves, the sound is better heard to windward than to leeward; it is only near the ground that the opposite is the case.

A similar converse case occurs in the case of a tunnel or tubular bridge. If a person at one end speaks to one at the other, it is found that the sound is well heard if the wind blows through the tunnel from the listener to the speaker, but not otherwise. If there is no wind, the waves are spheres with the speaker as centre; the energy travels radially from the speaker, and is mostly lost at the walls. If there is a wind through the tunnel, it is always quickest along the axis, being checked by friction near the walls. If the wind is with the waves, it carries their central parts on faster than their edges, they become more convex, the energy diverges more rapidly, and more is lost. But if the wind is against the waves, it keeps the central parts back; the waves become concave in front, advancing at right angles to their surfaces, and leave the sides of the tunnel altogether, after which there is no further appreciable loss of energy. Under these circumstances a listener with his ear near the axis of the tunnel can hear the sound at a great distance from the speaker.

131. Effects of the Distribution of Temperature in the Air.—Refraction of an exactly similar character and attended by the same effects is produced by the variation of the temperature of the air with height above the earth's surface. Generally, in the day-time the temperature decreases as the height increases, and the velocity of sound is therefore greatest near the surface of the earth, and decreases as height increases. Under these conditions, then, sound is refracted in the same way as when it travels against wind. That is, for observers at the earth's surface, sounds do not travel far when the fall of temperature with increase of height is steady and well marked, as on a calm hot day. If the rate of fall of temperature upward becomes too great, the statical equilibrium of the air is upset and convection currents are set up. Under these conditions the sound waves are subject to so many reflections and refractions at the surfaces of separation of the various currents that they are quickly dispersed and broken up, and sounds are said to "carry" badly through the air.

Under certain conditions the air may be cooler near the earth's surface than at a higher level, and for a certain height there may be a rise of temperature with height. For example, in the evening after a very hot day the air near the earth cools more rapidly than higher up, and for a time a layer of air exists in which the temperature increases with height above the earth's surface. Under these conditions sound is refracted in the same way as when it travels with the wind, and sounds are found to carry to great distances along the ground.

When the conditions are such that the air is very still and the temperature is practically uniform for a considerable height, the sound waves travel through a perfectly homogeneous medium, and as there is very little dissipation of energy by reflection or refraction, sounds travel well, and may be audible at great distances.

132. Sound rays.—A ray is defined as a line drawn so that it cuts orthogonally all the wave-fronts that it meets. As disturbances are propagated perpendicular to wave-fronts, the sound may be supposed to travel along these rays. In homogeneous air the velocity of sound is constant, and therefore the waves diverging from a small source are concentric spheres and the rays are straight lines through the source. If the velocity varies from point to point, the rays are curved.

Fig. 104.

Let AB and CD (Fig. 104) be two short portions of rays near together, both taken in plane of paper, and AC and BD represent their wave-fronts. If these be not parallel they will cut in some point O and, since they are perpendicular to the rays, AB and CD can be regarded as circular arcs of which O is the common centre. Let R = radius of curvature OA of AB, $x = AC$, and let V, $V + v$ be the velocities along the two rays.

By similar figures

$$CD/AB = OC/OA = (R + x)/x.$$

But CD, AB are described in the same time.

$$\therefore\ CD/AB = (V + v)/V.$$

Hence $$(V + v)/V = (R + x)/R,$$

$$\therefore\ v/V = x/R,\ \ R = V/\frac{v}{x}$$

= Velocity of sound ÷ change of velocity per unit distance taken perpendicular to the ray.

If for instance the sound rays are horizontal, and the temperature falls 1° C. per 100 metres as we ascend, then since the velocity increases 60 cms. per sec. for every degree C. (Art. 43). $v/x = 6/1000$. Taking V as 34,000 cms. per sec., $R = 34{,}000 \times 1000/6 = 5{,}700{,}000$ cms. = 57,000 metres. Such a sound ray would rise nearly 10 metres while travelling a kilometre.

Changes of sound velocity due to wind are usually much more rapid than those due to temperature, and therefore the curvature produced is also greater.

133. Conditions favourable to the Propagation of Sound. —*The* condition essential for good propagation of sound waves is the *homogeneity* of the medium. If the medium is at rest, free from internal motion such as currents or eddies, and uniform throughout in temperature, density, and structure, the conditions are those most favourable for the propagation of sound waves in the medium. In the case of a medium containing small particles, the particles will not interfere with the propagation of wave motion through the medium, so long as the dimensions of the particles are small compared with the wave length. Thus, although smoke particles in mid air make it more or less opaque to light waves, they do not interfere in the least with the propagation of sound waves through the air. In the same way sound is readily transmitted through any screen or curtain which does not interfere with the continuity of the medium in which it is placed. For example, several screens of silk or loosely woven cloth interfere little with the propagation of sound in air, but a single sheet of *wet* silk wholly prevents transmission and acts as a reflecting surface.

It is found that foggy air is specially good for the propagation of sound. This is explained by the fact that fog can only form in homogeneous air, and foggy air is therefore specially homogeneous air containing fog particles which are much too small in dimensions to interfere in the slightest degree with the propagation of ordinary sound waves.

At the request of the Trinity Board, Tyndall conducted a long series of observations off the South Foreland with the object of determining the distance to which various sound signals could penetrate under all sorts of weather conditions. These observations proved that there is no relation between the visual clearness of the atmosphere and its transparency to sound. On some beautifully clear days sounds were extinguished at a distance of less than three miles, while on other occasions of much less optical transparency they penetrated to a distance of nearly thirteen miles. The presence of fog, falling snow, rain, or hail had no sensible effect in obstructing sound and often coincided with atmospheric conditions specially favourable to the transmission of sound. Tyndall concluded that the stifling of sound in air was due almost entirely to the scattering of the sound waves by repeated reflection and refraction at the surfaces of separation of air currents or of masses of air at different temperatures and in different hygrometric states. These masses of air Tyndall called *acoustic clouds*, and he attributed *aerial echoes* which he observed to reflection at the surface of these clouds.

EXAMPLES IX.

1. Explain why a brick wall will reflect waves of sound, but not waves of light.

2. Explain why a plane mirror one inch square will reflect waves of light, but not waves of sound.

3. Describe the process of reflection of longitudinal wave motion in a medium at the surface of (*a*) a denser medium, (*b*) a rarer medium.

4. In the case of reflection of longitudinal waves at the surface of a very dense medium most of the energy in the incident wave is reflected, while in the case of reflection at the surface of a very rare medium most of the energy is transmitted. Why is this?

5. Explain by a diagram how refraction takes place at the separating surface of two different media.

6. Show that the laws of reflection and refraction of sound waves are the same as the laws of reflection and refraction of waves of light.

7. The velocity of longitudinal waves in hydrogen is 1280 metres per second and in oxygen 320 metres per second. Find the index of refraction between oxygen and hydrogen for these waves.

8. What experiments would you perform to illustrate the fact that sound can be reflected from hot air?

9. Explain any method by which the ticking of a watch may be made audible to a person at the other end of a large room.

10. Explain why a watch is used rather than a tuning-fork in experiments on the concentration of sound by reflection or refraction. Also why is it not necessary that the mirror should be polished?

11. A person is walking between two parallel walls which are near together, and hears a prolonged echo of each footstep. Explain how the echo is produced.

12. Describe an experiment illustrating the use of sound-boards in making the vibrations of a wire audible.

13. A cannon is placed 550 yards from a long perpendicular line of smooth cliffs. An observer at the same distance from the cliffs hears the cannon-shot four seconds after he sees the flash. If the velocity of sound is 1100 ft. per second, when will he hear the echo from the cliffs?

14. An echo repeated six syllables. The velocity of sound is 1120 ft. per second. What was the distance of the reflecting surface?

15. Describe an experiment to show that sound may be refracted. Can sound be dispersed?

16. A strong wind is blowing. When I am some distance on the windward side from a sounding whistle I can easily hear it. When I am the same distance away on the leeward side I find difficulty in hearing it. Why is this?

17. Explain why in the evening after a hot day sounds "carry" very well along the surface of the ground.

18. Why is foggy weather sometimes specially good for the propagation of sound?

19. How do you account for the fact that the distance at which a loud sound (such as the discharge of a cannon) is heard varies considerably from day to day?

20. Explain as fully as you can what takes place when a wave of condensation travelling along a tube arrives at the open end. What difference would it make if the tube were full of air, while the space outside was filled with hydrogen?

CHAPTER X.

STATIONARY UNDULATION.

134. In Chapter IV. we considered what would be the effect if two systems of waves travelled simultaneously in the same direction through the same air. Let us now consider what is produced in any region of the air if two sources vibrate together which, vibrating separately, would send through the region, in exactly opposite directions, waves of the same length, amplitude, and wave-form. We will first suppose that this wave-form is harmonic.

Let each of the lines *Y*, *Z*, 11, 12, etc., in Fig. 105, represent the same region of air in different conditions. Suppose that there are two similar vibrating springs *A* and *B* (not shown), one to the left and the other to the right of the region. Let the top line *Y* represent, on the plan explained in Art. 1, the condition in which the air in this region *would have been*, at a given instant *T*, if *A* had been vibrating exactly as it is, while *B* was at rest, and let the second line *Z* represent the condition in which the same air *would have been*, at the same instant *T*, if *B* had been vibrating exactly as it is, while *A* was at rest. *Y* represents a progressive wave-system travelling from left to right, while *Z* represents an exactly similar wave-system travelling from right to left. The actual condition of the air in the region represented, at the instant *T*, will be (very nearly) the *resultant* of the two conditions represented by *Y* and *Z*.

This condition is represented in the third line marked 11. The height of the letter *C* or *R* at each point, which denotes the degree of condensation or rarefaction there, is

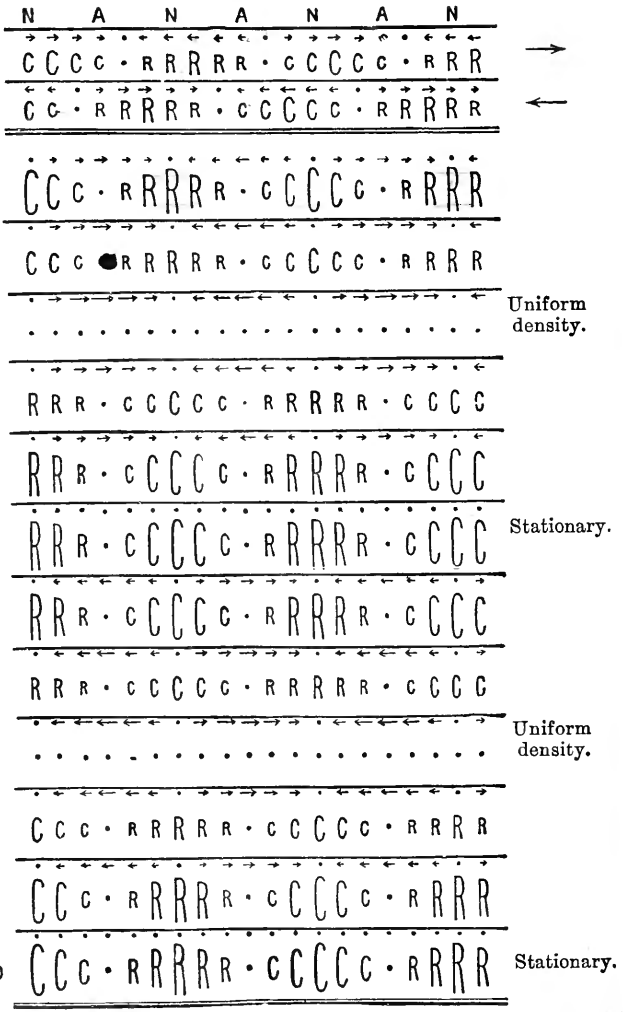

N A N A N A N
Y
Z
11
12
1
2
3
4
5
6
7
8
9
10
Uniform density.
Stationary.
Uniform density.
Stationary.

Fig. 105.

the sum of the heights of the letters at the corresponding points of *Y* and *Z* if they are both *C* or both *R*, and the difference of their heights if one is *C* and one *R*; and the length of the arrow over each letter, which denotes the velocity with which the air is moving at each point, is the sum or difference of the lengths of the arrows at the corresponding points of *Y* and *Z*, according as these are in the same or opposite directions.

On examining line 11, we see that we have a condition of the air quite different from any kind of progressive undulation. In every progressive undulation the velocity of the air is greatest where the condensation or rarefaction is most extreme, but here the greatest velocity occurs at the parts which are neither condensed nor rarefied, and the greatest density-differences occur where the air is at rest. So that, though there are regions where the air is condensed and others where it is rarefied, these do not correspond with the "condensations" and "rarefactions" of progressive undulation. In progressive undulation a "condensation" means a region where the air is all condensed, and all moving one way. In the condensed region in line 11 half the air is moving one way and half the other.

Next let us consider what will be the condition of the air in this region an instant later than the moment for which *Y* and *Z* are drawn. We can represent the condition in which the air would have been an instant later if *A* had been vibrating alone by shifting the line *Y* a little to the right, and the condition in which it would have been if *B* had been vibrating alone by shifting the line *Z* the same distance to the left. Suppose each shifted the distance of between two letters, and find the resultant condition again. This time we get line 12. We notice that, though we have supposed lines *Y* and *Z* altered only in position, the resultant condition has not been displaced either to right or left, but has changed in degree; the condensations and rarefactions have become everywhere less pronounced, while the velocity of the air has everywhere increased. We see, in fact, that in this mode of vibration a given condition of the air does not move along as it does in progressive undulation.

The greatest degree of rarefaction which existed at the moment shown in line 11 is not to be found in line 12 a little to right or left of its former place; it no longer exists anywhere. And, though there is nowhere so great a degree of rarefaction as in line 11, the greatest degree of rarefaction which exists anywhere in line 12 is in the same position as the greatest degree of rarefaction in line 11.

At certain points, those under the *A*'s in the top line, the degree of condensation which would be due to one of the progressive wave-systems *Y*, *Z* is exactly equal to the degree of rarefaction which would be due to the other, so that the actual condition of the air at these points, as shown in line 11, is one of average density. It is easy to see that, if each of the lines *Y*, *Z* is shifted onward the same distance, to represent the conditions which would be due to the two wave-systems a moment later, the rarefactions and condensations under the *A*'s are still equal, so that this condition of average density is a permanent one at these points, which are called **Antinodes.**

We see also that there are certain other points, those under the *N*'s in the top line, where the *velocity* of the air which would be due to one of the wave-systems *Y*, *Z* is equal and opposite to the *velocity* which would have been due to the other, so that at these points the air is at rest; and, as before, we see that this is a permanent condition at these points, which are called **Nodes.**

The antinodes are called by some writers "ventral segments or loops." This name is more properly applied to the whole region between one node and the next, an antinode being the middle point of a ventral segment.

The existence of points fixed with respect to the air, at which a definite condition of the air is always to be found, is the most striking peculiarity of this mode of vibration; hence the name **Stationary Undulation.**

To get a more exact idea of this condition, we will trace the changes in the region shown in the figure through some further stages, at each stage advancing *Y* and *Z* one letter as before. The condition of the air at these stages is shown in the lines 1 to 10. These complete a cycle, as the next stage after that shown in line 10 is line 11 again.

135. Characteristics of Stationary Undulation.—By comparing the successive lines of Fig. 105 we can make out the following important points :—

(1) **Nodes** occur at the points where the maximum condensations of one imaginary progressive wave-system (*i.e.* the wave-system which one of the sources would produce if it vibrated alone) arrive at the same moments as the maximum condensations of the other. **Antinodes** occur where a maximum condensation of one imaginary system arrives at the same moment as a maximum rarefaction of the other.

(2) The distance between two successive nodes, or two successive antinodes, is half the wave-length of the waves which the sources would produce separately. From a node to an antinode is one-fourth of this wave-length.

(3) The period in which the stationary undulation goes through all its changes is the period in which the waves which the sources would produce separately would advance their own wave-length, *i.e.* it is the vibration period of either source.

(4) Condensation and rarefaction do not move along as in progressive undulation; they simply appear and disappear again, to be succeeded by the opposite condition in the same place.

(5) The nodes are not places of greater *average* density than the rest of the air, but of greatest variation of density; each node is a point of maximum and minimum density in turn. The average density is the same at nodes as elsewhere.

(6) There is an instant, twice in each complete vibration, when all the air is stationary at the same moment (line 4 or 10). This may be called the stationary instant. At a stationary instant every point has the maximum degree of condensation or rarefaction which it ever has, and this is greatest at the nodes and diminishes to zero at the antinodes, alternate nodes being condensed and rarefied. After the stationary instant all the air, except at the nodes, begins to move from the condensed nodes towards the rarefied ones (lines 5, 11); its velocity at any one instant is greatest at the antinodes and diminishes to zero at the nodes. The velocity increases everywhere (line 6 or 12), the velocities at the different points always keeping the same

ratio, so that it is always greatest at the antinodes. Meanwhile the condensation and rarefaction everywhere diminish, but fastest at the nodes, so that the degrees of condensation and rarefaction at different points remain in the same ratio. The velocity increases, and the condensation and the rarefaction diminish, till we reach an instant (line 7 or 1) when the air has everywhere the same density, which is that of the external air. This may be called the moment of uniform density. At this moment the air has at each point the maximum velocity which it ever has at that point. The air continues to move in the same direction, but with diminishing velocity, and the nodes towards which it is moving, which were previously the rarefied ones, now become condensed (line 2 or 8) and those which were previously condensed become rarefied. The velocity everywhere diminishes, and the degree of condensation and rarefaction everywhere increases (line 3 or 9), till all the air comes to rest at the same moment, and we have a stationary instant again. Then all the movements begin again, but in the reverse direction, and so on.

(7) At any given moment all the air between two consecutive nodes is moving in the same direction, and all the air between two consecutive antinodes is in the same condition (all rarefied or all condensed).

(8) Since the velocity of the air at each point varies harmonically, the air at each point moves harmonically, and its amplitude at each point is proportional to the maximum velocity of the air there; this amplitude is therefore greatest at the antinodes and zero at the nodes. Each particle of air passes its mean position at the moment (line 1 or 7) when it has its maximum velocity.

136. Energy of Stationary Undulation.—The total energy in the stationary undulation is the sum of the energies of the undulations which the sources would produce separately, but it is in a different form. For, while in any progressive undulation half the energy is at any one moment kinetic and the other half potential, in the stationary wave-system the whole energy keeps changing from one form to the other. When the air is in the condition shown in

line 1, Fig. 105, there are no differences of density, and the whole energy is kinetic, depending on the velocity with which the air is moving. At this moment most of the energy is near the antinodes, where the air is moving fastest. When the air is in the stage of stationary undulation shown in line 4, there is no kinetic energy, as all the air is at rest; it is all potential, depending on differences of pressure, and none is at the antinodes. In intermediate stages the energy is partly in one form, partly in the other.

137. Fig. 106 shows in the same form as Fig. 32 the movements which take place in a region of stationary undulation. The vertical lines in each row represent, as before, plane surfaces, seen edgewise, which were equidistant in the undisturbed air, and the air is to be supposed moving in such a way that the same air always remains between the same two planes. The lines vibrate harmonically about their mean positions; those at the antinodes have

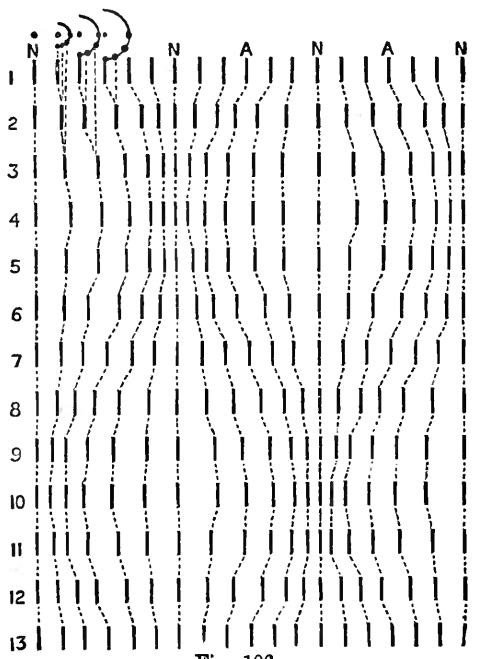


Fig. 106.

the greatest amplitude of vibration, and those at the nodes have zero amplitude. They all cross their mean positions together, but those on opposite sides of the same node are moving in opposite directions. The numbers of different stages correspond with those in Fig. 105. It is easy to see, from the successive lines, how the air sways backwards and forwards between fixed planes, the nodes.

A comparison of Figs. 32 and 106 shows clearly the fundamental difference between progressive and stationary undulation. In the former all parts of the air move the same distance, but at different times. In the latter they move at the same time, but different distances.

138. **Cheshire's Disc.**—The movement of which Fig. 106 shows successive stages may be shown passing through these stages by a modification of Crova's Disc (Fig. 33), designed by Mr. Cheshire and published in *Nature* (1892).

Exp. 55.—*Make a Cheshire's Disc.* Describe a circle of $\frac{1}{8}$ inch radius in the middle of a circle of millboard 8 inches in diameter.

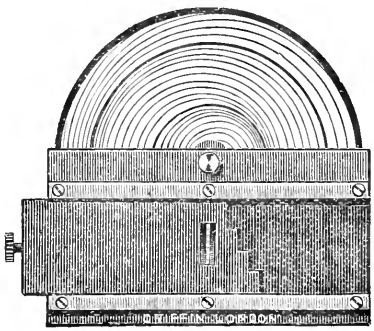


Fig. 107.

Divide the circumference of the small circle into twelve equal parts, numbering the points of division 1 to 12. Draw a diameter from 3 to 9, and draw lines at right angles to this diameter from 2 to 4, 1 to 5, 12 to 6, 11 to 7, 10 to 8, so dividing the diameter into six parts, not all equal. We will call the points of division of the diameter *a*, *b*, *c*, etc., so that, including its ends, there will be seven marked points on the diameter, marked respectively 9, *a*, *b*, *c*, *d*, *e*, 3. Taking these points as centres, in this order, describe ink circles, increasing the radius $\frac{1}{8}$ inch each time. When you have described the cirole with centre 3, go back to *e*, *d*, etc., to 9, then *a*, etc., back again, making 20 or more circles in all. Mount this disc exactly like the one in Fig. 33, and, on rotating it, the portions of the circles seen through the slit will execute the movements of stationary undulation.

Fig. 107 shows a form of Cheshire's Disc mounted in a frame which fits like a slide carrier in an optical lantern, and can be used in any lantern with an open top. By projecting various parts and lengths of a radial slit the vibrations corresponding to the notes of organ pipes (Arts. 145, 157) can be shown on the screen.

The movement can also be well shown by the spiral wire of Exps. 11, 43, 44, as described in Exp. 71.

139. Stationary Undulation caused by Reflection.—It does not often happen that the condition of stationary undulation is due to the simultaneous vibration of two sources. Much more usually it is due to reflection of waves from a single source. In this case the two imaginary component wave-systems are the waves which the source would produce if no reflection occurred, and the reflected wave which each of these would produce if it arrived by itself at the reflecting surface. The real condition of the air between the source and the reflecting surface (say a tuning-fork and a wall) can be found by adding these imaginary conditions, and it is evidently nearly stationary undulation along a line from the fork at right angles to the wall, if the circumstances are such that an incident wave is reflected without much spreading and consequent loss of intensity. This, as we saw in the last chapter, requires that the waves shall be short compared to the reflecting surface; a shrill whistle, or a squeaker, such as is used in many toys, is therefore better than a tuning-fork. If such a squeaker is blown with a steady pressure of air (so as to give always waves of the same length) at a distance of a few feet from the wall of a room, there is stationary undulation between it and the wall.

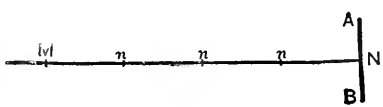


Fig. 108.

At the wall itself, AB (Fig. 108), there is a node, for the part of the reflected wave which is just starting back

is the reflection of, and similar in condensation to, the part of the incident wave which arrived an infinitely short time ago, and therefore differs infinitely little in condensation from the part of the incident wave which is just arriving; and a node is a point where the degrees of condensation of the components are always equal. (See Art. 114 and Fig. 87.) Other nodes occur at *n*, *n*, *n*, every half wave-length of the incident waves from the wall along the normal.

It is only along a normal from the source to the wall that the direct and reflected waves are in exactly opposite directions, so that the condition of stationary undulation strictly exists only along this normal, but there will be a condition which is very nearly nodal at all points, not very far from this normal, which are at the same distance from the wall as the true nodes, and similarly for the antinodes, so that the nodes and antinodes are surfaces parallel to the wall. Their position depends only on the position of the wall and the length of the waves, not at all on the position of the source, which is not, unless by accident, *either at a node or at an antinode.*

140. Experiments on Stationary Undulation. — The existence of the nodes and antinodes of stationary undulation produced by reflection may be shown, and their positions found in two ways. In one method we detect the nodes, in the other the antinodes. The source of sound should be one of high frequency, so that the waves are short and manageable; a shrill whistle or bird-call is most convenient. To detect the nodes a sensitive flame is required; to detect the antinodes a small speaking tube may be used.

Exp. 56.—*Make a Bird Call.*—Obtain two short brass tubes, *P*, *Q* (Fig. 109), about 2 to 3 cms. long and 1-1½ cms. in diameter, one tube, say *Q*, to fit with friction inside the other. Cut out from thin copper foil two equal sized discs, *X*, *Y*, of diameter equal to the bore of *P*. Bore clean holes 1½ mm. in diameter through their centres. Solder *X* to the end of *P*. Cut the other disc *Y* down to triangular form, leaving only enough edge to enable you to solder it to *P* about half way up the tube. Fix a tube *T* into *Q* by means of a cork, push *Q* into *P* and blow air through *T*. A piercing note is emitted by the call. The pitch of the note depends upon the

distance between X and Y, and the pressure of the air in the supply tube. In order to maintain the pressure constant during an experiment the air supply should be obtained from a large weighted gas bag. The supply tube from the bag should be provided with a screw clip, and should end in a T-piece, one branch of which goes to the call and the other to a U-tube water manometer furnished with a graduated scale (Fig. 110). A convenient pressure to work with is that of 15 cms. of water, and the clip should be adjusted until this pressure is reached and maintained.

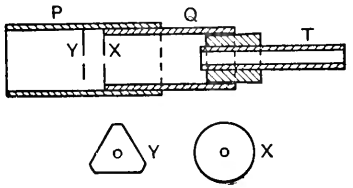


Fig. 109.

Exp. 57.—*Set up a Tyndall's Sensitive Flame.* A sensitive flame is produced by burning, at a burner consisting of a plug of steatite perforated with a single pinhole, coal-gas at a pressure equal to that of 20-25 cms. of water. The gas should be obtained from a weighted gas bag, the supply being adjusted by the tap on the bag. It is not necessary to keep the pressure absolutely constant. It will be found that, as the tap is turned on, the flame, which is like a much elongated candle flame, increases in length to about 40 or 50 cms., and then suddenly shortens to half that length, flaring at the top and producing a loud noise. If the supply is adjusted so that the flame is just on the point of flaring, it is very sensitive to movement of the air just above the burner, which makes the flame flare as long as the movement lasts, but it is not at all sensitive to changes of pressure in the air apart from movement. Test the flame by hissing and talking and shaking a bunch of keys. Such a flame, placed in a region of stationary undulation, flares everywhere except at the nodes, and thus the nodes, if they are at all perfect, may be very exactly determined.*

Exp. 58.—*Find the position of the nodes formed in the system of stationary undulation produced by reflection of short sound waves at a screen* (**Rayleigh's experiment**). Mount the bird-call about 30 cms. above the table (see Fig. 110) and fix a vertical piece of wood (a drawing-board) about 200 cms. away from it and facing it. Sound the bird-call. Starting with the sensitive flame close up to the board gradually withdraw it. The flame flares until it reaches a node, when it shoots up straight. Note its position. Withdraw further; the flame flares again until the next node is reached, and so on. Find the mean distance between successive nodes.

---

*Some sensitive flames behave in an exactly opposite manner, flaring at the antinodes.

This is, of course, a method of finding the length of the waves from the source, which is twice the distance between two consecutive nodes (Art. 134). For very short waves it is one of the best methods. From the results of the above experiment find the frequency of the bird-call by the formula $V = n\lambda$. For example, if the internodal distance is 5 cms. the wave length is 10 cms., and if $t = 13°$ C. the velocity of sound = 34000 cms. per sec., and therefore the frequency of the call = 3400.

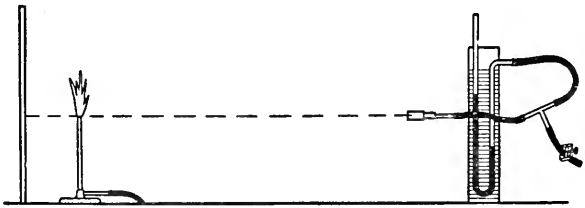


Fig. 110.

This form of sensitive flame requires a gas-bag or gas-holder, since the ordinary gas supply is at too low a pressure, but sensitive flames may be more simply obtained. If gas at the ordinary supply pressure is allowed to issue from a pin-hole burner, and a piece of wire gauze fixed a little distance above the jet, the gas may be lit above the gauze without lighting that between the gauze and the burner, and the flame above the gauze will be blue at the bottom and yellow above. If we increase the distance of the gauze from the burner, the blue part increases and the yellow diminishes, and by trying different distances a position may be found at which the flame is sensitive, the yellow tip entirely disappearing while any vibratory movement of the air, of high frequency, is taking place above the burner. A wide glass tube or lamp-chimney round the flame, standing on the gauze, makes the flame still more sensitive and the effect more visible, as a much longer flame is then obtained, which shortens to less than half its length, and becomes much less luminous, when there is any vibration in the air. These flames, like the other, are unaffected at a node, but disturbed at any other point in a stationary undulation.

Sensitive flames may also be used for showing to an audience the phenomena of reflection and refraction of sound—for instance, its concentration at the focus of a concave mirror. See Arts. 117, 128.

Exp. 59.—*Make and use a speaking-tube to detect the nodes and antinodes.*—Take two pieces of glass tubing about 15 cms. long and 15 mms. bore, and connect them by thin rubber tubing. Mount one tube vertically on a piece of wood so that its upper extremity is level with the bird-call of Exp. 58. Place the end of the other piece of tubing in the ear. Move the vertical tube from the screen towards the call. A series of points are found where the sound is fainter than at intermediate points; these are the antinodes, for the movement of the air backwards and forwards across the opening, without change of density, sends no waves up the tube. When the open end of the tube is at a node, each change in the density of the air sends a corresponding wave up the tube, and loud sound is heard.

Exp. 59 is not a very good method, for it is clear from Fig. 105 that the absence of change of density at the antinodes depends on the exact equality of the condensations of one component wave-system and the rarefactions of the other. Now the rarefactions of the reflected waves are produced by the rarefactions of the incident waves, and do not correspond in wave-form with the condensations of the incident waves unless the incident waves are symmetrical, which never is the case in practice. So that unsymmetrical waves, like those from a squeaker or bird-call, do not, when reflected "with change of sign," form perfect antinodes at all, though they do form true nodes, or places of no movement. So the antinodes are indicated, not by silence, but only by faintest sound, and the nodes by loudest sound; neither is easy to determine. A sensitive flame, which directly determines the nodes, is much better.

141. Imperfection of Stationary Undulation formed by Reflection.—As reflected waves are not quite as intense as the incident ones, stationary undulation produced by reflection is more or less imperfect. It is convenient to consider the incident wave-system as the sum of two systems, one of the same intensity as the reflected waves, the other making up the actual intensity. The first of these, with the reflected system, gives true stationary undulation, so that the actual condition of the air is the sum of a stationary undulation, and a very feeble progressive undulation in the direction of the incident waves.

Very perfect stationary undulation occurs in organ-pipes, but there it is complicated by resonance. We consider it in the next chapter.

EXAMPLES X.

1. Describe experiments with a wave machine illustrating (*a*) the transmission of a pulse of condensation or rarefaction, (*b*) the reflection of a pulse of compression or rarefaction, (*c*) stationary wave motion.

2. What would happen if two trains of sound waves exactly alike were to meet one another in the air, moving in opposite directions?

3. Compare, in tabular form, progressive and stationary longitudinal undulation.

4. Describe an instance of the interference of sound and explain how the phenomenon may be used to measure the frequency of the note from a shrill whistle.

5. A whistle making 2000 vibrations per second is placed a metre from a wall. At what points between the whistle and the wall are antinodes to be found? (Take $V = 34000$ cms. per second.)

6. A whistle is sounded near a wall, and the average distance between the nodes is 3·4 cms. Determine the frequency of the whistle.

7. How would you measure the frequency of vibration of a body which gives a note beyond the upper limit of audibility?

EXAMINATION QUESTIONS II.

1. Describe some two or three experiments to illustrate the laws of forced vibrations.

2. What is meant by resonance? Give some instances. If between an observer and, say, a band, a great number of resonant bodies of the same pitch, say tuning-forks, were to be placed, what would you expect the observed effect to be?

3. A train of waves impinges at the rate of n per second on a system which vibrates with frequency n. Explain by the consideration of a special case how the system is set vibrating and point out

what limits the amplitude. What happens if the frequency of the system is slightly different from that of the waves? How would you illustrate resonance on a piano?

4. If the handle of a vibrating tuning-fork is held against a wooden board, the amount of sound produced is considerably increased. Explain carefully why this happens. Is the time during which the fork goes on vibrating altered, and, if so, how and why?

5. Explain how the harmonics of a musical note may be investigated experimentally.

6. Describe in detail how you would demonstrate the fact that a shrill note is of great frequency.

Can air-waves of any frequency whatever produce the sensation of sound in the ear?

How has this question been answered experimentally?

7. A musical note is sounding in a large open space. Describe the nature of the motion of a particle of air a few feet distant from the instrument emitting the note. How would the motion be affected (i) if the pitch were raised to the octave, (ii) if the note became louder, (iii) if it were replaced by another of the same pitch and loudness but of different quality?

8. What is a Musical Sound? Describe exactly what it is that determines the intensity, the pitch, and the quality of a musical sound. Give experimental evidence for your answer.

9. Describe experiments to show that the impression of a musical interval as judged by the ear depends solely upon the ratio of the frequencies of vibration of the two notes concerned, and not upon the difference of their frequencies.

The frequencies of two notes being 400 and 600, what is the frequency of a note that would appear to the ear to lie midway between them?

10. The question has been much discussed whether resultant tones have an external physical existence. Explain exactly what is meant by their having an external physical existence, and why the question is very difficult to answer experimentally. Does their external existence seem to you to have been proved in any case, and, if so, in what case?

11. Explain why the note of a bicycle bell seems different in pitch when the cycle is approaching you from that which it has when receding from you.

12. Show that if a sound source of n vibrations per second moving with uniform velocity v_1 in air moving with velocity v_2 is heard by an observer moving with velocity v_3, then the number of vibrations per second heard by the observer will be $n \frac{V \pm (v_3 - v_1)}{V \pm (v_2 - v_1)}$, where V is the velocity of sound and v_1, v_2, v_3 are taken to be all in the same straight line. Draw a curve showing how the number of vibrations per second reaching the observer will vary with the time when $v_3 > v_1$ and the observer starts behind the source.

13. Explain how to set up an experiment and describe carefully the observations you would make in order to determine the laws of reflection of sound.

14. Explain how an echo is produced. A man cracks a whip near a line of vertical palings, and after each crack a shrill note is heard. Explain this.

15. Describe some experiments illustrating the refraction of sound waves.

16. Describe carefully how a longitudinal wave consists of alternate compressions and rarefactions. What distinction do you draw between travelling waves and stationary waves?

17. A small whistle is made to produce a note of too high a pitch to be audible. How can the wave length in air and the frequency be experimentally determined?

18. Give an account of Helmholtz's theory of consonance and dissonance of the chief musical intervals.

19. What do you mean by the terms "free period," "forced oscillation," "resonance"? Obtain an expression for the loudness of a note produced by the incidence of an oscillation of a given period on a stretched string or a fork.

CHAPTER XI.

LONGITUDINAL VIBRATIONS OF AIR COLUMNS AND RODS.

142. Wave in a Tube.

Exp. 60.—Take a wide tube, say about 3 inches in diameter and 6 inches long, and stand it on a table, so that its lower end is closed. A bottle of about the right dimensions forms a very good substitute. Hold a strip of paper, an inch wide, with its free end over the open mouth of the tube, and tap the upper side of the paper sharply, near the free end, with a penholder, so that the paper moves suddenly towards the tube, but does not reach it. In addition to the noise which would be produced in any case by striking the paper, another sound will be heard which is not produced when the paper is held at a distance from the tube and struck in the same way. This additional sound will be found to be always the same for the same tube, and quite independent of the size of the paper or the way in which it is held and struck, and most persons will recognise that it is a musical note of definite pitch, which can be matched on the piano or by the voice, while the tap of the penholder on the paper is a mere noise, and has no definite pitch.

Any large wide-mouthed hollow vessel, a jug for instance, may replace the tube in this experiment, and the sound produced is so characteristic of hollow vessels that any sound which produces the same effect on the ear is commonly termed a "hollow sound."

The sensation of definite pitch is found, in all cases which can be investigated, to depend on condensations and rarefactions reaching the ear at *regular* intervals (Art. 60). Now there is nothing regular about the movements of a piece of paper which has been struck; this is shown by the sound being a mere noise. The tube, then, has in some way the effect of converting the single wave produced by

striking the paper into a succession of waves starting at regular intervals. And it is not the tube itself which does this, but the air contained in it, for, if we try the experiment with a metal and a pasteboard tube of the same size and shape, we find no difference in the hollow sounds they give back.

The way in which this happens is as follows:—The condensation produced by tapping the paper travels down the tube to the table, where (Art. 116) it is reflected, and a condensation travels up the tube again. When it arrives at the open end, a slight condensation starts off through the outside air, but (Art. 116) by far the greater part of the energy travels down the tube again in the form of a rarefaction. This is reflected, still as a rarefaction, at the closed end, and, when it reaches the mouth again, a slight rarefaction starts off through the air, while a condensation travels back along the tube, and so on. Only a small fraction of the energy leaves the tube each time the wave reaches the mouth, so that the wave may travel hundreds of times up and down the tube, changing its sign each time it reaches the mouth, before it becomes imperceptible. A condensation starts off from the mouth at the end of every fourth single journey of the wave, and a rarefaction half way between each two condensations, and, as these travel away through the air at the same speed as the wave travels in the tube, we have waves, travelling away through the air, whose wave-length is four times the length of the tube.

A similar action will take place if we tap a strip of paper over the upper end of a tube which is also open at the lower end. In this case the condensation produced by the sudden movement of the paper travels down to the bottom of the tube and there starts a slight condensation off through the outside air, while a rarefaction travels back up the tube. When this reaches the upper end, a rarefaction starts off through the air and a condensation travels down again, and so on. In this case a condensation starts away from the tube through the outside air at the end of every second single journey of the wave in the tube, so that the waves in the air are only twice as long as the tube. An

open tube, under these circumstances, gives waves of about the same length as a tube of half its length closed at one end (called a "closed tube").

Though a hollow vessel is often said to "resound" to a sudden blow like that of the penholder on the paper in Exp. 60, the sound produced is not an instance of resonant vibration in the sense in which we have used the term in Ch. VI., for the air in the tube in Exp. 60 was set in vibration by a single violent impulse, not by a succession of properly timed small impulses. But, as the air in the tube has, as we have seen, a natural period of vibration of its own, it can be set in resonant forced vibration by impulses of the same period.

143. Resonant Vibration of Air Columns.—Suppose we take a tube open at the top and closed at the bottom, and make a flat spring vibrate over the mouth. Each movement of the spring towards the mouth of the tube sends a condensation down the tube, and each movement away from the mouth a rarefaction. When the first condensation returns to the top, it would of itself start a rarefaction down. If at this moment the spring is moving upwards, a rarefaction produced by its movement travels down the tube together with the rarefaction, which would in any case be produced by the arrival of the condensation at the top; we have a rarefaction of nearly double the amplitude of the first wave. When this returns to the top, it would of itself send a condensation down, and, if the spring is at this moment moving downwards, this condensation will be increased by that due to the spring, and so on. Evidently the effect of a large number of impulses can be added up in this way if the period of the spring is exactly such that, each time the wave in the tube returns to the mouth, it finds the spring moving in the opposite direction to that in which it was moving the previous time.

The condition that the air in a closed tube may be set in resonant vibration by a vibrating spring is, therefore, that the spring must make an odd number of half-vibrations in the time that a pulse takes to travel twice the length of the tube.

In the case of an open tube, a condensation sent down returns as a rarefaction, which would of itself start a condensation down again. In order that this may be increased by the movement of the spring, the spring must be at that moment moving downwards again, so that it must have completed a whole number of (double) vibrations while a pulse has travelled twice the length of the tube.

In trying the experiment it is more convenient to use a tuning-fork than a spring, because, unless the spring is very firmly fixed, it shakes its support, and then it does not vibrate very long or regularly.

A closed tube, then, will be set in resonant vibration by a fork which makes either 1, 3, 5, or any other odd number of half-vibrations while a pulse travels twice the length of the tube. In any of these cases each pulse of condensation or rarefaction, on reaching the mouth, gives off a small fraction of its energy in the form of a wave of its own kind, which travels away through the outside air, while the rest of the energy travels back down the tube as a pulse of the opposite kind, increased by the wave which the fork was sending down at the same moment. The wave in the tube keeps on increasing in intensity till the energy which is sent off at each return to the mouth is equal to that received from the fork.

As the tube sends a pulse off for each one that the fork sends down the tube, the waves start from the tube with the same frequency as from the fork, and are of the same wave-length, but they are of much greater intensity, so that the sound heard, though of the same pitch as that heard when the fork is sounded without the tube, is very much louder. It is not at first clear how this increased loudness can be produced, as of course the tube cannot send out more energy than it receives from the fork, but we shall see presently that, when the air in the tube is in resonant vibration, the air near the mouth moves up and down, keeping time with the fork, and under these circumstances the fork communicates its energy much more rapidly to the air, as explained in Art. 56. Of course the fork comes to rest much sooner when it makes the tube resound, but while it lasts the sound is much louder.

144. Simple case of Resonance with a Bottle

Exp. 61.—*Illustrate Resonance with a Fork and a Bottle.* Provide yourself with half-a-dozen medicine or jam or pickle bottles of various sizes, one or more tuning-forks, and a jug of water.

Singling out one bottle, strike a tuning-fork and hold it over the opening. Resonance may or may not occur. Find the natural pitch of the bottle by Exp. 60 or by blowing across the top. If the bottle is above the fork its note may be lowered to that of the fork by partially closing the top of the bottle with a piece of card-board. If the bottle is below the fork its pitch may be raised to that of the fork by partially filling the bottle with water. Any bottle about 4 – 9 ins. high and 1 – 3 ins. in diameter will do and ordinary tuning-forks are used. Adjust some bottles to one fork, some to another, and so on. (This experiment illustrates the use of resonance boxes to strengthen the sound from a fork. The internal dimensions of the box are such that the frequency of vibration of the contained mass of air is the same as that of the fork mounted on it.)

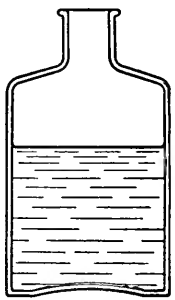


Fig. 111.

The above experiment admits of a simple mathematical treatment, which though not exact is very instructive. Let the bottle (Fig. 111) have a narrow neck. When the air is resounding to a fork held just above the bottle, we may imagine the air in the neck to constitute a piston which oscillates with great rapidity downwards and upwards alternately, compressing and rarefying the air below it. To complete the analogy the air below may be considered to act as an elastic spring. When such a spring, with its attached load, is vibrating, we know (Art. 8) that at maximum extension or compression all the energy is potential, and when they are passing through their normal positions all the energy is kinetic. In our case the piston has much less mass than the spring, but since the velocity of the air in the neck is much greater than that in the body of the bottle nearly all the kinetic energy will be located in the air in the neck, so that we may consider nearly all the effective mass of the air to be located in the neck. The formula for the period of vibration of a heavy body attached to the end of a light spring is

$$t = 2\pi \sqrt{\frac{\text{mass of the body}}{\text{force with which the spring acts on the body when the body suffers unit displacement from the position of rest.}}} \quad *$$

* *Properties of Matter*, Art. 95.

In our case this becomes

$$t = 2\pi \sqrt{\frac{\text{mass of air in the neck}}{\text{force with which the air in the body acts on the air in the neck when the air in the neck is displaced 1 cm. in or out.}}}$$

Let the cross-section and length of the neck be a and l respectively, V the volume of air in the bottle, and P and D the pressure and density of this air.

The mass of air in the neck $= alD$. In calculating the force per unit displacement we will suppose that the expansions and contractions of the air are adiabatic and that the pressure of the air is the same at all points. This last assumption is sufficiently approximate in the case of a wide bottle with a narrow neck. Initially we have a volume V of air at pressure P. Suppose that when the air in the neck moves inward unit distance the pressure is increased an amount p.

In adiabatic expansion or compression the volume elasticity is known to be equal to γP (see Art. 34), hence

$$p = \gamma P \times \text{change of volume per unit volume}$$

$$= \gamma P \frac{a}{V}.$$

Therefore the force with which the air in the bottle acts on the air in the neck due to unit displacement of the latter $= P\gamma \frac{a^2}{V}$.

Therefore the period of vibration of the air column (and of the fork)

$$= 2\pi \sqrt{\frac{alD}{P\gamma \frac{a^2}{V}}}$$

$$= 2\pi \sqrt{\frac{VlD}{P\gamma a}}.$$

Therefore if the same bottle is used with different forks we should expect to get

$$t \text{ of fork} \propto \sqrt{V},$$

i.e. $$\text{frequency of fork} \propto \sqrt{\frac{1}{V}}.$$

In an experiment to test this a large medicine bottle and two forks, g' and c'', were used. To obtain the exact places of resonance water was gradually poured into the bottle from a jug while the fork was vibrating in position. The height of the water was then marked with a piece of gummed paper. The volume of the air was found by filling up the bottle and emptying it down to the mark in a small medicine bottle graduated in teaspoonfuls.

The results were

For the g' fork $V = 15$ teaspoonfuls

,, ,, c'' ,, $V = 8$,,

$$\therefore \quad \frac{\text{frequency of } c'' \text{ fork}}{\text{frequency of } g' \text{ fork}} = \sqrt{\frac{15}{8}} = \sqrt{1{\cdot}88} = 1{\cdot}36$$

which is near the correct ratio 1·33.

Exp. 62.—Having tuned a bottle to a fork, speak near the bottle a note of the same pitch. Observe the response.

145. Condition of the Air in a Resounding Tube.—If we produce at the mouth of the tube a single condensation, as in the experiment with the strip of paper, a real wave travels up and down the tube, and would be seen to do so if the air was visible. When a fork vibrates continuously at the mouth of the tube, producing condensations and rarefactions alternately, it is convenient to speak as if these actually travelled down the tube, while previous waves, reflected from the lower end, were travelling up again. It has, however, been explained (Art. 52) that this is merely a convenient way of saying that the real condition of the air is one which can be found by adding two such wave-systems, and does not mean that these systems really exist in any physical sense. So that, if we could make the air in a tube, which is resounding to the tuning-fork, visible, we should not *see* waves travelling down and other waves travelling up, or indeed anything moving continuously along the tube either way; what we should see would be simply the process of stationary undulation described in the last chapter, since this is the condition which we find when we add two imaginary equal wave-systems in opposite directions.

It is very important to keep in mind that the real physical condition of the air in a resounding tube is the condition of stationary undulation, and that the waves travelling up and down are a mathematical fiction, not a physical fact. To describe the condition of the air as one in which waves are travelling along it in opposite directions is as physically incorrect (and as mathematically correct) as to describe the condition of a man who is standing still by saying that he is walking forwards and walking backwards at the same time.

All harmonic stationary longitudinal undulation is of the same kind, so that the general description of this process given in Art. 134 applies to the air in any resounding tube if the vibrations are harmonic. To make it a complete account of the movement of the air in a resounding tube it only remains to state where the nodes and antinodes are situated in the tube. This is easily found in any given case. Suppose, for instance, that we hold over the mouth of a tube, closed at the bottom, a fork which makes $2\frac{1}{2}$ vibrations while a pulse would travel up and down the tube, so that a pulse travels $\frac{4}{5}$ of the length of the tube while the fork makes a vibration. Fig. 112 shows the position of the fork at the moment when it is producing a maximum of condensation, and the positions which each maximum of condensation and rarefaction previously produced would occupy at that moment if it was solitary. The maximum condensation produced one period ago would be $\frac{4}{5}$ of the way to the bottom, and the one before that would have reached the bottom and come $\frac{3}{5}$ of the way up again, while the maximum rarefaction which started $2\frac{1}{2}$ periods ago would have just reached the top, so that the maximum condensation due to it would be just ready to start down with that now being produced by the fork. Those pulses which started still earlier would in the same way have positions coincident with later ones, so that the points marked are all the places where there would be maxima of condensation and rarefaction if each pulse travelled independently. The actual condition of the air can be inferred from the positions of these imaginary pulses. *N*, *N*, *N*, where an imaginary maximum of condensation travelling down passes another travelling up, are nodes (Art. 134);

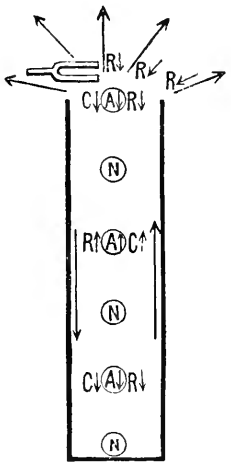


Fig. 112.

A, *A*, *A*, where imaginary pulses of opposite kinds cross, are antinodes. In Fig. 112 the imaginary waves travelling down the tube are shown on the left, and the imaginary reflected waves travelling up again on the right, the directions in which the air would be moving in such waves being shown by short arrows. The actual condition of the air in the tube (*i.e.* nodal or antinodal) with the actual direction of movement of air is shown by encircled letters in the middle column. The moment chosen is the "uniform density instant." (See Fig. 105.)

In the same way we find that forks which make respectively $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$ vibrations while a pulse travels twice the length of a closed tube throw the air into the conditions of stationary undulation represented by *a*, *b*, *c* of Fig. 113, and forks making $\frac{7}{2}$, $\frac{9}{2}$, $\frac{11}{2}$ or any higher odd number of half-vibrations in the same time will produce a similar condition with still shorter intervals between the nodes. And by the same method we find that forks which make respectively 1, 2, 3 vibrations while a pulse travels twice the length of an open tube throw the air into the conditions of stationary undulation represented by *d*, *e*, *f* of Fig. 113, and forks making 4, 5, or any higher number of complete vibrations in the same time would produce a similar condition.

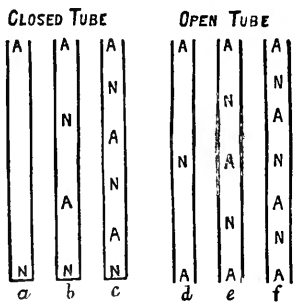


Fig. 113.

In either open or closed tubes, the number of half-segments into which the tube is divided by nodes and antinodes is equal to the number of half-vibrations made by the spring (and therefore by the air in the tube) while a pulse would travel twice the length of the tube; the number of half-segments is therefore odd in a closed pipe and even in an open one. A closed end is always a node, and an open end an antinode. The middle point of an

open tube must be either a node or an antinode; in a closed tube it is neither.

The mouth of the tube being an antinode, the air there simply moves in and out, and may be considered as a piston vibrating at the end of the tube like that in Fig. 36. Condensation and rarefaction are therefore produced in turn in the air just outside the mouth, and travel away (by the process of progressive undulation) in all directions through the external air. The condensation just outside the mouth is at its maximum when the air just inside the mouth is moving outwards most rapidly, which is at one of the moments of uniform density of the stationary undulation in the tube; the maximum degree of rarefaction just outside the tube occurs at the other uniform density instant (Art. 134). It is this latter instant which is represented in Fig. 112, and the letters and arrows round the mouth are intended to indicate that the air there is rarefied and moving inwards towards the mouth, while the rarefied condition is travelling away in all directions. It will be seen that this condition of the air outside the mouth might be considered as a continuation into the external air, with diminished amplitude, of the imaginary return waves which have been reflected at the bottom. The changes of density produced in the air outside, though it is to them that the sound we hear is due, are very small compared to those which occur in the air in the tube itself.

As a condensation starts off through the external air each time the air just inside the mouth of the tube moves outwards, the length of the waves in the air outside is the distance a pulse travels in one period of the stationary undulation. And we showed in Art. 135 that the period of a stationary undulation was four times the time required for a pulse to travel a half-segment (from a node to the nearest antinode). So that the waves produced in the external air are always four times the length of a half-segment of the stationary undulation in the tube. If we examine Fig. 113, keeping this in mind, we can easily make out the following table, in which l is the length of the tube, and t the time required by a pulse to travel this length.

Kind of Tube.	Closed.				Open.			
Mode of Vibration. (See Fig. 115.)	Slowest or Fundamental. *a*	2nd. *b*	3rd. *c*	&c.	Slowest or Fundamental. *d*	2nd. *e*	3rd. *f*	&c.
No. of half-segments	1	3	5	&c.	2	4	6	&c.
Length of half-segment	l	$\frac{1}{3}l$	$\frac{1}{5}l$	&c.	$\frac{1}{2}l$	$\frac{1}{4}l$	$\frac{1}{6}l$	&c.
Length of waves in external air	$4l$	$\frac{4}{3}l$	$\frac{4}{5}l$	&c.	$2l$	l	$\frac{2}{3}l$	&c.
Period of vibration	$4t$	$\frac{4}{3}t$	$\frac{4}{5}t$	&c.	$2t$	t	$\frac{2}{3}t$	&c.
Ratio of frequency to slowest vibration	1	3	5	&c.	1	2	3	&c.

It can be understood from the above table how notes of different pitch can be obtained on the ordinary tin whistle or flute by opening and closing holes in the tube or barrel of the instrument. The note given by these instruments is produced by the longitudinal vibration of the air column in the barrel, and the pitch of the note is determined by the mode in which the column is made to vibrate. By opening and closing the holes in the barrel with the fingers, and by evoking where necessary some of the higher modes of vibration for a given constraint a considerable range of pitch can be adequately covered.

In order to specify the state of the air at any point in a column of air in longitudinal vibration, it is convenient to remember the following points, which have already been established.

In any internode, the half wave-length from node to node, the phase of vibration of the particles is the same at all points, but the amplitude of vibration increases from zero at the nodes to a maxmium at the antinode. Also in adjacent internodes, that is, on opposite sides of a node, the phases of vibration are exactly opposed.

In the half wave-length from antinode to antinode the character of the strain (compression or rarefaction) is the same at all points, but the range or "amplitude" of the strain increases from zero at the antinodes to a maximum at the node. The range or "amplitude" of the strain at any point is determined by the extent of the change in density or the change in pressure which occurs at that point. Also in adjacent half wave-lengths, measured from antinode to antinode, that is, on opposite sides of an antinode, the strains are opposite in character.

In any quarter wave-length, measured from antinode to node, the strain is one of compression when the particles are displaced towards the node, and one of rarefaction when the particles are displaced away from the node. Also motion in the direction of displacement means an increasing strain, and motion in the direction opposite to that of displacement means a decreasing strain.

At any point in the column the strain is zero when the displacement of the vibrating particles is zero, and as the displacement varies through a complete vibration, the strain also varies through its complete periodic cycle of changes. At points, however, where the amplitude of displacement is a maximum (antinodes) the range or amplitude of the strain cycle is zero, and at points where the amplitude of displacement is zero (nodes) the amplitude of the strain cycle is a maximum.

These points are all indicated in Fig. 114 for a particular mode of vibration of the column of air in a pipe open at both ends.

Some of the statements given in the table (p. 277) are only approximately true, for the mouth of a tube is not strictly an antinode.

The stationary undulation in the tube is not divided from the progressive undulation which starts from it by a definite line; the reflection of the wave at the open end does not take place exactly at any one point, so that there is an intermediate region, partly in the pipe and partly outside, where the undulation is not exactly either stationary or progressive. There is thus no true antinode at the

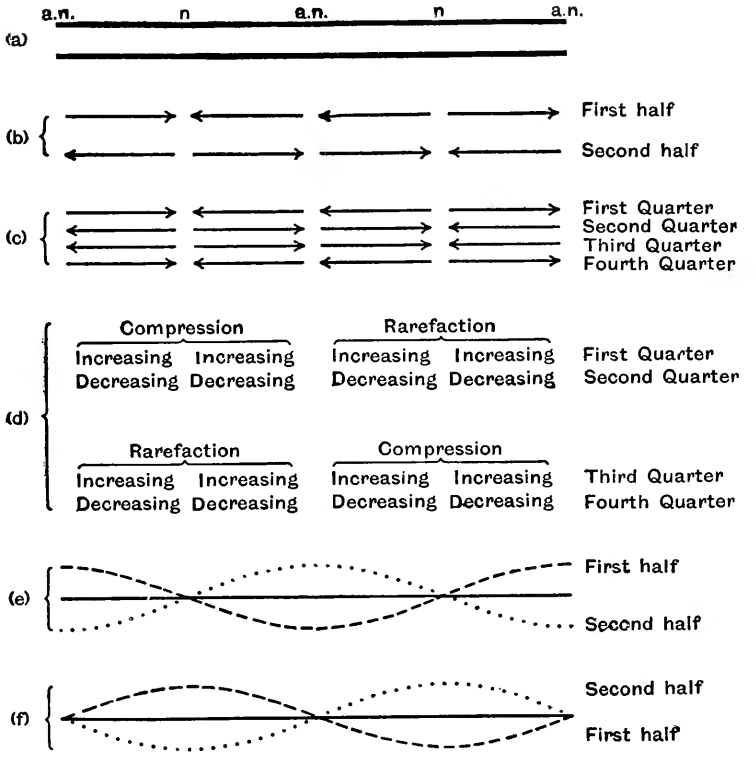


Fig. 114.

(*a*) Column of air in a pipe open at both ends in the mode of vibration corresponding to the production of its first harmonic.

(*b*) Arrows indicating the direction of displacement at all points along the column during a complete vibration.

(*c*) Arrows indicating the direction of motion at all points along the column during a complete vibration.

(*d*) Nature of the strain at all points along the column during a complete vibration.

(*e*) Curves showing the amplitude of displacement at different points along the column for a complete vibration. Displacements to the right are shown as positive ordinates.

(*f*) Curves showing the range or "amplitude" of the strain at different points along the column for a complete vibration. Compression strains are shown as positive ordinates.

mouth, nor is the distance from the mouth to the first node quite as great as the distance from a node to an antinode in the tube. The difference is shown, both by experiment and calculation, to be about equal to $\frac{1}{3}$ of the diameter of the tube, so that we must add this quantity to the length of a closed tube, and twice this quantity to that of an open one, to get the length of an exact number of half-segments. Strictly, therefore, these increased lengths should be substituted for l in the table.

The length of the waves from a closed tube vibrating in its slowest mode is thus $4(l + a)$ where a is about ·3 or ·4 of the diameter of the tube; from an open tube it is $2(l + 2a)$. The waves from the open tube are therefore not exactly half as long as those from a closed tube of the same length, but rather more; the note produced by the open tube is rather less than an octave higher than that of the closed one.

It should be noted that the correction given by $\cdot 3d$ or $\cdot 4d$ is strictly applicable only when the open end is the right section of a cylindrical pipe and is not obstructed by a flange or constriction of any kind.

The slowest resonant harmonic vibration possible for a pipe is called its fundamental vibration, and the others are called its harmonics, or overtones. We see from the table that the overtones of a closed pipe have frequencies which are odd multiples of that of its fundamental, while the overtones of an open pipe have frequencies which include every exact multiple of its fundamental. The presence of the even harmonics renders a note full and strong so that the quality of the notes given by closed and open pipes are greatly different, the latter giving by far the sweeter notes.

Exp. 63.—*To find by resonance the wave-length of a note (in air).* Very nearly fill a tall glass cylinder C (Fig. 115) with water. Support a metal or glass tube B about 1" wide) vertically by a clamp A (on a retort stand) so that its lower end dips in the water.

Hold the sounding body, a tuning-fork, say, giving the note to be experimented on, close to the upper end of the tube, and adjust the length of the tube outside the water so that the column of air within the tube responds to, and strongly reinforces the note.

This is conveniently done by holding the tube with the left hand, loosening the clamp, and letting the tube down until the upper end is near the water. Sound the fork, bring it with the right hand over and close to the upper end of the tube, and raise the tube with the left until resonance is heard, then clamp the tube somewhat loosely. Resound the fork, and twist the tube to move it a little up or down, adjust in this manner until the maximum resonance is obtained. The fork should be sounded several times, and the position carefully confirmed. Finally clamp firmly, and measure the distance (l) between the water surface and the top of the tube.

Fig. 115.

Repeat the whole experiment half-a-dozen times. Calculate the mean value of l.

Finally measure the diameter (d) of the tube, and note the temperature (t) of the air.

Then at temperature t the wave-length

$$\lambda = 4(l + 0{\cdot}3d).$$

A *second position* of resonance (of weaker intensity, however) may be found by lifting the tube further out of the water (about $3l$). If necessary the tube can be lengthened by a paper extension. (Make this by rolling stout paper round the experimental tube, slide it off, and fix the edges with stamp paper or gum.) Obtain, as before, half-a-dozen values for the position of maximum resonance. Calculate the mean value of L.

Since in the first case $\frac{1}{4}\lambda = l + 0{\cdot}3d$, and in the second case $\frac{3}{4}\lambda = L + 0{\cdot}3d$, therefore the difference $L - l = \frac{1}{2}\lambda$.

Since velocity of sound = frequency × wave-length we can use the results of the above method to find—

(1) The velocity of sound, if the frequency of the fork is known.

(2) The frequency of the fork, if the velocity of sound is known.

Exp. 64.—*Find the velocity of sound in air at 0° C. by means of a tuning-fork and resonance column.* Proceed as in Exp. 63, using a fork of known frequency.

EXAMPLE.—Frequency of fork, 546. Temperature 10° C.

Mean length of resonance columns = 14·6 cms. and L = 45·6 cms.

$\therefore$ Half wave-length = 31·0 cms.,

$\therefore$ velocity of sound at 10° C. = 546 × 62 = 33,800 cms./sec.,

$\therefore$ velocity of sound at 0° C. = 33,800 − 60 × 10 = 33,200 cms./sec.

Exp. 65.—*Express the end correction of a resonance tube in terms of the diameter.* Measure the diameter, d, of the tube of the last experiment. Let xd be the end correction, then

$$l = \tfrac{1}{4}\lambda - xd,$$
$$L = \tfrac{3}{4}\lambda - xd,$$
$$\therefore\ l = \tfrac{1}{2}\{L - l\} - xd;$$
$$\therefore\ x \text{ can be calculated.}$$

By this method Blaikley determined the correction, at the open end to be given by $\cdot 288d$, where d denotes the diameter of the tube.

EXAMPLE.—The tube in the example of Exp. 64 had a diameter of 3·0 cms. Then, to find the end correction, we put

$$14{\cdot}6 = \tfrac{1}{2}\{31{\cdot}0\} - 3x,$$
$$\therefore\ 3x = 15{\cdot}5 - 14{\cdot}6 = {\cdot}9,$$
$$\therefore\ x = {\cdot}3.$$

The end correction is therefore $\cdot 3d$.

Fig. 116.

Exp. 66.—*Find the frequency of a tuning-fork (use g', a', c'') by means of a resonance column.* Obtain the wave-length at temperature of the experiment (Exp. 63). Calculate the velocity of sound at this temperature, and then the frequency from the formula

$$\text{frequency} = \frac{\text{velocity}}{\text{wave-length}}.$$

Find the frequencies of the forks of pitches g', a', c'' respectively.

EXAMPLE.—Fork g'. Temperature, 15° C. Diameter of tube, 4·5 cms. Observed lengths of first resonance column 19·7, 19·4, 19·3, 19·6, 19·6, 19·5 cms. Mean length of resonance column, 19·5 cms.

Corrected length $= 19{\cdot}5 + 0{\cdot}3 \times 4{\cdot}5 = 20{\cdot}9$ cms.

$\therefore$ wave-length $= 83{\cdot}6$ cms.

Now velocity of sound at

$15° = 33200 + 60 \times 15 = 34100$ cms. / sec.

$\therefore$ frequency of fork $g' = 34100/83{\cdot}6 = 408$.

If the apparatus of Exp. 63 is not available the wave-length can be determined from a single narrow cylindrical jar or tube by holding the fork over the mouth of the tube and pouring in water from a jug. It is rather difficult

to get the exact position of resonance in any case so that many readings should be taken. This method is therefore rather tedious, but it is of great historical interest.

Another suitable form of apparatus is shown in Fig. 116.

The tube T and the reservoir R, mounted on a suitable stand, communicate, as shown, by a length of wide rubber tubing. The height of the water in the tube can evidently be raised or lowered by raising or lowering the reservoir.

The air in a tube closed at both ends may also be set in resonant vibration by a vibrating rod, as in Kundt's experiment (Art. 159). The modes of vibration possible in this case are like those of an open tube with nodes and antinodes interchanged. As no waves are given off from a tube closed at both ends, the resonance is not audible in this case.

146. Pipes other than Cylindrical.—So far, we have considered only pipes of uniform bore. Pipes of varying diameter can also be thrown into resonant vibration; in this case they divide into segments whose natural vibration periods are equal, but whose lengths are unequal. A conical pipe, stopped at the apex, vibrates with nodes and antinodes like those of a uniform cylindrical tube; but curiously the closed apex takes the place of an open end or antinode, and is a quarter wave-length from the next node, and a half wave-length from the next antinode, precisely as if it were an open end. Hence the frequencies of a conical tube open at the wide end and closed at the apex are the same as of a cylindrical tube open at both ends.

147. Vibration produced by Air Blast.—The air in a tube may be set in resonant vibration in many other ways. One is to send a blast of air across an open end of the tube.

It is not very clear how this produces resonant vibration, and various explanations are given. It is sometimes stated that the rushing noise produced by the blast striking the edge of the hole has among its harmonic components a vibration of the frequency to which the tube resounds; but, if that was the cause, a tube ought also to resound when a rushing sound is produced near its mouth by blowing across the edge of something else, without the

blast itself reaching the tube, and this does not occur. The following is perhaps nearer to the true explanation. A very slight difference in the direction of the blast of air determines whether the air goes into the tube, so producing a condensation, or simply passes across the opening, in which case it exhausts air from the tube, by an action similar to that of spray and scent diffusers. If, as is usually the case, this exhausting action first takes place, the air inside the mouth is rarefied, till the pressure inside the tube becomes so much less than that outside that the air blast is deflected inwards, so producing condensation; and so on. These conditions travel down the tube, and are reflected, and each condensation, as it reaches the mouth again, deflects the air blast outwards, so that its action increases the rarefaction which would in any case be produced by a condensation reaching the mouth; and similarly for a rarefaction. The stronger the blast, the more rapidly it exhausts or condenses the air, and the larger the number of rarefactions and condensations which start down the tube before the first returns, after which the action of the blast is simply to increase the waves each time they return to the mouth.

On this theory of the maintenance of the vibration of the air column in an organ pipe it is evident that the source of energy which maintains the continued vibration of the column is the kinetic energy of the air blast.

When the pipe is blown gently, so as just to sound steadily, it gives its fundamental note (with its characteristic overtones), but when blown strongly it may be made to give (as the predominant note) the second or even a higher harmonic. This is possibly explained by assuming that when the pipe is very strongly blown the action of the blast in producing compression or rarefaction is so vigorous that the sheet of air is driven out or into the tube before the compression or rarefaction last initiated by it has had time to travel up to the other end of the tube and back again. This means that the air column cannot vibrate in the fundamental mode, but that by vibrating in the second possible mode and so reducing the interval of time necessary between the inward and outward deflections of the blast, it may again bring the motion of the blast into step with its vibration and derive from it the energy necessary for the maintenance of the vibration. It is, however, difficult to produce any but the fundamental mode of vibration in wide tubes.

It is also found that when an organ pipe sounding its fundamental note is blown more strongly, the pitch of the fundamental note rises appreciably before the first overtone is obtained. This is probably due to the action of the air blast being so increased that it is driven in or out of the tube a little sooner than it should be to coincide in its action with the return of the pulse reflected from the top end. The result of this is that the lower limit of the air column in vibration is slightly raised, and the column is thereby shortened. This goes on until the fundamental mode of vibration breaks down and the second harmonic mode is established.

The blast of air may be across the end, as in whistling with the barrel of a key or in blowing the Pandean pipes, or across a hole in the side, as in a flute, or directed by means of a passage so that the jet strikes a sharp edge, as in a whistle, or organ pipe with a flute mouthpiece (Fig. 117).

Instead of producing any required mode of vibration by adjusting the strength of the blast, we can produce one mode to the exclusion of others by opening additional holes in the side of the tube at points which are antinodes for the particular mode of vibration we require, for no mode of vibration is possible which has not an antinode at every opening. This principle is used in the flute and tin whistle.

Exp. 67.—Hold a test-tube by the closed end and strike the open end lightly on the knee. This causes a blast of air across the mouth. The note given out by the vibrating column may be heard for a second or two.

Exp. 68.—Withdraw a cork from a long bottle. Note the musical sound produced.

148. Non-harmonic Vibration.—The impulses given by the wavering air-blast to the air-column are periodic but not harmonic, and may therefore be considered as the sums of harmonic pressure-changes of frequencies which are multiples of the real movement of the blast. (Art. 89.) Each such pressure-change would produce resonant vibration of its own period if the air column had a free-vibration period not very different. The vibration produced is therefore not harmonic, and may have harmonic components of any frequencies (multiples of that of the

vibration) which are nearly equal to possible free-vibration frequencies of the air column. Thus the vibration in a closed tube, caused by a blast, may have harmonic components whose frequencies are odd multiples of its own, but not even ones; that of an open tube may have components which are any exact multiples of its frequency. The harmonic components of the waves sent out are the same as those of the air column itself.

Any air column, open or closed, if set in resonant vibration by a single tuning-fork, vibrates harmonically like the fork, and under these circumstances the resonant sounds from an open and a closed tube are exactly alike. But, when either tube, if not very wide, is set in resonant vibration by a blast of air, it usually vibrates non-harmonically, and in that case the sounds are of different quality, because, as explained above, the closed tube gives waves whose harmonic components are all odd multiples of their fundamental, while the open tube has even multiples as well. Very wide tubes are not easily set in non-harmonic vibration by a blast of air, so that there is not so much difference in the quality of the sounds from open and from closed pipes when they are wide as when they are narrow.

There is an important difference between the vibration produced by a fork and that produced by a blast of air. The air column in a tube is of small mass, and there is considerable loss of energy in each vibration owing to the waves sent off; forced resonance, of a period different to its natural vibration, is therefore easily produced (Art. 80). A fork has a natural period not easily altered, and accordingly sets an air column in resonant vibration of this period, even when the natural period of the air column is considerably different. The air blast, on the contrary, has no natural period of vibration, and is controlled by the return of the waves it produced so that the period of the resonant vibration is exactly that in which a pulse would travel the length of four half-segments of the tube.

EXAMPLE.—Two notes forming a major third, a little out of tune, are sounded on narrow open pipes. Show why the imperfection is more easily perceived than if the same notes are sounded on wide pipes.

The frequencies should be n and $5n/4$; let the second be $\frac{5}{4}n + x$ where x is small. The overtones present are $2n$, $3n$, $4n$, $5n$, etc., also $5n/2 + 2x$, $15n/4 + 3x$, $5n + 4x$, etc. The $5n$ and the $5n + 4x$ may be near enough to be discordant. With wide pipes the overtones are much less important.

149. Organ Pipes.—The most important application of the vibration of air columns in the construction of musical instruments is found in the organ pipe. An organ pipe usually takes the form of a cylindrical metal tube or a wooden tube of square section. One end of the tube is specially constructed, as described below, so that when air is blown through it from a bellows the air column in the tube is maintained in vibration. This end of the tube is in free communication with the air, and acts, with certain limitations, as an open end; the other end may be open as in *open pipes*, or closed by a tightly-fitting piston as in *stopped pipes*. The length of the pipe is determined by the wave-length of the note to be produced, and the cross-section dimensions (which have no influence on the pitch of the note) are always small compared with the wave-length.

Fig. 117.

Fig. 117 shows a section of a wood organ pipe in which the construction of the lower end of the pipe is exhibited. The tapering mouthpiece *m* is made to rest in a corresponding socket, leading by a suitable channel to the wind chest fed by the bellows. It will be seen that the construction is such that the stream of air from the wind chest into the tube is forced through the narrow slit *s* as a thin ribbon-like sheet of air. This sheet is directed towards the thin razor edge which forms the upper boundary of the rectangular opening or *embouchure* which puts the air column in communication with the outer air and makes this end of the pipe an open end.

When the blast is properly adjusted (found by trial) the pipe "speaks" or sounds; that is, the air column in it is thrown, by resonance, into stationary vibration. The fundamental note is sounded when the blast is moderate; by blowing harder the harmonics are successively obtained. In every case the open end and mouthpiece are antinodes.

In tuning an organ pipe it is necessary to adjust in some way the length of the air column. In the case of stopped pipes this is readily done by means of the piston closing the upper end; the

length of the air column can be increased or diminished and the pitch lowered or raised correspondingly by moving the piston up or down. In the case of open pipes there is generally a side opening covered by an adjustable plate near the open end of the pipe, and by adjusting the position of this plate the length of the vibrating column can be adjusted as may be required. Another method is to bend the top edge of the pipe inwards if the pipe is sharp and outwards if the pipe is flat.

It is practically impossible to determine the length of the effective air column in a given organ pipe. The correction for the embouchure end is uncertain, and the correction for the other end of an open pipe is also uncertain if it departs from the circular section, or if it carries any special tuning device.

An organ usually consists of several "stops," each "stop" consisting of several octaves of pipes. The organist uses these stops at will by pulling in or pushing out plugs which are fixed around the keyboard. For details of the names of the stops and their characteristics more advanced treatises must be consulted.

150. Reed pipes.—An air column may also be thrown into resonant vibration by means of a *reed*. The term "reed" is applied sometimes to the whole and sometimes to a part of an arrangement consisting of a strip or tongue, usually of thin metal, fixed at one end to a plate, so that it covers a rectangular opening in the plate, as shown in Fig. 118. The tongue may be either a little larger than the opening, in which case it is a *striking reed*, or a little smaller, when it is a *free reed*. In either case the tongue is bent so that its free end stands a little away from the plate, and a blast of air is blown through the opening from the side on which the tongue is fixed. As the air increases in speed, it carries the tongue with it, and so blocks the opening; when the rush of air stops, the tongue springs back. Thus a succession of puffs of air escape through the opening, producing waves. These may either be allowed to escape into the air, as in the

Fig. 118.

harmonium, in which case the frequency of the puffs is that of the natural vibrations of the tongue; or they may escape into a pipe whose length is such that it can be set in resonant vibration of the same frequency, as in the clarinet and several kinds of organ pipes. In this case, the tongue being of small mass as well as the air column, each forces the vibrations of the other; the vibration period is a compromise between that natural to the reed and that natural to the air.

The tongue of a free reed is not a freely vibrating spring, being affected by the changing pressure of the air blast; its frequency is not independent of this pressure, as is often stated, but increases with it: this is easily shown on a concertina. The striking tongue, which rebounds from the plate, has its frequency still more increased by an increase of air pressure. The striking form is the one practically used to cause vibration in tubes.

Fig. 118 shows a reed in conjunction with a conical tube, the box which contains the reed being provided with windows for observing the vibration. The air is blown from below into this box, passes through the reed in the direction away from the reader, and escapes behind the metal plate into the conical tube. The vibration period of the tongue is adjusted by a sliding wire which allows a shorter or longer portion to vibrate.

An air blast against an edge of a hole in a tube sets the air column in vibration whether the other end of the tube is open or closed; in either case the hole across which the air is blown is an antinode, and the air vibrates in and out through it, but the blast sucks out in one half-vibration the air that it has just blown in in the other; there is no continuous current of air along the tube, and if smoky air is used for the blast the air in the tube remains clear for a long time. The case is quite different when a tube is made to vibrate by a reed at one end, as in the clarinet. Here the air never passes out of the tube through the reed, so that the tube must be open at the other end; and as a condensation, returning to the reed end, does not escape there, it is reflected as a condensation, so that the reed end is a node. A reed tube is therefore always a closed tube, and, if cylindrical, can give only the odd harmonics of its fundamental. As pointed out above, this may be avoided by making the tube conical, as in the French horn, in which, as in many other instruments, the lips of the performer take the place of a mechanical reed.

151. Vocal Sounds.—The vowel sounds produced by the voice are due to the vibrations of two cartilaginous plates, the *vocal chords*, placed at the top of the windpipe, edge to edge, with a narrow slit between them; air blown through this slit from the lungs keeps the plates vibrating. The apparatus is really a free reed. The vocal chords have muscles attached to them, which can vary the frequency of the vibration, and the pitch of the sound produced. The different vowel sounds are produced by varying the size and shape of the mouth cavity, but it is uncertain what effect this produces. Some believe that the mouth reinforces by resonance certain harmonics in the sound produced by the chords, a sound containing among its harmonics the same multiples of the fundamental being recognised as the same vowel, whatever the absolute frequency of the fundamental. This is the *relative pitch* theory. Thus a note whose harmonic components are the fundamental and its octave is said to give the sound Ō, while the fundamental with the first five harmonic overtones gives the sound Ā. On this view each vowel is a note of particular quality. Another view is that each vowel is distinguished by the addition to a note which may have any pitch, of one or two other notes whose absolute frequencies determine which vowel is heard; thus the sound Ō requires a note of frequency about 980 added to the louder and lower note. The note which determines the vowel is not a harmonic of the note which constitutes the greater part of the sound. This is the *fixed pitch* theory. The two views are often held together, each vowel sound being considered to require certain harmonics of the fundamental note, and also certain notes of fixed frequency independent of the fundamental. There seems no direct evidence that these notes are produced, and if they are, they probably exist only for a short period at the beginning of the sound. The consonant sounds are produced by the tongue, teeth, and lips. Most of them are *noises*. Others, *e.g.* *P*, *J*, *S*, are notes with accompanying noises. Several sounds usually called consonants, *e.g.* *L* and *M*, are really vowels with their quality modified by the resonance of the cavities of the nose, or of parts of the mouth cavity partitioned off by the tongue.

152. Effect of Change of Temperature.—As the vibration period of an air column depends on the time taken by a pulse to travel its length, the vibration frequency is proportional to the velocity of a pulse. Rise of temperature therefore increases the vibration frequency of the air column, the frequency being proportional to the square root of the absolute temperature (Art. 43). The increase of frequency is really a little less than this, because the pipes lengthen with rise of temperature; this effect is greater with metal than with wood pipes. In reed pipes the increase of frequency with rise of temperature is much less than with flute pipes, because the stiffness of the tongue diminishes as the temperature rises, so that, while the rise of temperature shortens the natural period of the air column, it lengthens that of the reed. Reeds without pipes, as used in the harmonium and concertina, diminish slightly in frequency with rise of temperature.

153. Vibration of Liquid Columns.—Liquid columns may also be set in resonant vibration.

Exp. 69.—Immerse entirely a common tin whistle in water in a jar, and connect it by a tube to a high pressure water supply (such as the ordinary water pipes of a house). The water in the whistle is set in resonant vibration. Not much sound is heard, as sound does not pass easily from water to air (Art. 115) but the trembling of the jar is easily felt. If you put your ear under the water, the sound can be well heard.

154. Vibration of Solid Rods.—As condensations and rarefactions travel along rods of elastic material exactly as along the air in tubes, and are reflected at free ends of rods exactly as at open ends of tubes, and at fixed or loaded points of rods exactly as at closed ends of tubes, rods can be set in resonant stationary undulation exactly like that of air columns, but different means must be adopted to give the successive impulses. The simplest way is to draw a resined cloth along the rod.

Exp. 70.—Set rods in longitudinal vibration. Hold the rods at the middle with one end and stroke with the other. Try wooden rods using resined leather, or glass rods using wet dusters.

Resin, like other viscous substances, adheres the more strongly to surfaces over which it moves the more slowly it travels along them. The cloth sticks to the rod, pulling the part with which it is in contact along with it, and so producing condensation in front and rarefaction behind. These conditions travel to the ends of the rod, or to any point of it which is fixed or loaded, and are reflected ; and thus travel up and down the rod, and as they pass any part of the rod that part moves a short distance. As a wave passes the cloth, if it is one in which the particles of the rod move in the same direction as the cloth, the relative velocity of the cloth along the rod is diminished, and the cloth adheres more strongly to the rod, and gives the surface a pull in the direction in which it is already moving, while a contrary action takes place if the wave is one in which the particles of the rod are moving the opposite way to the cloth. Each wave is therefore increased each time it passes the cloth, and the rod is set in resonant vibration.

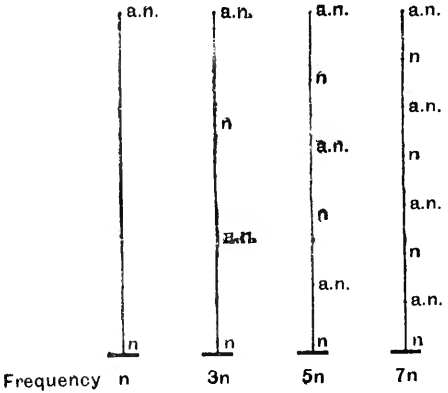


Fig. 119.

The resonant vibration of a rod fixed or loaded at one end is exactly similar to that of the air in a closed tube, for the waves are reflected in just the same way (Fig. 119). The vibration of a rod not firmly fixed anywhere, for instance held in one hand and rubbed with the other, is similar to that of the air in an open tube. In the vibration of a rod clamped in the middle, each half vibrates like any other rod clamped at one end, but the two halves keep time

with each other, so that points at equal distances from the middle always move in opposite directions at the same time. A rod fixed at both ends (which may be a stretched wire) vibrates like the air in a tube closed at both ends. If pieces of lead are fixed by clamps to two points on a stretched wire, and a piece of sandpaper, or resined cloth, drawn along the part of the wire between them, a loud sound is produced, whose pitch depends on the distance between the pieces of lead, but not on the tightness of the wire. This sound is due to the resonant longitudinal vibration of the part of the wire between the lead blocks, which are nodes. The blocks need not be fixed, except to the wire, as the waves are almost totally reflected on arriving at a portion of the wire of so much greater density than the rest (Art. 114).

155. Compound Modes of Vibration of a Rod.—In general when a rod is excited to longitudinal vibration in any way it does not vibrate in any *one* of the modes described above, but in a complex mode compounded of some or all of the modes of which it is capable. In fact it is extremely difficult to so excite a rod as to set it in one definite mode of vibration. One particular mode may be made predominate, but the vibration is in general the resultant of all the modes of which the rod is capable under the conditions of its vibration. Hence when a rod is apparently sounding some particular harmonic this note is generally only the predominant component of a compound note including all the harmonics possible under the conditions of vibration. Hence, when a rod fixed at one end is apparently sounding its fundamental note, the note heard is really a compound of the fundamental note, in predominance, and the second, fourth, sixth, and higher harmonics. The harmonics or overtones present in this way with the fundamental note determine the quality of the compound note heard.

In the case of a rod fixed at one end the overtones are evidently only the odd harmonics. A note of this kind is generally of somewhat harsh quality, for it lacks the strength and fulness given by the even harmonics which include the octave and double octave of the fundamental note.

The frequency of longitudinal vibration of a rod may be calculated when its elastic constants and linear dimensions are known (see Art. 158).

Owing to the intensity of the stresses brought into action the frequency of a rod in longitudinal vibration is very high, so high that the pitch of the overtones soon passes beyond the limits of audibility. For example, the frequency of vibration for the fundamental note of a rod of glass 20 cms. long fixed at one end is about 5,000 per second. The frequencies for the corresponding overtones are therefore 15,000, 25,000, 35,000, and so on. Of these probably only the first is within the audible limit for ordinary observers.

The longitudinal vibration of rods has practically no application in the construction of musical instruments.

156. Owing to the great number of successive impulses whose energy may be added to cause a resonant vibration, the movement of the air in a resounding tube, or of the material of a rod in resonant vibration, may be very large compared with the ordinary movements of progressive undulation. In an air column the movement of the air may amount to an actual wind, capable of carrying along cork filings or other light powders. By drawing, with one hand, a resined cloth along a steel bar, we can make the bar lengthen and shorten to an extent which it would require a direct pull of many tons' weight to effect. Thick glass rods capable of supporting a ton or more may easily be pulled to pieces in this way.

157. Experimental Illustrations of the Vibrations of Air in Pipes.—Any of the modes of resonant vibration of an air column may be imitated by means of the spiral wire of Exp. 11.

Exp. 71.—Suspend a short heavy pendulum, of adjustable length, vertically over one end of the spiral, so that the bob of the pendulum hangs between the horizontal rods and on a level with them. Disconnect the pair of strings supporting that end of the spiral from the rods and tie them to the pendulum bob, so that the end coil of

the spiral now hangs from the bob, while the others hang from the rods. The other end of the spiral must be fixed if the vibration in a closed tube is to be represented; free for an open tube. If the pendulum, which represents the tuning-fork used with an air column, is set swinging in the direction of the length of the spiral, only an irregular movement of the coils results until the period of the pendulum is adjusted so that it fulfils the condition necessary in order that periodic impulses may cause resonant vibration of a rod or fluid column: the condition explained in Art. 143. The spiral then begins to vibrate quite regularly, with definite nodes where the coils do not move, and forms a good illustration of the process of longitudinal stationary undulation. With a little practice, the hand, moved regularly and rapidly backwards and forwards, may replace the pendulum bob, and gives better results.

The motion of the air in an organ pipe during vibration is readily shown by a simple experiment due to Savart. A light ring (Fig. 120), covered with tightly stretched paper or thin membrane, and carried like a scale pan by three light cords, is covered with a thin layer of fine dry sand and lowered into an open pipe in vibration.

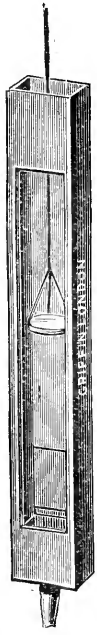


Fig. 120

At an antinode, where there is maximum motion and displacement of the air particles, the membrane moves up and down very rapidly, too rapidly for the motion to be directly observed, but the rattling of the sand particles on the membrane caused by the movement can be distinctly heard.

At a node where there is practically no displacement the membrane is at rest and no rattling sound can be heard. As the membrane is moved from a node to an antinode the rattling at once begins, at first very faintly, but gradually attaining a maximum at the antinode. If the membrane is moved beyond this point to the next node the sound gradually dies away and ceases when the node is reached. If one side of the organ pipe is made of clear glass, as in Fig. 120, the agitation of the sand at an antinode can be distinctly seen.

Exp. 72.—Sounding the fundamental note of an open organ pipe, suspend the membrane at the middle of the pipe. The bellows used for a blow-pipe will also do to blow the organ pipe. To ensure a steady blast, weight the boards of the bellows, and partly close the indiarubber tube with a clip. Work the treadle slowly, and as little as necessary. The middle point is a node and no rattling is heard. Blow the pipe more strongly until it sounds its first overtone. The middle point now becomes an antinode, and the sand is violently agitated and can be clearly heard rattling on the membrane.

The variations of pressure which accompany the strains of compression and rarefaction produced in the air column during vibration are most conveniently exhibited to a large audience by means of *Koenig's manometric flames.* (Art. 74.)

In order to investigate the variation of pressure at a point in the air column in an organ pipe the manometric capsule is fitted to the wall of the pipe, either directly so that the membrane *m* forms a portion of the inner wall of the pipe, or by means of a receiving capsule fitted over the outer face of the membrane and communicating with the air in the pipe by means of a tube and a mouthpiece which can be inserted in a hole bored at any point in the pipe. Fig. 121 shows an organ pipe fitted directly with three manometric capsules fed by one supply tube and communicating with the three burners shown on the stand. The behaviour of the burners can then be simultaneously examined by means of the rotating mirror shown at the right of the figure.

When a manometric capsule is fixed at an antinode in a vibrating air column where there is no variation of pressure the flame of the attached burner is quite steady. When, however, the capsule is applied at a node where there is maximum change of pressure the flame jumps up and down with a frequency equal to that of the air column. At any point intermediate between a node and an antinode there is also periodic disturbance of the flame, but the amplitude of the disturbance is smaller than at the node, and the teeth of the image seen in the rotating mirror are smaller and less marked the further the point is from the node. Thus the position of the node and antinodes for any particular node of vibration can be easily determined.

Exp. 73.—Fit an open organ pipe with three manometric capsules and burners, as shown in Fig. 121, at the points which divide its length into four equal parts, and make the pipe sound its fundamental note. It will be found that the middle flame indicates a

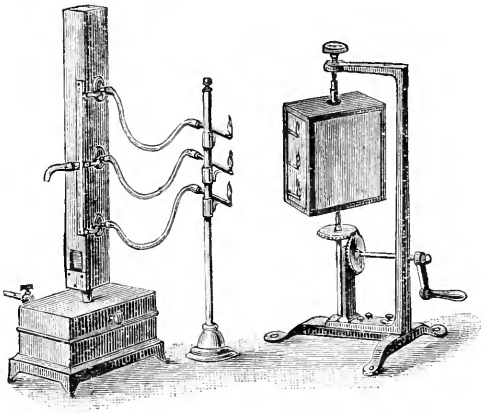


Fig. 121.

node at the middle point, but that the two other flames are practically undisturbed. Now make the pipe sound its first overtone: the top and bottom flames are disturbed, each indicating a node, and the middle flame is quite undisturbed, showing that it communicates with an antinode. These results confirm the descriptions already given of the modes of vibration of the column of air in an open pipe.

The position of the nodes in a vibrating column can also be exhibited by means of fine dust, such as lycopodium powder or fine cork dust. If a thin line of dry lycopodium powder is arranged along a perfectly dry tube fixed horizontally, and the air column is set in vibration by resonance, the dust at the antinodes is violently displaced and collects in little characteristically shaped heaps at the nodes. The mode of vibration of the air column is thus clearly indicated and the wave-length of the note can be measured. (See Art. 159.)

The same method can be used for a vibrating liquid column, a heavier powder such as precipitated silica being used instead of lycopodium or cork dust.

The magnitude of the variation of pressure and the corresponding amplitude of displacement which takes

place during the longitudinal vibration of an air column in an organ pipe have also been determined experimentally.

If a manometer or pressure gauge of the ordinary U-tube shape and containing water as the indicating liquid is applied at a node in the vibrating column by fitting the horizontal branch of the tube through a hole in the wall of the pipe, the manometer will be subject to a pressure which varies, with high frequency, above and below the atmospheric pressure. These variations will be so rapid that the water in the manometer cannot respond to them and will simply indicate the normal pressure about which the variations take place. If, however, the end of the tube fitted into the pipe is covered with a valve opening inwards only, then air from the pipe will pass into the manometer until the pressure in the latter is equal to the *maximum* pressure in the pipe. Similarly, if the valve is made to open outwards only, air passes from the manometer into the pipe until the pressure in the manometer is equal to the *minimum* pressure in the pipe.

By this means Kundt found that in a closed tube about a foot long the manometer indicated a variation of pressure at the node of about one-thirtieth of the atmospheric pressure. It can be calculated that this variation of pressure corresponds to a maximum amplitude of displacement at the antinode of about ·4 cm., that is, at the antinode the air particles, in this case, moved up and down through a total distance of about ·8 cm. or one-third of an inch.

The nodes of a rod in stationary undulation may be shown either by scattering sand on it or by putting a number of card or wire rings on the rod and holding it nearly, but not quite, horizontally. When the rod is set in resonant vibration, the rings slip down it till they reach a node.

The resonant vibration of gas columns and rods affords a means of determining the velocity of sound in the respective gases and solids.

158. Indirect Methods of Determining the Velocity of Sound.—From the results given in this chapter it will be obvious that the velocity of sound in any material (in the limited sense explained below) may be determined if the frequency of longitudinal vibration of a column or rod of that material is known. Thus, if a rod of any material of

length l, clamped at its middle point and set in longitudinal vibration, is found to have a frequency denoted by n, then, since $V = n\lambda$ (Art. 19) and λ here equals $2l$, we have $V = 2nl$. This value of V, however, is strictly the velocity of longitudinal wave motion along the rod as given by $V = \sqrt{Y/D}$, and although this is the only type of "sound" waves which can travel along the rod, it differs from the longitudinal wave motion in the material as a free medium which constitutes the true sound wave and which travels with the velocity given by

$$V = \sqrt{(E + \tfrac{3}{4}N)/D}$$

as explained in Art. 39. Further, if the material of the rod is of grained structure, like wood, the value obtained for V will evidently depend on how the rod is cut relatively to the grain.

Exp. 74.—*Find the velocity of sound in the substance of a rod or wire.*

(I) ROD. Take a rod of glass, oak, deal, or brass about 5 or 6 feet long. Grasp the rod at its centre by the left hand. Rub it by the right hand with a well resined leather. (A glass rod requires a cloth moistened with water, or, better, alcohol.) Obtain the frequency (n) of the note emitted (the simplest method is by a sonometer and a fork of known frequency, Art. 177). Measure the length, l, of the rod.

The note (fundamental) obtained is that of the rod when vibrating longitudinally and having a node at the middle and antinode at each end. Hence the wave-length, $\lambda = 2l$. Hence velocity, $V = 2ln$.

(II) WIRE. Take a length of steel wire (piano wire) or copper wire. Stretch it over two bridges 9 or 10 feet apart. The stretching force need not be known: the note emitted is independent of this. Proceed as in (I).

When giving the fundamental note there is a node at each end and an antinode in the middle.

The value of the velocity obtained in the determination above is that in a wire or tube. The velocity in an unlimited medium is about 1·1 times the velocity in a rod.

The Young's modulus of the substance of a rod or wire can be calculated if we know the velocity of sound in the rod or wire and the density of the substance. For since

$$V = \sqrt{\frac{Y}{D}} \text{ and } V = 2nl$$

$$Y = 4Dl^2n^2.$$

Although the results of this method in the case of a solid are of considerable indirect interest, they are of little value for the determination of the true velocity of sound in the solid. The value of the velocity of sound waves in an extended solid medium is, however, of theoretical interest only, and has not been studied experimentally.

In the case of air, however, the results of the method (Exps. 63 and 64) are important. The velocity of sound given by determining the frequency of vibration of an air column of known length in a pipe is the true velocity of sound in air along the pipe. The method can therefore be applied to determine the velocity of sound in air in pipes of different diameter, and by combining these results with the more extended results obtained by direct experiments, such as those of Regnault (Art. 47), the velocity of sound in free air can be determined. This velocity is of practical interest and importance.

Exps. 63 and 64 can be used to find the change of velocity due to a change of temperature in the air of the inner tube. In this experiment, mercury should be used to adjust the length of the air column in the inner tube instead of water, as, if water was used, the air would be mixed with a large proportion of water vapour at high temperatures.

The tube can be filled with other gases instead of air, and the velocities determined. For gases lighter than air, the open end of the tube must be turned downwards, and the length of the column adjusted by means of a sliding piston instead of liquid.

Owing to the small mass of the air column, its vibrations are easily controlled, so that it resounds nearly as loudly when it is a centimetre too long or too short as when it is of a length which would vibrate freely with the same frequency as the waves arriving from outside. Any single determination of the wave-length by this method is therefore very uncertain, though the average of a large number of independent determinations is fairly reliable.

A better way is to use a closed organ pipe whose length can be adjusted by a sliding piston. The tube is kept sounding by an air-blast, and the piston adjusted till the note is the

same as that of the fork, as shown by the absence of beats. In this case the air column is vibrating with its free-vibration frequency. But an organ pipe does not work well unless its mouth is much smaller than the cross section of the tube, and the narrowness of the opening increases the time required for the air to move in and out, so that an air column much shorter than a quarter of the wave-length of the fork vibrates with the same frequency as the fork. In an organ pipe, therefore, the distance from the mouth to the nearest node differs from a quarter of a wave-length by a considerable, and rather uncertain, amount, so that we cannot determine the wave-length of a note from a knowledge of any one length of the column which will give the note.

Two observations are therefore necessary. First blow gently, so that the air column vibrates in its fundamental mode (Fig. 113, *a*), and observe what length gives the same note as the fork. Then blow harder so as to make the column vibrate in its second mode (Fig. 113, *b*), and increase the length of the column till it again gives the note of the fork. In each experiment there is a node at the piston, and in the second experiment there is also a node at the point where the piston was in the first, since the frequency is the same in each case. So the difference between the lengths of the column in the two experiments is half a wave-length of the waves from the fork.

A whistle fitted by means of a cork into one end of a long glass tube 2 inches in diameter answers well for this experiment; it can be blown by foot-bellows.

The difficulty about the exact position of the point of reflection is avoided by another method, due to Kundt. This is described in the next Article.

159. Kundt's Dust Tube Experiment.—In this experiment, though a wave-length is actually measured, it is not really a method of determining the wave-length of a given wave-system, but of comparing the lengths of waves of the same frequency in different substances, and so the velocities of sound in those substances. One form of the apparatus is shown in Fig. 122. A wide glass tube *T*,

about 5 feet long and $1\frac{1}{2}$ inches in diameter, closed at one end by a tight-fitting cork K, rests in a framework up and down which it can be readily moved. The tube must be thoroughly dry and a fine even line of lycopodium or cork

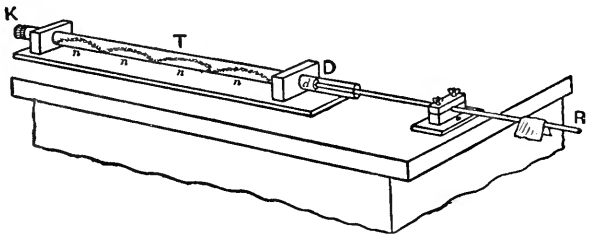


Fig. 122

dust arranged along its length. A rod RD of wood, metal, or glass, about 5 or 6 feet in length and $\frac{1}{2}''$ in diameter, is taken. At the end D a light disc d, of cardboard or cork, just smaller in diameter than the tube, is securely and rigidly attached to the rod by glue, sealing-wax, or a tack. The rod is then clamped horizontally and securely in a clamp of the form shown with the disc end penetrating some way up (9″ or 10″) the tube, the axis of the tube and rod being in line. When in position the tube should be twisted around its axis so that the lycopodium is drawn up on the side and is just on the point of slipping down.

A cloth or piece of leather, dusted with resin or moistened with alcohol, is drawn along the outer half of the rod, and when the rod squeaks it is in longitudinal stationary undulation, the middle being a node and the ends antinodes. The tube is then slowly shifted half an inch at a time until the dust begins to move when the rod squeaks. This is a sign that the air in the tube is being set in resonant vibration. The first movement occurs at the antinodes, the dust at these places sliding down towards the bottom of the tube and showing peculiar striations. Unless the tube is well adjusted, nothing further will be obtained, but with a little care the dust will be blown away from the antinodes and collect at the

nodes in narrow transverse patches. The corked end of the tube is a node, and the end of the rod, though an anti-node for the rod, is situated practically at a node for the air, for its longitudinal excursions are small. It will be found, therefore, that the nodes divide the distance from the cork to the disc in an exact number of parts of equal length. Measure the distance from a node to the mth node from it with a scale, and divide by $m - 1$. The quotient is the half wave-length in air of the note emitted by the rod. The length of the rod is the half wave-length in the rod of the same note, hence the ratio.

$$\frac{\text{Length of internode in tube}}{\text{length of rod.}} = \frac{\text{velocity of sound in air}}{\text{velocity of sound in rod}}$$

so that if either of the velocities is known the other can be calculated. Or if the frequency of vibration of the rod is determined by the vibroscope, siren, sonometer, or by calculation,* the value of the absolute velocity of sound in the air and in the rod can be found.

By filling the tube with different gases in turn, the relative velocities of sound in them can be found: they are proportional to the spaces between consecutive nodes; if liquids be used instead of gases a heavier powder, such as fine silica, must be used. As the walls of the tube are not perfectly rigid, the velocity of sound along the liquid in a tube is not nearly so great as in a large volume of the same liquid (Art. 39), so the results are not of great value.

Rods of different materials may be tried in place of the glass one, the tube being always filled with air. The velocities of sound in the different rods are proportional to the numbers obtained by dividing the length of each rod by the length of the segments into which the ridges divide the air space when that rod is sounding; for these numbers are the ratios of the velocities of sound in the rods to velocity in air.

* If Young's Modulus for the glass, and its density, are known, the velocity of a travelling wave in it can be found from Art. 35, and the period of the stationary undulation of the rod, when vibrating with one node as in this experiment, is the time that a travelling wave would require to travel twice the length of the rod.

Although the above is the arrangement usually described, there is some difficulty in holding the rod firmly enough by one point. The experiment is much easier if the rod is firmly held in two clamps, at distances of a quarter of the length of the rod from each end. The rod is set in vibration by drawing a resined cloth along the part of the rod between the clamps. The clamps are nodes, the ends of the rod and its middle points antinodes.

For any of these purposes, except the comparison of different rods, the rod and disc may be dispensed with, and the wide tube itself made to vibrate longitudinally. This is done by holding it by the middle, and drawing a cloth, moistened with alcohol, along one half. The tube is closed at both ends by corks, one at least of which must be adjustable in position as in the last experiment. When the distance between the corks is exactly or nearly an exact number of half wave-lengths in air of the frequency of the vibrations of the glass, the air is thrown into resonant stationary undulation, the corks being nodes (very nearly, like the disc in the other form).

Exp. 75.—*Find the velocity of sound in air and in a solid rod with a Kundt's dust tube.*

EXAMPLE.—In such an experiment, using a deal rod 186 cms. long, clamped in the middle, the distance between 7 nodes when the temperature was 13° C. was 80·5 cms. The frequency of vibration of the rod (found by a monochord and a c″ fork) was 1270. Find (1) the velocity of sound in the air at 0° C., (2) the velocity of sound in the rod.

$$\text{Distance between 7 nodes} = 80{\cdot}5 \text{ cms.}$$

$$\therefore \text{ half wave-length in air} = \frac{80{\cdot}5}{6}.$$

$$\therefore \text{ velocity of sound in air at } 13°\text{C.} = 1270 \times \frac{80{\cdot}5}{3}$$

$$= 34000 \text{ cms./sec.}$$

$$\therefore \text{ velocity of sound in air at } 0°\text{C.} = 33200 \text{ cms./sec.}$$

$$\text{Wave-length in the rod} = 186 \times 2 = 376 \text{ cms.}$$

$$\therefore \text{ velocity of sound in the rod} = 1270 \times 376$$

$$= 478000 \text{ cms./sec.}$$

The Dust tube can be used to find the velocity in any gas other than air by filling the tube with a gentle stream of gas passed in through a small tube thrust through the cork *K*.

To find the effect of temperature on the velocity of sound surround the wide tube with a steam jacket, and pass a current of steam through it for some time until

the temperature within the dust tube is practically 100° C. Obtain the dust figures as before and deduce the velocity of sound in air at 100° C. Compare this with the value found for the velocity at ordinary temperatures and confirm the relation

$$\frac{V_T}{V_{T'}} = \sqrt{\frac{T}{T'}}$$

160. Application of Kundt's Dust tube to find the Ratio of the Specific Heats of a Gas.

The velocity of sound in a gas can be determined by the dust tube and then equated to $\sqrt{\frac{\gamma P}{D}}$ (Art. 40), whence the value of γ can be found. In this way Rayleigh and Ramsay found the γ of Argon and Helium to be 1·66.

A variation of the method was used by Kundt and Warburg to find the γ of mercury vapour. The rod was supported at its points one quarter of its length from the ends, so that when stroked in the middle it gave the octave of its fundamental. One end of the rod penetrated a dust tube containing air, while the other penetrated a dust tube containing mercury vapour, which was kept in a furnace. Quartz sand was used in this tube to show the nodes. They found for mercury vapour $\gamma = 1{\cdot}66$, thus verifying its monatomic nature.

EXAMPLES XI.

[In the following examples the velocity of sound in air is to be taken as $33{,}200 + 60t$ cms. per second, t being the centigrade temperature. Unless otherwise stated, the temperature is to be taken as 0° C.]

1. Explain in what way the air vibrates in an open organ pipe sounding its fundamental note. How would you show the state of motion of the air in the pipe?

2. What is the relation between the wave-length in air for a given note, and the length of the closed organ pipe which resounds to it? Account for the difference in quality of notes of the same pitch from a closed and from an open organ pipe.

3. In the case of a closed organ pipe state clearly in what manner and direction the air particles move when the pipe sounds its fundamental note. How is the motion in the pipe produced?

4. If you blow across the open end of a key you can frequently obtain a shrill note. What connection is there between the length of the key and the shrillness of the note? By blowing very strongly, a note of much higher pitch may be obtained. Explain this.

5. Describe an organ flue-pipe. If two such pipes of the same length are sounded, the one open, the other closed, how do the notes differ from each other?

6. A brass tube closed at one end readily responds to a tuning-fork at the ordinary temperature, but not when heated considerably. Explain this. Could it be made to respond otherwise than by altering its length?

7. A small closed organ pipe is connected by a long tube full of air with a bag of coal-gas. On forcing the air and gas gently through the pipe so as to elicit its fundamental note, how would you expect the pitch of the note to be affected when the coal-gas reached the pipe?

8. A stopped organ pipe 4 ft. long gives when filled with a certain gas the same note as an open pipe 5 ft. long filled with air. Calculate the speed of sound in the gas. Temp. 20° C.

9. Give the lengths of the three shortest closed tubes, and of the three shortest open tubes, which would resound to a tuning-fork making 200 vibrations per second.

10. Give in each case the four slowest vibration frequencies possible in a tube 3·32 metres long (*a*) when open at both ends, (*b*) when open at one end, (*c*) when closed at both ends. [Neglect the correction for radius.]

11. If a tube makes 340 vibrations per second when the temperature is 16° C., what is its frequency, in the same mode of vibration, when the temperature is 51° C.? [Neglect expansion of tube.]

12. The air in a closed tube 34 cms. long is vibrating with two nodes and two antinodes, and its temperature is 51° C. What is the wave-length of the waves produced in the air outside the tube if the temperature of that air is 16° C.?

13. A closed tube 15 cms. long resounds, when full of oxygen, to a given fork. Give the length of a closed tube, full of hydrogen, which will resound to the same fork.

14. If the velocity of sound in hydrogen is 126,000 cms. per second, and in air 33,300 cms. per second, what is the length of the waves which will be produced in the surrounding air by blowing an open organ pipe, a metre long, with hydrogen, the pipe being also full of hydrogen?

15. A vertical tube 1 metre long and 4 cms. in diameter is gradually filled with water, while a tuning-fork, making 500 vibrations per second, is held over the upper end. At what positions of the water surface will the tube resound (taking the correction for diameter of the pipe (·3d) into account)?

16. What must be the diameter of a closed tube 2 ft. long in order that it may resound to the lowest note given by an open tube 4 ft. long and 8 ins. in diameter?

17. Where must a conical tube, closed at the apex, be cut in two in order that each part may resound to the same note? What is the ratio between the frequency of this note and the note to which the whole tube would resound?

18. (*a*) A Galton's whistle (Art. 101) in the form of a very narrow pipe, closed by a movable plug, was used to find the frequency of the highest note audible to a certain person. The shortest length giving an audible note was 4·5 mm. Find the frequency of this note. (*b*) The lengths (corrected) of the closed pipes of a double whistle giving difference tones (Exp. 41) are 7·2 and 6·0 cms. Find the frequency of the difference tone. Take $V = 34000$ cms./sec.

19. When rods of the same length, but of different materials, are held in the middle and rubbed with a resined glove in the direction of their lengths, explain why musical notes are produced and why they are of different pitch.

20. A wooden rod 5 ft. long held in the middle and rubbed with resined leather gives the same note as an open organ pipe 4 ft. 3 ins. long. Find the speed of sound in the rod. Temp. 12° C.

21. A wooden rod 1 yd. in length floats (when compelled to float vertically) with 2 ins. of its length out of water, and if rubbed longitudinally, without being firmly fixed anywhere, the lowest note it can be made to give has a frequency of 256. Find Young's Modulus for the wood in poundals per square foot.

22. A brass wire 2 metres long and 1 sq. mm. in sectional area weighs 16 grams, and when it is hung up by one end, and 20 kilog. are suspended from the other, it elongates by 6 mm. What note will it give when rubbed in the direction of its length?

EXAMINATION QUESTIONS III.

1. A vibrating tuning-fork is held near the mouth of a cylindrical tube 32 cms. long closed at one end, and the tube is found to "speak." Explain clearly why this happens.

Assume the velocity of sound in air to be 340 metres per second, find the time of vibration of the fork

2. A cylindrical tube 100 cms. long, closed at one end and of 1 cm. internal radius, is placed upright and filled with water, and a tuning-fork of frequency 510 is sounded continuously over its open end. Assuming the velocity of sound in the air to be 340 metres per second, describe exactly what you would expect to observe if the tube were gradually emptied.

3. An open organ pipe emits a fundamental note of frequency 256 when sounded in air. Assuming the velocities of sound in air and coal-gas to be 350 and 500 metres per second respectively, find the pitch and wave-length of the note emitted by the organ pipe when sounded in an atmosphere of coal-gas.

4. State how the frequency of the note emitted by an organ pipe depends upon (1) its length, (2) the pressure of the gas in which it is sounding, (3) the temperature of the gas, and (4) the nature of the gas.

Does the frequency depend upon the cross section of the pipe?

5. The disc of a siren possesses 32 holes and it is making 1050 revolutions per minute. Find the length of the open organ pipe which, sounding its fundamental, will emit the same note.

6. The end of one of the prongs of a tuning-fork is held over the mouth of a tube which can be raised or lowered in water. When the mouth of the tube is at a given height above the water the sound of the fork appears to swell out loudly. Carefully explain this.

Would the height be different if (*a*) the temperature of the air were higher, (*b*) if the air in the tube were replaced by carbonic acid gas? Give reasons for your answer.

7. State, and carefully account for, the difference in the mode of vibration of the air at the middle point of a closed and of an open organ pipe.

Describe a simple method of experimentally exhibiting this difference.

8. Describe (*a*) the motion of the air in an organ-pipe that is sounding its fundamental tone; (*b*) that of the air outside the pipe by which the sound is transmitted. Also explain (best with the help of a diagram) the connection between the motion of the air inside and outside the pipe.

9. Describe the motion of the air in any particular position in a closed and in an open organ pipe, each sounding its first harmonic.

10. State and explain carefully how you suppose the air in an open organ pipe which is sounding its fundamental note to be moving, defining the terms *wave-length*, *frequency*, *amplitude*. What is the relation between the length of the pipe and the wave-length of the note sounded?

11. How does the pitch of the note sounded by an open organ pipe depend on the length of the pipe and the velocity of sound in air? Show how the note is affected by changes of temperature.

12. Describe a mouth (or flue) organ pipe. What is the cause of the note which such a pipe emits when air is driven into it at gentle pressure? How is this note related to others which are sometimes produced when the pressure is increased?

13. Explain how the pitch of a note emitted by an organ pipe, closed at one end and open at the other, varies with the length of the pipe. Find, also, the frequencies of the harmonics given by such a pipe when that of the fundamental is 256 vibrations per second.

14. Describe the various modes of vibration of the air in a narrow pipe open at both ends. Taking the fundamental mode, indicate, by carefully drawn figures, the motion at various parts of the pipe at some one instant, and the distribution of pressure in the tube at some one instant.

15. Describe carefully the states of the air, (*a*) at a node, (*b*) at the middle of a vibrating segment, in an organ pipe during a single vibration.
What would be the effect of boring a hole through the side of the pipe at either place?

16. What is meant by resonance? Show how the phenomenon of resonance may be used to measure the velocity of sound in a gas.
Point out any analogy which appears to you to exist between an echo and the resonance of an open or closed tube.

17. Describe and show how to test the mode of vibration of the air in an open organ pipe. How is its length related to the wave-length of the fundamental note of the pipe? With a tuning-fork and a pipe, how would you determine the velocity of sound in air?

18. What must be the length of a tube, closed at one end, that a vibrating tuning-fork held over its mouth may receive its maximum reinforcement of sound from the vibrations of the air in the tube? Explain clearly how the sound-wave is propagated in the tube.

19. How would you find the frequency of vibration of a given organ-pipe? Describe carefully the apparatus you would use; and give an account of the experiment.

20. Explain, by means of diagrams or otherwise, the motion of the air in sounding organ pipes (*a*) when open at both ends, (*b*) when closed at one end and open at the other. Calculate the frequency of vibration of a closed pipe 11 ft. long. (Velocity of sound in air = 1,100 ft. per second.)

21. A tuning-fork, making 1,028 vibrations per second, is sounded opposite one end of an open tube 3 ft. $2\frac{1}{4}$ ins. long, which reinforces the note of the fork. Calculate the positions of the various points where maximum movement and maximum pressure-change occur in the air column in the tube. (N.B.—Velocity of sound in air may be taken at 1,100 ft. per second.)

22. Describe an experimental method of exhibiting the variations of pressure at the nodes of a vibrating air column.

23. If you are required to prove that in warm air sound travels quicker than in cold, how would you do it? What is the law connecting velocity of sound and temperature in the case of a dry permanent gas?

24. A sounding organ pipe is warmed by means of boiling oil from 16° to 127° centigrade. What is the effect on the note which it emits?

25. Describe carefully some accurate method of determining the velocity of sound in hydrogen.

26. Describe a simple experiment by which you could prove that the velocity of sound through coal-gas is not the same as the velocity through air.

27. A rod is held in the middle and rubbed longitudinally with a resined rag till it emits a note. Describe carefully the way in which it is vibrating. How can the velocity of sound in oak be compared with that in deal?

28. A metal rod is clamped at the centre and made to vibrate longitudinally. Explain the nature of the wave-motion produced, and state what forces are engaged in the transmission of the wave along the rod. How may the velocity of sound in the metal be determined?

29. Describe some method of measuring the velocity of sound in a rod of glass.

30. Explain the difference in quality of the notes given out by open and closed tubes when set in vibration by an air blast. Is there a corresponding difference in the sounds heard when the tubes resound to a tuning-fork?

CHAPTER XII.

TRANSVERSE UNDULATION.

161. Transverse Wave in a Cord.

Exp. 76.—Take a long rope AB (Fig. 123) and fasten one end B firmly to a wall at a point about six feet from the ground. Hold the other end A in your hand, about four feet from the ground, and stretch the rope till the part nearest to your hand is about horizontal. It will not be quite straight; but that may be neglected at present. Raise your hand suddenly a few inches, so that A is at A'. The immediate result of this is that a short portion $A'C$, close

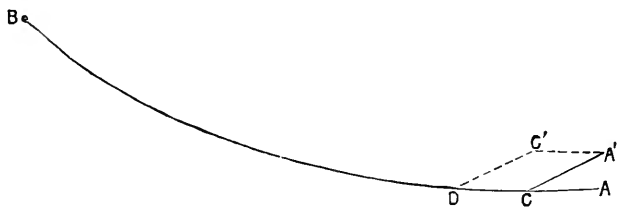


Fig. 123.

to your hand, is in an altered condition, not rarefied or condensed, but sloping instead of horizontal, the rope having the form $A'CB$. The point C is now acted on by two forces, due to the stretched condition of the string, along CA' and CB, and the resultant of these is upwards. C therefore begins to move up, and it stops only when it has moved as far as A moved, so that a portion of the rope $A'C'$ is horizontal again, while the sloping condition exists in another portion $C'D$, the rope having now the form $A'C'DB$. D then begins to move up in the same way, and so the sloping condition which was first produced in $A'C$ travels all along the rope.

Now this continuous movement of a condition of slope along the rope has been effected by each portion of the rope moving in turn a short distance in a direction at right angles to the rope: A to A', C to C', and so on. Further, each portion moved only while it formed part of the sloping section; as soon as the sloping condition had passed it, it was at rest again.

The whole process is in many respects very similar to the transmission of a pulse of condensation or rarefaction, as described in Art. 15. In both cases a *condition* of altered relative position of the particles travels continuously, while each particle in turn moves a short distance and then stops. In both cases the altered relative position exists where the particles are moving, and when the particles come to rest they are in their original position relative to their neighbours, though not in space. In both cases the velocities with which the particles at different points are moving are proportional to the difference between their actual and their ordinary relative position. The chief differences are (1) that in Exp. 76 the "altered relative position" of the particles is altered relative *direction*, not altered relative *distance;* slope, not condensation or rarefaction; (2) the short movement which each particle executes in turn is at right angles to the direction in which the condition travels, not in that line, as in Art. 15. For this reason this kind of motion is called *transverse progressive undulation*

Ripples and waves on the surface of a liquid are familiar examples of tranverse wave motion. In the case of ripples only the particles near the surface layer of the liquid are involved in the motion; these particles vibrate up and down in vertical paths, which are approximately straight lines at right angles to the direction of transmission. In the case of larger waves particles below the surface are involved to a depth determined by the size of the waves. These particles vibrate in closed curves lying in a vertical plane through the direction of transmission at any point. The wave motion in the case of these waves is therefore not strictly transverse wave motion, but under the conditions of transmission the transverse component of the vibratory motion of the particles is the one that determines the character of the waves, and the wave motion may therefore be considered as a case of transverse wave motion.

If a piece of cork is placed on the surface of water, along which regular surface waves are being transmitted, it will be noticed that it is not carried along by the waves, but oscillates up and down at a particular point in a closed vertical curve with a definite period. This period is the period of the wave motion, and may be determined by direct observation of the motion of the piece of cork. The surface of the liquid assumes the well-known *trough* and *crest* wave form characteristic of ripples and waves on a liquid surface, and the wave-length is the distance from crest to crest or from trough to trough.

162. Specification of a string.—A "string" in acoustics is usually understood to mean a cord or wire or filament of any material. Unless otherwise specified, it is supposed to be uniform in material and section throughout its length, and to be, therefore, of constant mass per unit length. It is further assumed to be perfectly flexible, or rather, its stiffness is supposed to be negligible, and during vibration the changes in length which it undergoes are also assumed to be negligibly small. These conditions are found to be fairly well satisfied by any ordinary cord or wire of not too great thickness, under a sufficiently high pull. The limit of thickness depends on the material and on the length of the wire, and cannot here be definitely specified; but, as an example, it may be noted that a steel wire of not more than, say, No. 20 gauge (·9 mm. in diameter) fulfils the specified conditions quite satisfactorily. The thin catgut strings of a violin are almost perfectly flexible.

163. Velocity of Transverse Waves along Strings.—It will be shown below that the slope produced by the movement of A (*i.e.* the angle between the changed direction and the original one) is proportional (as long as it is small) to the velocity with which A was displaced, but that the velocity with which the sloping condition travels along the rope does not depend on how A moved, but only on the mass of each unit length of the string and on the force with which the string is stretched. Even without investigating the exact relation, it is evident that, the more tightly the string is stretched, the greater the resultant force on C, and therefore the quicker C will move up to C', and the sooner D will begin to move, and so on. So that the velocity of a

transverse wave depends on the force with which the string is stretched. The velocity of a longitudinal wave, on the other hand, is the same whatever the tightness of the string, as explained in Art. 35. Even if a string or wire is stretched to the point of breaking, the velocity of a transverse wave along it is always much less than that of a longitudinal one.

I. To find a relation for the velocity of transverse waves along a string, let us consider a very long flexible string AB (Fig. 124) stretched horizontally by a force of f dynes at each end, so that a small portion A is in equilibrium

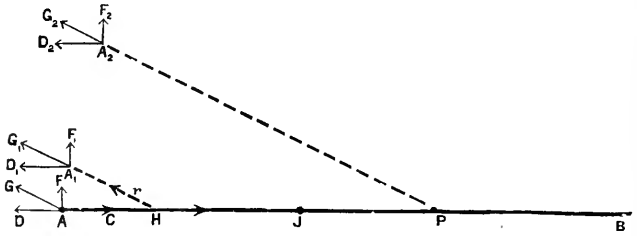


Fig. 124.

between two forces of f dynes, one AC along AB, and the other AD in the opposite direction. Apply at A another force AF of t dynes in an upward direction, and let the resultant of AF and AD be a force r in the direction AG. A will then move in the direction AF, and it can never be in equilibrium until the part of the string close to A is in the direction opposite to AG. When a portion of the string has thus assumed the position A_1H, the forces on a small portion at H are a force r in the direction HA_1, and a force f in the direction HB. These are equal and parallel to the forces r and f on A when the force was first applied; H therefore begins to move upwards exactly as A did, and another portion HJ becomes equally sloping, and so on.

As the forces on A_1 would accelerate it if A_1H was more horizontal than A_1G_1, and retard A_1 if A_1H was less horizontal than A_1G_1, A moves in such a way that AH is

always opposite to AG, and similarly for each succeeding portion.

Thus, as point after point of the string begins to move sideways, A continues to move in the direction AA_1 in such a way that the part of the string between A and the point of the string which is just beginning to move is parallel to the line GA. Let A_2 be the position A has reached when the sloping condition has extended to P. AA_2P is really isosceles, but in practice AA_2 is very short compared with the other sides, and we may consider AA_2 as perpendicular to AP. Hence $\frac{AA_2}{AP} = \frac{A_2F_2}{A_2D_2} = \frac{t}{f}$.

If V is the velocity with which the sloping condition extends along the string, $\frac{Vt}{f}$ is the velocity with which the sloping part of the string moves upwards. If m is the mass of one centimetre of string, $\frac{Vtm}{f}$ is the momentum of each centimetre of the sloping portion, and, as V fresh centimetres are added to this portion per second, $\frac{V^2tm}{f}$ is the increase of momentum per second. This is equal to the force which produces it. Hence $t = \frac{V^2tm}{f}$ or $V = \sqrt{\frac{f}{m}}$

$$= \sqrt{\frac{\text{dynamical measure of the force with which the string is stretched}}{\text{mass of unit length of the rope}}}.$$

Just as in the corresponding proof for a longitudinal wave, every variation of t produces a corresponding change of slope in the part of the string close to A, which slope is always proportional to t, and every change of slope produced in the part close to A extends along the string with the velocity $\sqrt{\frac{f}{m}}$.

The reader who has not studied dynamics may be surprised that a constant force applied to A gives A instantly a velocity which does not afterwards increase, not an increasing velocity. It must be remembered that the effect of a force is to cause a momentum

which increases as long as the force lasts. If the mass in movement is constant, its velocity must increase, but if, as in the cases considered above, the mass in movement keeps increasing as long as the force lasts, the velocity of the part in motion may be constant.

II. The following is another method of finding the velocity of transverse waves along a flexible string:—Let ab (Fig. 125) be a hump on a string AB travelling from B

Fig. 125.

to A with velocity V. If we impress on the whole string a velocity V in the opposite direction it is evident that the position of the hump will not change and that every point of Ab as it comes to the hump will glide over the hump and then travel on in a direction aB.

Now every portion of the string is under tension, the stretching forces on any portion of the string being equal, and in the same straight line if the string is straight. If the string is not straight the resultant of the two stretching forces on any piece of the string must be equal and opposite to the force which is causing the distortion of that particular element of string.

Let us now consider the motion of a small element of string, XY (Fig. 126), of length l as it crosses the top of the hump. Let m = mass of unit length of the string and f the stretching force in the string.

In order to calculate the forces on XY we shall imagine that the shape of XY is a circular. Let O be the centre of curvature of XY, *i.e.* the centre of the circle passing through XY, and let R be the radius of this circle. Since XY is travelling with velocity V, the centrifugal force F acting on XY is $\frac{ml.V^2}{r}$. It is this force which balances the resultant of the two stretching forces f, f, which are acting along the string perpendicular to OX and OY respectively.

Draw a triangle of forces ABC where AB, BC are parallel and proportional to f, f. Then CA, the third side, is parallel and proportional to F.

To find the magnitude of F we have, since ABC, XOY are practically similar triangles,

$$\frac{F}{f} = \frac{CA}{AB} = \frac{XY}{OX} = \frac{l}{r}.$$

But $$F = \frac{ml \,.\, V^2}{r},$$

$$\therefore \quad \frac{ml \,.\, V^2}{fr} = \frac{l}{r},$$

$$\therefore \quad V^2 = \frac{f}{m},$$

i.e. $$V = \sqrt{\frac{f}{m}},$$

a result identical with that obtained above.

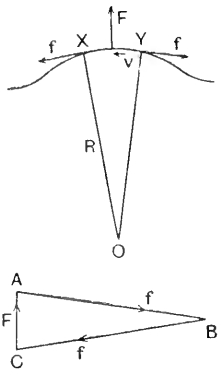


Fig. 126.

164. Reflection of Transverse Waves. — When the sloping condition arrives at the fixed point B (Fig. 124) it is reflected, and travels back again to A. The direction of the slope is the same in the reflected as in the original wave, but the movement of each particle of the rope while it forms part of the reflected wave is in the opposite direction to its motion while it formed part of the original wave. This corresponds to the reflection of a condensation or rarefaction at the closed end of a tube, and is reflection with change of sign.

If such a transverse wave arrives at a free end of a rope, it is also reflected, but in this case the slope is reversed, and the motion of the particles is not. This corresponds to the reflection of condensations and rarefactions at the open end of a tube. The wave produced by cracking a whip is reflected in this way at the free end.

Every movement which we cause A to execute at right angles to the length of the rope is repeated in turn by each particle. So that, until the waves reach B, the past displacements of A are the present displacements of the successive points of the rope. The rope is in fact a displacement curve representing the history of the movements of A.

If we move A up and down harmonically, the rope itself is thus thrown into harmonic waves, which travel along it away from A. The continuous line in Fig. 127 shows the form of the rope at an instant when A has been vibrating harmonically for some time; the arrows show the relative velocities of the material of the rope at different points.

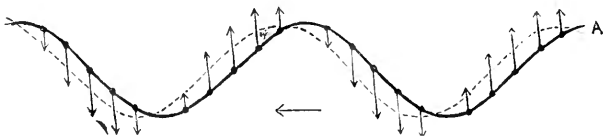


Fig. 127.

These velocities are proportional at each point to the slope of the rope there, and are in opposite directions at points where the rope slopes oppositely. An instant later every point of the rope has moved a short distance in the direction of the arrow attached to it, and the form of the rope is the dotted curve. Thus, while each particle of the rope has moved up or down, the form of the rope has moved to the left.

If the stretching force is different in different parts of the string, the velocities with which the waves travel along different parts are proportional to the square roots of the stretching forces as shown above. The number of waves which pass any point in a second is of course equal to the number that start in a second, and is the same in all parts of the string. The waves are therefore longer where they travel quicker, the length of a wave varying as its velocity varies as it goes along, as in longitudinal waves.

165. Transverse Stationary Undulation.—Next suppose we move both A and B (Fig. 124) up and down harmonically with the same frequency. A series of harmonic waves, of equal wave-length, will start from each end towards the other. After these wave-systems have met in the middle of the rope, the principle of superposition shows that the actual displacement at any given moment, of every part of the rope, can be found by adding the displacements which would be due at that moment to the wave-systems separately.

Let X, Fig. 128, be the form which the rope would have, at a given moment T, if only the waves from A travelled along it, and let Y be the form which it would have at the same moment if only the waves from B travelled along it. The actual form of the rope at the moment T is that found

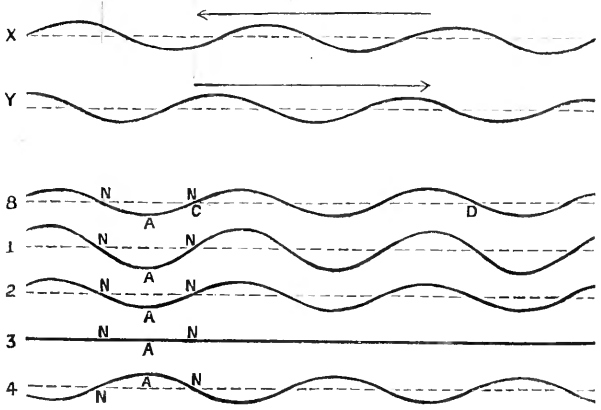


Fig. 128.

by adding the ordinates of X and Y; it is shown in line 8. The forms which would be due an instant later to the waves from A and from B respectively can be found by shifting X to the left and Y an equal distance to the right. If we suppose each advanced $\frac{1}{8}$ of a wave-length, and add the ordinates again, we get line 1. By advancing X and Y again each $\frac{1}{8}$ of a wave-length, we get line 2, and so on. (Only 5 out of the 8 such stages of a complete cycle are shown, the remaining 3 being simply 1, 2, and 3 inverted.)

We see that the actual movement of the cord is one in which its form is always a harmonic curve, and that, twice in each complete cycle, there is a moment when every part of the cord has, simultaneously, its maximum displacement, and is therefore at rest (stationary instant) while twice in each cycle the curve becomes a straight line. We see also that there are certain fixed points—**nodes**—

through which the cord always passes, so that the displacement at these points is always zero, but that it is at these points that the greatest changes of slope occur. Also that between these there are other points—**antinodes**—where the greatest displacements occur, but where the cord is always parallel to its original position, so that there are no changes of slope. The movement of the cord is thus one of **transverse stationary undulation.**

The distance from one node to the next is half a wave-length of either of the wave-systems which would be caused by the movement of A or B alone, and the stationary undulation goes through the complete cycle of movements in the time in which either of the progressive undulations, due to A or B alone, would advance one wave-length, or twice the distance between two nodes.

Thus, if l is the distance between two nodes and V the velocity of a transverse travelling wave, $2l/V$ is the period in which the stationary undulation goes through its changes, and $V/2l$ is its frequency or the number of times it does this is a second. As

$$V = \sqrt{\frac{\text{dynamical measure of stretching force}}{\text{mass of unit length}}},$$

this frequency

$$= \frac{1}{2l}\sqrt{\frac{\text{dynamical measure of stretching force}}{\text{mass of unit length}}}$$

$$= \frac{1}{2l}\sqrt{\frac{f}{m}}$$

where f is the dynamical measure of the stretching force and m is the mass of unit length of the string, both f and m being in the same system of units.

If, when the cord is in this condition of stationary undulation, we fix any two nodes, say C and D, the movement of the cord is of course unaffected, since C and D were stationary already. The condition of stationary undulation will therefore continue till the energy of the string has been partly communicated to the air in sound waves, and partly converted into heat in the string itself. When two points on a string are fixed, a solitary travelling

wave in the part between would run backwards and forwards between them, being reflected each time it reached either, and the stationary undulation of a cord fixed at two points may be conveniently considered as the resultant of two fictitious wave-systems travelling in opposite directions and continually reflected in the same way.

Exp. 77.—*Transverse waves on a rope.* Stretch a long rope (clothes-line) and fix its ends. Strike the rope near one end with the hand held edgewise. Observe that a depression is formed, travels to the other end, is there reflected, returns, is again reflected, and thus travels to and fro along the rope; the depth of the depression diminishes, its length does not alter. Pull the rope tighter, observe that the wave travels more quickly. (The wire ropes that hold up a tall flagstaff are well adapted for the experiment.)

Exp. 78.—*Stationary waves.* Fix one end of a rope or light brass chain, or a rubber tube filled with sand. Move the other end to and fro by the hand at such a rate (found by trial) that the rope vibrates in two or more segments. Observe that each *vibrating segment* or *loop* (also called a *ventral segment*) is separated by a stationary portion, a *node*. Each part of the rope in any vibrating segment is moving in the same direction, but the parts of adjacent segments are moving in opposite senses. Thus the complete wave is formed by two adjacent segments, and the wave-length is twice the distance between consecutive nodes.

Increase the rapidity of movement of the hand, the rope will then divide into a larger number of vibrating segments, each of shorter length.

166. Transverse Vibration of a Flexible String.—Let a string, cord, or thin wire, AB (Fig. 129), be stretched tightly between the fixed points A, B.

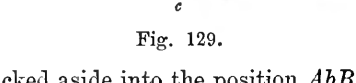


Fig. 129.

If it is plucked aside into the position AbB or AcB and released the string vibrates rapidly backwards and forwards between these positions with gradually decreasing amplitude until it comes to rest.

As explained above, the vibration is due mainly to the stretching force in the string and not to the elasticity of

its material. When the string is deflected into one of the lateral curved positions the action of the stretching force tends to restore it to its initial straight position and opposes its displacement from this position. For small lateral displacements the force resulting from displacement is proportional to the displacement, and the conditions are therefore such that the string is set in *transverse* vibration, and each point on it vibrates at right angles to the length of the string.

It will be noticed that the two fixed ends of the string at A and B are *nodes*, and as such are places of no amplitude of vibration but of maximum slope. As the points are fixed the amplitude is necessarily zero. Evidently also the vibrating string has an antinode at its middle point. The amplitude is here a maximum and the slope zero, for at the middle point the direction of the string is always parallel to its initial direction.

167. The Sonometer (or Monochord).—Experiments on the vibrations of strings are most easily carried out on a sonometer. As generally constructed the sonometer consists of a sounding box AA with holes in its sides, mounted

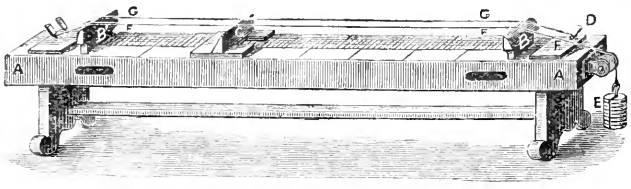


Fig. 130.

as shown in Fig. 130, and carrying the stretched strings to be experimented with on the covering board of this box. The instrument is usually arranged to carry two strings. As shown in the figure, one string GG can be stretched in the usual way between two wrest pins over fixed terminal bridges B, B. The other string FF is attached to a wrest pin at one end, but passes over a pulley at the other end and carries at its extremity a scale pan or weight carrier

for carrying weights, as shown at E. By means of these weights tension is applied to the string and can be adjusted and measured as required. This string also passes over the fixed terminal bridges B and B', but the bridge near the pulley must be so adjusted as not to cause a change in the tension of the string at that point.

The string GG, stretched between the two wrest pins, is usually required for purposes of reference as to pitch, and can be adjusted for pitch by altering its tension by means of a key applied to the wrest pin at D. It is convenient to use thin pianoforte steel wire for this string.

The fixed length of the strings in vibration is measured by the distance between the upright faces of terminal bridges B, B', and this distance, usually a metre, or half a metre, is divided by a scale marked on, or fixed to, the cover board of the sounding box, as shown in the figure. In order to experiment on different lengths of either string a *movable* bridge C is also provided. This bridge can be moved backward and forward along the length of the string, and by means of the scale any required length of string can be set in vibration. This bridge is so constructed that either string may be damped at any point without affecting the other. It is, however, more convenient to use two smaller bridges, one for each wire instead of a single large bridge.

The above is a costly and cumbersome instrument, and most of the experiments with strings can be carried out with the simple form of sonometer shown in Fig. 131. The base M is a deal board 36 ins. $\times$ 4 ins. $\times$ $\frac{3}{4}$ in. H is a block of hard wood $1\frac{1}{2}$ ins. $\times$ $1\frac{1}{2}$ ins. $\times$ 1 in. let into a hole in M and glued in position. A wrest pin W $1\frac{1}{2}$ ins. $\times$ $\frac{1}{4}$ in. diameter fits stiffly into M at an angle of 45°. B_1, B_2 are two hard wood bridges 3 ins. $\times$ $1\frac{1}{4}$ ins. $\times$ $\frac{3}{4}$ in. glued on the board with their upper edges exactly 30 ins. apart; these edges are faced with stout brass wire for the bridge wires to rest on. A_1, A_2 (only one of which is shown) are two smaller but slightly higher movable bridges of dimensions $1\frac{1}{2}$ ins. $\times$ $1\frac{3}{8}$ ins. $\times$ $\frac{3}{4}$ in. A paper scale graduated, by lines $\frac{1}{4}''$ apart, into 120 divisions lies between B_1 and B_2. S_1 and S_2 are two round-head iron screws. A piece of pianoforte

wire is looped around S_1 and the other end drawn through W, which is then turned to tighten the string. The other screw S_2 serves to support the wire which is to be weighted; when this second wire is used the sonometer is fixed vertically to the wall or the edge of the bench by a large screw P.

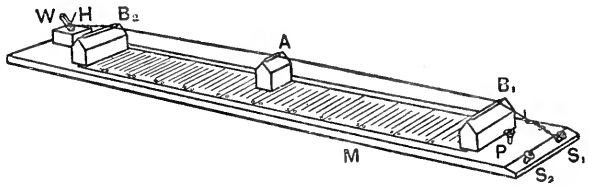


Fig 131.

168. Experiments with the Sonometer.—In Exps. 79-81 the sonometer should be placed horizontally on the bench. In Exps. 82-86 it should be placed vertically.

Exp. 79.—Pluck the stretched string near the centre, then release it. The string vibrates in one segment, the ends being nodes. This is the fundamental mode of vibration. Observe that the note is of unvarying pitch but of diminishing intensity. Hence the intensity is dependent on the amplitude, but the pitch (and frequency) are not, *i.e.* the vibrations are isochronous.

Exp. 80.—*Tune a wire to give a note in unison with a given note, say from a tuning-fork.* Sound the fork and pluck the stretched wire of the sonometer. Adjust the movable bridge until the notes emitted by the wire and fork seem to be the same in pitch. To make sure of this, sound first one and then the other. To a musical ear tuning is not a difficult matter, but the unmusical is easily deceived, especially with notes of a high pitch. The concord of a note (n) with its octave ($2n$) or fifth $\left(\frac{3n}{2}\right)$ is liable to be mistaken for that of two notes of the same pitch. Hence when the experimenter has found a point at which the agreement seems right he should move the bridge so as to make the vibrating segment first somewhat longer, then somewhat shorter. If in the former case the note of the wire is unmistakably lower in pitch, and in the latter higher, then the original position is nearly the correct one.

If these observations are not confirmed, one note is a harmonic of the other, hence the bridge must be moved to another part of the wire and tuning recommenced.

It is an assistance to use a rough "stethoscope" consisting of a wooden rod $\frac{3}{8}$ inch square and 8 inches long, fixed at one end into a slab of wood 3 inches square and $\frac{1}{2}$ inch thick. The end of the rod is put on the sonometer board, and the ear is placed against the surface of the slab. Sound from the sonometer is conducted along the wood, and is easily heard in a noisy room.

169. Use of Beats to Effect Exact Tuning.—With a wire and fork of moderate pitch beats are readily heard, especially when the sound is dying away, by pressing the shank of the fork on the board of the monochord. The two may be adjusted to agree so closely that a beat takes several seconds to complete itself. In obtaining unison by beats find the positions of the bridge, one on each side of the correct point, at which say one beat per second can be counted: the mean of these positions may be taken as the exact position of the bridge for unison.

170. Use of Resonance to Obtain Exact Tuning.—If, when the fork and wire are in tune, the fork is sounded and its shank pressed on the board of the sonometer, the string will be set in vibration; its note will be heard if the fork is removed and silenced. The movement of the string can also be detected by placing a finger just in contact with it; also small paper $\wedge$-shaped riders placed on the wire will be thrown off (see Figs. 135, 136).

The wire begins to respond to the fork only when unison is very nearly attained. This can be shown by first tuning the notes as nearly as possible, then gradually moving the bridge and noting the positions, one on each side of the correct point, at which the response cannot be detected.

In tuning a wire to a fork obtain first the beats, and when these have been reduced in rapidity look for the sympathetic vibration. The latter is not obtained easily if the sounding body cannot be placed on the sonometer. Beats may be heard with any two notes nearly in unison, but are not readily detected with loud or high notes.

Fainter beats and weaker response are sometimes obtained between the harmonics of the notes.

171. Verification of the Formula for the Vibration of Strings.—From the formula $n = \frac{1}{2l}\sqrt{\frac{f}{m}}$ several facts at once follow which are sometimes called the "laws" of the transverse vibration of strings.* The most important are:

(1) If the stretching force and mass of unit length remain constant, the frequency varies inversely as the length.

(2) If the length and mass per unit length remain constant, the frequency varies directly as the square root of the stretching force.

(3) If the length and stretching force remain constant, the frequency varies inversely as the square root of the mass of unit length.

The last of these may be put into two different forms for round strings or wires: for let d = diameter of a string, and D = density of the material of which it is composed,

then $$m = \pi \frac{d^2}{4}.D.$$

$\therefore$ the formula

$n = \frac{1}{2l}\sqrt{\frac{f}{m}}$ becomes $n = \frac{1}{dl}\sqrt{\frac{f}{\pi D}}$, from which follows:

(3*a*) If the length, stretching force, and material of several strings are the same, their frequencies are inversely as their diameters.

(3*b*) If the length, stretching force, and diameter of several strings are the same, their frequencies are inversely as the square roots of the densities of the materials of which the strings are composed.

These laws, being merely proportional, are true whatever units are used to measure the different quantities. In this they differ from the absolute formula $n = \frac{1}{2l}\sqrt{\frac{f}{m}}$, which cannot be deduced from these laws, as it contains more than they do.

* The absolute formula was first obtained by Brook Taylor in 1715, but the "laws" were discovered by Mersenne in 1636.

The "laws" are of course completely proved by the experiments which prove the absolute formula, but they may also be illustrated by less elaborate experiments which do not involve the measurement of an absolute frequency. These experiments are often said to "prove" the laws, but the proof is far from conclusive, and involves several assumptions.

172. Law 1. Relation between Frequency and Length of String.

Exp. 81.—Place the two movable bridges (A_1, A_2, Fig. 131) under the fixed wire of the monochord, and adjust their positions until the note given by the portion of the string between A_1 and A_2 is the octave of that given by the portion between A_1 and B_1. Now measure the lengths B_1A_1 and A_1A_2 with a millimetre scale. Show that $B_1A_1 = 2\,A_1A_2$. We know that if a note is an octave above another its frequency is double, therefore Law 1,

$$\text{viz. } n \propto \frac{1}{l} \text{ when } F \text{ and } m \text{ are constant,}$$

is proved. Check it by readjusting bridges to make one portion give a note a fifth above another. Show that the lengths are as 2 to 3. In this way, if we take in succession lengths $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$, and $\frac{1}{2}$ that of the full length of the wire, we get the succession of notes which constitute the diatonic scale on the key note as tonic.

Instead of using simple relations such as the octave and fifth, we could have verified the law by using tuning-forks of known frequency.

Having proved the law, we can use it to compare the frequencies of forks.

EXAMPLE.—An experiment like the above was performed to find the pitch of an unmarked fork. The lengths of string which gave notes in unison with a c″ fork and the unmarked fork were 31·8 cms. and 37·5 cms. respectively.

$$\frac{\text{Frequency of unmarked fork}}{\text{Frequency of c″ Fork}} = \frac{31{\cdot}8}{37{\cdot}5} = \frac{1}{1{\cdot}19}.$$

This is approximately $\frac{1}{1{\cdot}2}$ or $\frac{5}{6}$; ∴ the unmarked fork is an a fork.

It is easier to adjust to equality the notes given by two strings than the notes given by a string and a fork. Hence in the following experiments fix the monochord vertically and use the fixed string as a standard of pitch, and always adjust the weighted string to give a note in unison with it.

173. Law 2. Relation between Frequency and Stretching Force.

Exp. 82.—Hang in succession various loads on the loose wire, and each time adjust the length of the vibrating portion until it gives a note in unison with the fixed wire. Measure the lengths and show that $\frac{\text{stretching force}}{\text{square of vibrating length}}$, *i.e.* $\frac{f}{l^2}$, is constant.

Law 1 tells us that

$$n \propto \frac{1}{l} \text{ when } f \text{ and } m \text{ are constant.}$$

The above experiment proves that

$$\sqrt{f} \propto l \text{ when } n \text{ and } m \text{ are constant,}$$

and on combining these two relations we get

$$n \propto \frac{\sqrt{f}}{l} \text{ when } m \text{ is constant,}$$

i.e. when l and m are constant n is $\propto \sqrt{f}$, which is Law 2.

174. Law 3a. Relation between Frequency and Diameter of Wire.

Exp. 83.—Hang a certain load on a wire and find the length which gives the same note as the fixed wire. Replace the wire by another of different diameter but of the same material, and repeat the experiment. Measure the diameters of the wires with a screw gauge, and show from your experiment that the

lengths are inversely proportional to the diameters.

Law 1 tells us that

$$n \propto \frac{1}{l} \text{ when } d, f, \text{ and } D \text{ are constant.}$$

The above experiment proves that

$$d \propto \frac{1}{l} \text{ when } n, f, \text{ and } D \text{ are constant.}$$

The combination of these gives

$$n \propto \frac{1}{dl} \text{ when } f \text{ and } D \text{ are constant,}$$

i.e. when l, f, and D are constant, $n \propto \frac{1}{d}$, which is Law 3a.

Repeat the experiment with different stretching forces or at different pitches.

175. Law 3b. Relation between Frequency and Density of the Material of which the Wire is Composed.

Exp. 84.—Repeat Exp. 83, but in this experiment replace the first wire by another of the same diameter but of different material. To get the densities of the wires coil them up and weigh the coils in air and in water. Then density of material of wire

$$= \frac{\text{weight of coil in air}}{\text{loss of weight of coil in water}}.$$

Show from the sonometer experiments that the lengths are inversely proportional to the square roots of the densities.

Law 1 tells us that

$$n \propto \frac{1}{l} \text{ when } D, f, \text{ and } d \text{ are constant.}$$

The above experiment proves that

$$l \propto \frac{1}{\sqrt{D}} \text{ when } n, f, \text{ and } d \text{ are constant.}$$

The combination of these gives

$$n \propto \frac{1}{l\sqrt{D}} \text{ when } f \text{ and } d \text{ are constant,}$$

i.e. when l, f, and d are constant, $n \propto \frac{1}{\sqrt{D}}$, which is Law 3b.

Repeat the experiment with different stretching forces or at different pitches.

176. Law 3. Relation between Frequency and Mass of Unit Length of the Wire.

In case wires of the same diameter but of different material are not available, it is better to prove Law 3 as it stands.

Exp. 85.—Repeat Exp. 83, the second wire being now of different diameter and material to the first. Cut off equal lengths of the two wires and find their masses per unit length. Show from your results that the

lengths are inversely proportional to the square roots of the masses per unit length.

Law 1 tells us that

$$n \propto \frac{1}{l} \text{ when } m \text{ and } f \text{ are constant.}$$

The above experiment proves that

$$l \propto \frac{1}{\sqrt{m}} \text{ when } n \text{ and } f \text{ are constant.}$$

The combination of these gives

$$n \propto \frac{1}{l\sqrt{m}} \text{ when } f \text{ is constant,}$$

i.e. when l and f are constant $n \propto \frac{1}{\sqrt{m}}$, which is Law 3.

Repeat the experiment with different stretching forces or at different reference pitches.

177. The above experiments show that

$$n \propto \frac{1}{l}\sqrt{\frac{f}{m}}$$

or

$$n \propto \frac{1}{dl}\sqrt{\frac{f}{D}}.$$

The constant of the proportion can only be supplied if we know the absolute magnitudes of n, l, f, m or n, l, f, d, and D. See Art. 178.

EXAMPLE.—The following figures are quoted as an example of the experiments which may be performed to verify the above three laws. Two steel wires and one brass wire were used, and in each case the weighted wire was tuned to the fixed wire, which wire was in unison with a G fork.

Results:—	Stretching Force.	Length.
Fine steel wire (diameter ·48 mm.)	14 lbs.	42·1 cms.
	21 ,,	52·1 ,,
	28 ,,	60·5 ,,
Stout steel wire (diameter ·65 mm.)	14 ,,	31·2 ,,
	21 ,,	38·7 ,,
	28 ,,	44·9 ,,
Brass wire (diameter ·68 mm.)	14 ,,	26·6 ,,
	21 ,,	33·0 ,,

(1) To verify Law 2. Take the readings with the stout steel wire.

Stretching Force.	Length.	$\frac{\text{Stretching Force}}{\text{Square of Length}}$
14 lbs.	31·2 cms.	·0144
21 ,,	38·7 ,,	·0140
28 ,,	44·9 ,,	·0139

The figures in the last column are approximately constant: therefore Law 2 is verified.

(2) To verify Law 3a. Take the readings with the two steel wires.

Stretching Force.	Length of Fine Wire.	Length of Stout Wire.	Length of Fine Wire / Length of Stout Wire
14 lbs.	42·1 cms.	31·2 cms.	1·349
21 ,,	52·1 ,,	38·7 ,,	1·346
28 ,,	60·5 ,,	44·9 ,,	1·347
		Mean	1·347

Ratio of measured diameters of wires $= \frac{\cdot 65}{\cdot 48} = 1 \cdot 354$; therefore Law 3a is verified.

(3) To verify Law 3. The (equal) lengths of the wires were not measured, so that the table contains masses of equal length, not masses of unit length.

Wire.	Stretching Force.	Length.	Mass of Equal Length.	Product of Length and $\sqrt{}$ Mass of Given Length.
Steel (fine)	14 lbs.	42·1 cms.	·396 gm.	26·48
Brass	14 ,,	26·6 ,,	·990 ,,	26·46
Steel (fine)	21 lbs.	52·1 cms.	·396 gm.	32·77
Brass	21 ,,	53·0 ,,	·990 ,,	32·83

Therefore Law 3 is verified.

178. Absolute Measurement of Frequency by the Sonometer.—If the wire used is of such flexibility that the theoretical relation $n = \frac{1}{2l}\sqrt{\frac{f}{m}}$ is practically true, the sonometer may be used for the absolute measurement of the frequency of a note or vibrating body. The method, however, is hardly as accurate as the methods detailed in Ch. V.

Exp. 86.—Fix the sonometer nearly vertical, so that the hanging wire just does not rest against the lower fixed bridge. Put on a load of about 20 lbs. Place the movable bridge under the wire and adjust it till the note given out by the wire is in unison with the note whose frequency is to be determined. If the length is inconveniently long or short alter the stretching force. Carefully measure the length, l, of the vibrating portion and mark where it rests on the bridges. Cut the wire off at the marks and weigh the portion between. Let the mass be M, then since

$$n = \frac{1}{2l}\sqrt{\frac{f}{m}}, \text{ we get } n = \frac{1}{2}\sqrt{\frac{f}{Ml}}$$

from which n can be found by substituting for f, M and l.

Example.—Find the frequency of a G fork from the following readings. 33·2 cms. of steel wire stretched by 21 lbs. gave a note in unison with the fork. The mass of this length of wire was ·4258 gm. Then since 21 lbs. = 21 × 445000 dynes*

$$n = \frac{1}{2}\sqrt{\frac{21 \times 445000}{33{\cdot}2 \times {\cdot}4258}} = 406.$$

179. Sonometer Methods of Comparing Frequency.—A note of unknown frequency may be conveniently compared with that of a standard fork of higher or lower frequency by means of a sonometer wire. A length, l, of the wire is adjusted so that the note it emits is in unison with the fork; the length is then altered, either by shortening or lengthening it, until, for a length l', its note is in unison with the given note. The frequencies corresponding to the lengths l and l' of the wire are inversely proportional to these lengths, so that the ratio of the frequency of the source of the unknown note to the frequency of the fork is as l is to l'.

Another convenient method for determining the frequency of a given note by means of a sonometer wire is as follows. Find the length of the wire which gives a note in unison with the given note; let this length be denoted by l. Then find the length (longer) which gives a note which beats with the given note, say, four times a second; let

* The stretching force must always be expressed in absolute units—*dynes* or *poundals* (1 pound = 32 poundals, 1 gram = 981 dynes), ∴ 1 lb. = 453·6 grams = 453·6 × 981 = 445000 dynes.

this length be denoted by l'. Then, if n denote the frequency for the given note, we evidently have the relation

$$n/(n-4) = l'/l,$$

from which n can be calculated.

EXAMPLE.—Suppose that the note of which the frequency was to be determined was in unison with the note given by 40 cms. of the wire and made 12 beats in 3 seconds with that given by 42 cms. of the wire. Then

$$\frac{n}{n-4} = \frac{42}{40} = \frac{84}{80} \text{ or } n = 84.$$

It will be clear that this method is most suitable for notes of low pitch; when n is too great the values of l and l' are so nearly equal that very accurate measurement would be necessary to get the true value of their ratio. There is also some difficulty in making an accurate determination of the number of beats per second. These methods are, in fact, convenient for rapid approximate determination of frequency, but are not suitable for accurate work.

180. The Harmonics of a String.—We saw in Art. 165 that, if we had any stretched string, we could, by sending harmonic waves of length $2l$ along it from both ends, throw it into a condition of stationary undulation with nodes l apart, and that if the mass per unit length of this string is m, and the force with which it is stretched f, the frequency of its stationary undulation is $\frac{1}{2l}\sqrt{\frac{f}{m}}$. Also that, if we fix any two nodes xl apart (x being any whole number), the part of the string between them will continue to vibrate with the same frequency in x loops separated by nodes.

At each stationary instant the shape of such a string is a motionless harmonic curve having x single bends. So that, if we took a similar string, similarly stretched, fixed in it the form of a harmonic curve having x single bends of length l, and then let it go everywhere at once except the two ends (that this is impracticable does not matter for

our present purpose), it also would vibrate in the same way and with the same frequency. (This can also be proved directly, without considering the movement as the resultant of two fictitious travelling waves.) The more bends we made, the shorter l would be, and the greater the frequency, the frequency being proportional to the number of bends.

If, for instance, we could bend a string into the form of the continuous harmonic curve A, Fig. 132, and then let go, it would vibrate from that position to that of the dotted one and back, with a frequency $\frac{1}{2l}\sqrt{\frac{f}{m}}$, l being in this case the whole length of the string. We will call this frequency n. If we could bend the same string into the form of the continuous line B, Fig. 132, and let go, it would vibrate in

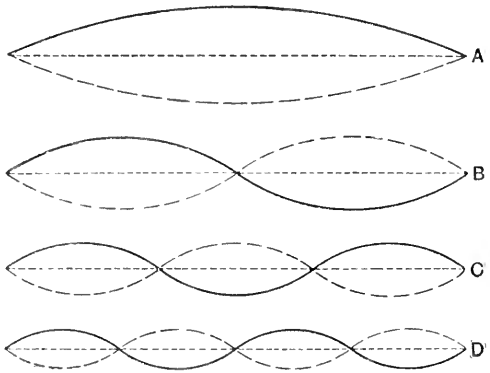


Fig. 132.

two loops, with a frequency $2n$, since l is now only half the length of the string. Similarly for three or more bends C and D, Fig. 132. In any case, if the string starts from a harmonic form, every point of it vibrates harmonically with the same frequency.

It should be noticed that when a string vibrates in segments, each segment vibrates in the mode already

described (Art. 166) for a string as a whole, also that the adjacent segments of a string vibrating in segments are always in opposite phases.

When the string vibrates as a whole (in one "segment") it is vibrating in its simplest mode, and the note which it emits is called the *fundamental note* of the string. When it vibrates in two segments, the frequency is twice that for the fundamental note, and the note emitted is the octave of the fundamental note. This note is usually called the *first harmonic*. Similarly, when the string vibrates in three segments, the frequency for the note produced is three times that for the fundamental note, and the pitch of this, the *second harmonic*, is a fifth above the first harmonic. It may, in fact, be stated as a general result that when a string vibrates in p segments it gives a note for which the frequency is p times that for the fundamental note, and this note is the $(p-1)$th harmonic.

The modes of vibration of a string in segments by which it gives the harmonics of its fundamental note are sometimes called the *harmonic modes* of vibration of the string.

Exp. 87.—*Study the harmonic modes of vibration of a string.* Stretch a fine string, six or eight feet long, between two stout nails.

(1) Pluck it between the fingers near the centre, then release it. The string forms one vibrating segment, the fixed ends being nodes. This is the fundamental mode of vibration. The wave-length is twice the length of the string. The note heard is of unvarying pitch, but of diminishing intensity. Hence the intensity is dependent on the amplitude, but the frequency is not, *i.e.* vibrations of different amplitudes are isochronous.

(2) Press the centre of the string with a folded strip of paper, pluck one side, let go, the string vibrates in two segments, the fixed ends and centre being nodes. The wave-length is equal to the length of the string.

(3) Press the paper strip at $\frac{1}{3}$ (length of string), pluck the shorter segment, let go; the string vibrates in three segments. Wave-length $= \frac{2}{3}$ (length of string).

(4) Press at $\frac{1}{4}$ (length of string) and get four vibrating segments and so on.

If paper-riders, Λ-shaped, are placed on the string when at rest, they, when the string is vibrating, are tossed off at the antinodes, but remain on at, or near, the nodes (Figs. 135, 136).

181. Compound Modes of Vibration of a String.—In general, when a string is bowed or excited in any way, it does not vibrate in any *one* of the modes described above, but in a mode compounded of some or all of the modes of which it is capable. In fact, it is extremely difficult to so excite a string as to set it in one definite mode of vibration. One particular mode may be *predominant*, but the vibration is in general the resultant of all the modes of which the string is capable under the conditions of its vibration. Hence, when a string is apparently sounding its fundamental note or a particular harmonic, this note is generally only the predominant component of a compound note made up of all the harmonics (including the fundamental note) possible under the conditions of vibration. It is the presence of these harmonics which determines the *quality* of the note.

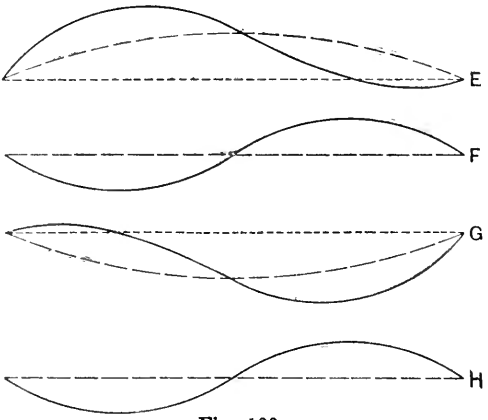


Fig. 133.

This compound mode of vibration of a stretched string will be more easily understood if it is realised that in the compound mode the motion of any point on the string is merely the resultant of all the motions proper to the component modes present. For example, suppose that we could bend the string into the form *E* (Fig. 133),

whose ordinates are the sums of the ordinates of A and B (Fig. 132). When we let it go, the movement of each point of the string will be the sum (in the sense explained in Art. 11) of the movement which it would have executed if it had been bent into the form A and that which it would have executed if it had been bent into the form B. In other words, the string vibrates in two loops from side to side of an imaginary line which vibrates like A, in the same way as the string B vibrates in two loops from side to side of a straight line. Fig. 133 E, F, G, H shows four stages in this movement at intervals of one quarter of the period of the fundamental, the broken line being the imaginary line which moves as the string A did. The movements of different points of the string are not similar, nor, usually, harmonic, but the motion of each point is the sum of two harmonic vibrations of frequencies n and $2n$.

The same would be true if we began by bending the string into a form which was the sum of any number of harmonic curves which, like A, B, C, and D, have an exact number of single bends in the length of the string. Now, when we pull a point of the string to one side, the form the string assumes is that of two straight lines meeting at an obtuse angle at that point (Fig. 134). This is not a form which looks likely to be the sum of a number of harmonic curves, but Fourier's Theorem, or an easy extension of it, shows that any of the angular forms which can be produced by pulling a point of the string to one side is a form which might be made by adding the ordinates of harmonic curves of which the length of the string contains an exact number of single bends. It shows further that, of the infinite number of such curves, all are required except those which cut their axis line at the point at which the string was pulled aside. Those which cut the axis very near this point are required only of small amplitude.

Suppose, for instance, we pull the string CD (Fig. 134) to one side at a point E, $\frac{1}{5}$ of the way from D to C. The form CED is one which might be produced by adding the ordinates of harmonic curves which have 1, 2, 3, 4, 6, 7, 8, 9, 11, etc., half-waves in the length CD, those which have any multiple of 5 half-waves being omitted, because they would cross the

axis under the point E. So that, if n is the frequency with which the string would vibrate if bent into the form of a single harmonic half-wave, the motion of each point of the string, when it has been pulled into the form CED and

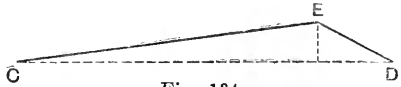


Fig. 134.

released, is the sum of harmonic movements whose frequencies are n, $2n$, $3n$, $4n$, $6n$, etc. Vibrations of these frequencies are the harmonic components of the real movement of the string.

Similarly, if we pull a string aside at a point $\frac{a}{b}$ of its length from one end, $\frac{a}{b}$ being a fraction in its lowest terms, any harmonic curve which has xb single bends in the length of the string (x being any whole number) cuts the axis at the point where the string was pulled aside. The harmonic components of the vibration produced when the string is let go are vibrations whose frequencies are all multiples of n which are not multiples of bn, n having the same meaning as above.

If we strike a string, as in the piano, the harmonic components of the vibration produced have the same frequencies as if we had pulled it to one side at the same point, but their relative amplitudes are different. A hard hammer, which touches only a very short piece of the string, and remains in contact with it a very short time, produces a vibration whose high-frequency harmonics are intense, and conversely.

Next, suppose we touch the vibrating string at some other point with a light object such as a feather or camel's-hair brush. This stops all modes of vibration of the string except those which have a node at the point touched, and the subsequent motion of the string is the sum of those harmonic components of its previous movement which have a node at this point. If, for instance, after plucking

the string CD at $\frac{1}{5}$ of its length from one end we touch it at a point $\frac{1}{3}$ of its length from one end, all harmonic vibrations become impossible except in 3, 6, 9, 12, 15, etc., loops. Of these its motion included all except that in 15 loops, so that the subsequent motion is the sum of the harmonic vibrations which it could execute in 3, 6, 9, 12, 18, etc., loops, all multiples of 3 which are not multiples of 5 being included. The harmonic components of this vibration have of course frequencies $3n$, $6n$, $9n$, $12n$, $18n$, etc.; n having the same meaning as before

Similarly, if a string which has been plucked or struck at a point $\frac{a}{b}$ of its length from one end is then touched at a point $\frac{c}{d}$ from one end ($\frac{a}{b}$ and $\frac{c}{d}$ being fractions in their lowest terms), the harmonic components of the subsequent vibration have frequencies which include all the multiples of n which are multiples of dn but are not multiples of bn.

The ends of a stretched string must be fastened to some solid body, for instance to pegs or screws in a board, and when the string vibrates its pull on the pegs varies and the board vibrates also, and so produces air waves. The movements of the air so produced correspond nearly to those of the board, and the movement of the board, though it does not correspond closely to that of the string, is the sum of the movements which the harmonic components of the vibration of the string would produce separately, so that the movement of the board has a harmonic component corresponding in frequency to each harmonic component of the movement of the string. (The relative amplitudes of the components may be very different for the board and string, since some components may cause resonant vibration of the board, and not others. There may also be components of the vibration of the board which are not components of that of the string.)

The string itself also produces waves in the air, but for the reason explained in Art. 25 they are insignificant in intensity compared with those from the larger moving surface of the board, and a vibrating string would hardly

be audible if it could be fastened to absolutely immovable points.

The character as well as the frequency of the non-harmonic motion of a point on a vibrating string may be determined by the use of a vibration microscope (Art. 185).

182. Experimental Study of the Harmonic and Compound Modes of Vibration of a Stretched String.—Use the stretched wire of the sonometer for the following experiments :—

Exp. 88.—Observe the different qualities of the notes given out by the string, first, when plucked at the centre and, secondly, when plucked near the edges. Observe also the effects of different methods of sounding the string. Try plucking gently between the fingers, plucking sharply with the thumb, and plucking with the thumb-nail (or a piece of metal). The difference in quality is due to the difference in the number, frequency, and amplitudes of the harmonic produced.

In the next experiment we shall use the paper scale on the base of the instrument. The graduations are described in Art. 167.

Exp. 89.—(*a*) Bow the string at the centre. It vibrates in one segment (Fig. 129).

(*b*) Now bow at 30 (*i.e.* at a distance of a quarter of a length of the wire from one end). The predominant vibration will be in two

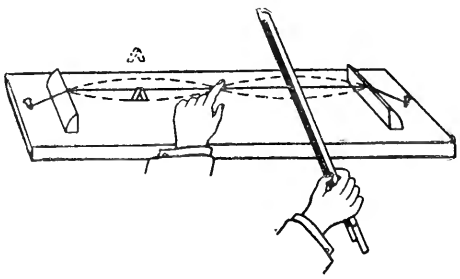


Fig. 135.

segments or loops. While bowing, touch the string lightly with a feather or a twist of blotting-paper at 60, as shown in Fig. 135. The vibration in two segments can be plainly seen, and it is evident to the ear that the octave is developed.

Even when the amplitude of vibration is very small the existence of an antinode at the middle of the unbowed segment can be demonstrated by placing a small V-shaped paper rider at this point on the string before bowing. When the string is bowed the vibration set up "throws" the rider, as shown in Fig. 135. It will be found that even if the string is held firmly at the middle point both segments are set in vibration by bowing one of them at its middle point. This shows that there is transmission of energy through the node from one segment to the other. It is also an illustration of the principle of resonance, for, since the period of vibration for each segment is the same, the impulses transmitted through the node from the segment are timed exactly as they should be to set the other segment in vibration.

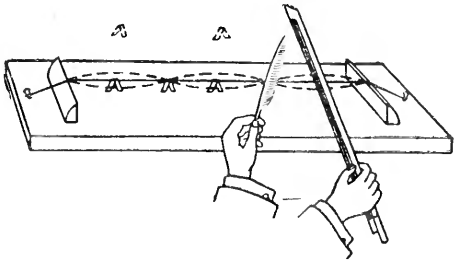


Fig. 136.

(*c*) Bow the string at 20 or 60 and damp at 40 or 80. The string vibrates as in Fig. 136, the fifth above the octave being produced.

(*d*) Bow at 15 or 45 and damp at 30 or 60, the double octave is produced.

Proceeding in this way, if a fine steel wire is used, nearly a dozen harmonics can be observed.

The fact that the usual mode of vibration of a string is the resultant of all the simple modes possible under the conditions of its vibration can be readily detected by the musically educated ear. Thus, if the compound note given out by a vibrating string be carefully listened to, a competent observer can detect not only the fundamental tone, but also the first harmonic an octave above the fundamental, the second harmonic a fifth above the first, the third harmonic an octave above the second, and possibly some higher harmonics. The presence of these harmonics

in the compound tone indicates the presence of the corresponding mode of vibration as a component of the compound vibration of the string. These harmonics are more easily detected by the ordinary observer by means of Helmholtz *resonators* (Fig. 73).

Exp. 90.—*Illustrate resonance on a piano.* (1) Strike the note c'' so as to catch the pitch to remember it. Then, keeping the digital of c'' down, strike that of c' sharply and then raise it. This allows the piano mechanism to damp it. The wire c'' will then be heard vibrating, being set in resonance by the octave present in c'. If c' is repeatedly struck half a dozen times before being damped out the resonance is louder. Repeat the experiment, keeping down g'', c''', e''', etc., instead of c'', and notice with which resonance occurs.

(2) Press down the pedal of the piano so as to free the wires. Sing a note to the piano. The note is taken up by the strings and echoed back. The wires analyse the note sung, each string taking up the component of frequency equal to its own, so that the quality of the note is well reproduced.

183. Resonant Vibrations of Strings. Melde's Experiment.—The harmonic components of the air-waves, and therefore the quality of the sound they produce, thus depend on the point at which the string is pulled aside or struck, but we cannot make the string vibrate harmonically or produce harmonic waves by pulling or striking it. But

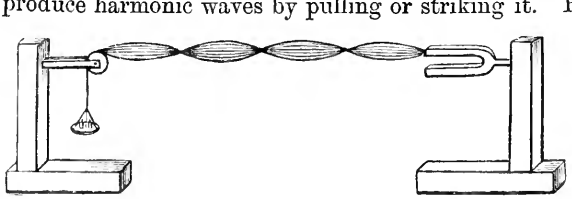


Fig. 137.

it may be set in *resonant* vibration by harmonic impulses agreeing in frequency with any of its modes of vibration. For instance, if we attach a light silk cord or very thin wire to the extremity of one prong of a large tuning-fork, as shown in Fig. 137, it will be found that when one of certain definite loads is placed in the scale-pan and the fork excited by drawing a violin bow across one prong, the

thread or wire is thrown into strong stationary undulation, the number of loops depending on the combined weight of the pan and "weight." This occurs only if the combined weight or stretching force is such that one of the modes of free stationary undulation of the thread is nearly of the *same* frequency as the fork.

Let n = frequency of the fork ;

the frequency of the $(p - 1)$ harmonic of the string is

$$\frac{p}{2l}\sqrt{\frac{f}{m}},$$

therefore if $$n = \frac{p}{2l}\sqrt{\frac{f}{m}}$$

the string will be set in vibration with p loops.

Since n, l, m are constant, it follows that

$$p\sqrt{f} = \text{a constant}$$

or $$f p^2 = \text{a constant},$$

i.e. the weights which are required to make the same string vibrate in 1, 2, 3, etc., loops are inversely proportional to the squares of these numbers.

EXAMPLE.—The following numbers obtained in a lecture experiment with a low fork and a string between one and two metres long support the above formula :—

p	f expressed in gram wts.	fp^2
1	680	680
2	170	680
3	75·5	679·5
4	42·5	680
5	27·2	680

The fork may also be placed so that it vibrates in the direction of the length of the thread, instead of at right angles to that direction, so that the fork pulls the thread instead of shaking it up and down. In this case the thread is tightest when the prong is the end of its swing away from the pulley and loosest when the prong is nearest the pulley, so that the thread makes a quarter of a vibration while the prong makes half a vibration, *i.e.* the frequency

of the string is half that of the fork. In this case, therefore, there is resonant vibration when the stretching force is such that one of the modes of vibration of the thread has half the frequency of the fork, *i.e.*, with the same notation as above,

$$\frac{n}{2} = \frac{p}{2l}\sqrt{\frac{f}{m}}.$$

As before, fp^2 is a constant, but the stretching force now required to give any particular number of loops is only one quarter of that required in the first case.

It will be noticed that in the first case the fork end of the thread has a slight transverse motion, so that the effective length of the string is slightly greater than the actual length, and that in the second case the fork end of the string has no transverse motion, but a small longitudinal motion, determined by the amplitude of the vibration of the fork.

Melde's experiments may be considered in another and rather instructive manner. They are really simple illustrations of the principle explained in Art. 165 and illustrated by Fig. 128.

The vibration of the prong of the fork acts as a source of transverse waves, which are transmitted along the stretched string to the pulley, and there reflected back again along the string. Interference takes place between these trains of direct and reflected waves, and, under proper conditions, the string is thereby set in *stationary vibration.*

In the second arrangement above the frequency of the prong *as a source of transverse waves along the string* is n', where $2n'$ is its actual frequency of vibration. The velocity of the waves along the string is not constant, being given by $V = \sqrt{f/m}$. Now, if λ denote the wave-length of the transverse waves in any case, we have $v = n'\lambda$, or $\lambda = v/n'$. Hence, if the stretching force is so adjusted that V has a value such that $\lambda/2$ is equal to the length of the string or to some aliquot part of it, the interference between the direct and reflected travelling waves sets up stationary vibration of the string as a whole or in a definite number of segments of length $\lambda/2$. That is, if $\lambda/2$ or $v/2n' = l/p$, where l denotes the length of the string and p is an integer, then the string takes up stationary vibration in p segments.

In the arrangement of Fig. 137 the wave-length of the travelling waves will evidently be $v/2n'$, and if $v/4n' = l/p$ as above, then the interference between the direct and travelling waves will set the string in stationary vibration in p segments.

It should be noticed that these experiments are exactly analogous to the much more familiar experiment, Exp. 63. In this experiment the adjustment to obtain stationary vibration is made by adjusting the length of the air column. The adjustment in Melde's experiments could also be made by adjusting the length of the string under a constant tension.

184. Successive Configurations of a Vibrating String.— If a string is bent initially into the form of a sine curve its successive positions are sine curves, the ordinates always decreasing or increasing in the same ratio for the same interval of time. If, however, the string is plucked into a form like *CED*, Fig. 138 (1), the successive positions of the string are not in any way similar to (1). They can, however, readily be found from (1) by a method due to Thomas Young and already foreshadowed in Art. 165. Young deduced the shape of the string at any instant by the superposition of the waves of the two equal sets of oppositely travelling wave-systems, of which we have shown in Art. 165 that the stationary system is due.

Thus, if the string is initially bent into the state given by *ECD*, Fig. 138 (1), the shape of the top halves of the travelling waves is obtained by drawing a perpendicular from *E* on *CD*, bisecting this perpendicular at *H* and joining *CH*, *HD*. In (1) the top halves of the waves of the travelling systems are coincident, and the ordinate of the string at any point of *CD* equal to twice the ordinate of the curve *CHD* at the same point. Successive configurations of the string at future times are obtained by shifting the travelling waves equal distances to the right and left and compounding their ordinates. Thus, when one-twelfth of a period has elapsed since the initial position shown in (1), one wave-system has travelled a distance to the right equal to one-sixth of *CD*, the other system has travelled an equal distance to the left. The positions of

the primary waves are shown by the dotted curves of (2). The sum of these curves gives the full curve, which is the configuration of the string at this instant. After another twelfth of the period the primary waves have moved

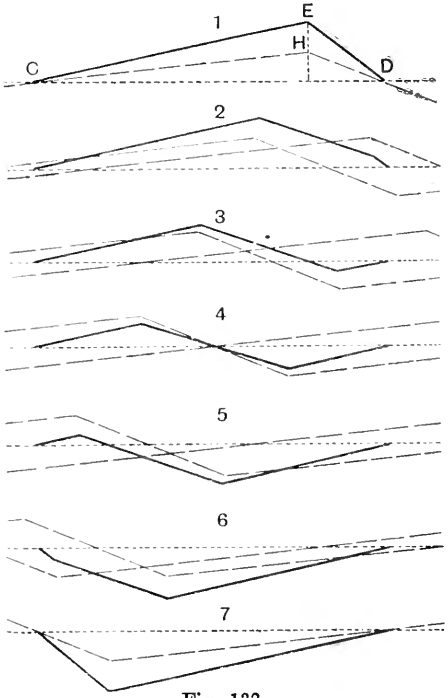


Fig. 138.

another sixth of *CD* to the right and left, and their sum shown in the full line of (3) gives the configuration of the string at this instant.

The positions of the string at intervals of $\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$, $\frac{6}{12}$ period after (1) are shown in (4), (5), (6), and (7), and it is easily seen that the string reverts to (1) by retracing the figures (6), (5), (4), (3), (2) in turn.

The inclination of the string at the supports C and D alternates between two constant angles, namely, the angle ECD and the angle EDC. From this it follows that the pressure on the supports alternates between two constant values, *i.e.* the pressure remains high for a certain time, then suddenly changes to low and remains so for another certain time, and then suddenly reverts to the high pressure. Now very little of the sound in the air, due to a stretched string, comes directly from the string itself; most of it comes from the vibrations of the supports produced by the vibrating string. It therefore follows that the air-waves by which we hear the sound of the string are produced by the alternation of these high and low pressures on the supports.

185. Experimental Study of the Motion of a Point on a Vibrating String.—In order to study the motion of a vibrating point on a string (and incidentally any other vibrating point), Helmholtz designed the *vibration microscope* (Fig. 139). It consists of a large tuning-fork G fixed

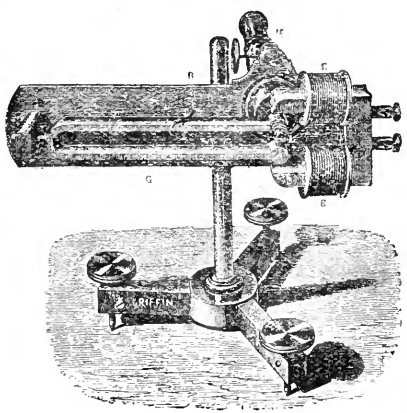


Fig. 139.

to a support, which also carries an electromagnet E, which actuates the fork (Art. 62), and a microscope M mounted with its axis perpendicular to the plane of vibration of the fork and its object glass L carried by one prong of the fork. The string under investigation is placed vertically

and a bright point on it is viewed by the microscope. If the fork is vibrating and the string is at rest, motion of the bright point yields a vertical line such as AB (Fig. 140).

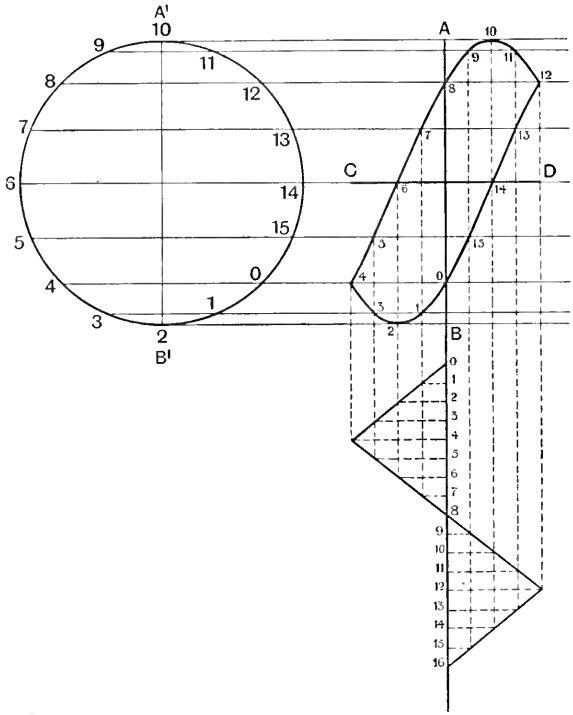


Fig. 140.

If the fork is at rest and the string is vibrating, the bright point yields a horizontal line such as CD. The lengths of AB and CD depend of course on the extent of the amplitude of vibration.

When both fork and string are in vibration the bright point yields a curve (Fig. 140) whose shape depends on the ratio between the two frequencies and on the phase difference. If the "interval" between the two frequencies is simple the curve repeats itself over and over again. The curve is reminiscent of Lissajous' figures, but whereas in the case of these figures both motions are simple harmonic and the figures are used only to determine the "interval" between the two frequencies, in the present case only one motion (that of the fork) is simple harmonic, and the curve is used to find the details of the motion of the point on the string, the only use of the fork being to provide a time scale.

To show how this is done let us take the simple case when the tuning-fork and the string are in exact unison. The curve in Fig. 140 represents a possible form of the figure obtained. Draw the vertical line AB bisecting the figure, and draw horizontal tangents to the figure cutting this line in AB. Some distance to the left draw $A'B'$ parallel to AB and between the same tangents. On $A'B'$ describe a circle. From the point O, where AB cuts the curve, draw a horizontal to cut the circle in O and, beginning at O, divide the circle into any number of equal parts, say 16. From the points of division draw horizontal lines cutting the curve in the points 0, 1, 2, 3... etc.

Since the motion of the fork is harmonic we know that the pieces 01, 12, 23... etc., of the curve are described in equal times. On a prolongation of AB mark off 16 equally spaced points 0, 1, 2, 3...etc., and from these points draw horizontals to meet verticals from the points 0, 1, 2...15 of the curve. The line through the intersections gives the time-displacement curve of the point of the string, from which the speed of this point is easily inferred. It is plain that in the example taken both the forward and backward speeds of the point were constant and equal. This only happens if the point is the middle of the string. If any other point be viewed, the forward and backward lines in the time-displacement diagram are unequally sloped, showing that the forward and backward speeds are unequal.

186. Correction for Rigidity in a Transversely Vibrating String.— If the string is perfectly flexible the frequency of vibration of a string is given by

$$n_p = \frac{p}{2l}\sqrt{\frac{f}{m}},$$

where $p = 1$ for the fundamental note and 2, 3, 4, etc., for the successive harmonics. If the string is not perfectly flexible its rigidity has two effects. One effect is to cause internal friction in the wire, which dissipates mechanical energy into heat, and so accelerates the dying away of the vibration; this, however, has no effect on the frequency of vibration. The other effect is one which increases the force, bringing back the wire to its central position. It thus causes an increase in the frequency. Considering this second effect alone, Donkin* has worked out that the frequency of the fundamental tone and overtones of a circular wire, not perfectly flexible, stretched over bridges is given by

$$n_p = \frac{p}{2l}\sqrt{\left\{\frac{f}{m}\right\}}\left\{1 + \frac{p^2\pi^3 r^4 Y}{8l^2 f}\right\},$$

where r = radius of cross section of the wire,

and Y = Young's modulus of the material of the wire; †

thus the frequencies of the fundamental and the first two overtones are respectively

$$n_1 = \frac{1}{2l}\sqrt{\left\{\frac{f}{m}\right\}}\left\{1 + \frac{\pi^3 r^4 Y}{8l^2 f}\right\},$$

$$n_2 = \frac{2}{2l}\sqrt{\left\{\frac{f}{m}\right\}}\left\{1 + \frac{4\pi^3 r^4 Y}{8l^2 f}\right\},$$

$$n_3 = \frac{3}{2l}\sqrt{\left\{\frac{f}{m}\right\}}\left\{1 + \frac{9\pi^3 r^4 Y}{8l^2 f}\right\}.$$

The ratios between these are

$$\frac{n_2}{n_1} = 2\,\frac{1 + \frac{4\pi^3 r^4 Y}{8l^2 f}}{1 + \frac{\pi^3 r^4 Y}{8l^2 f}} \quad \text{and} \quad \frac{n_3}{n_1} = 3\,\frac{1 + \frac{9\pi^3 r^4 Y}{8l^2 f}}{1 + \frac{\pi^2 r^4 Y}{8l^2 f}},$$

so that $\frac{n_2}{n_1} > 2$ and $\frac{n_3}{n_1} > 3$, *i.e.* each tone is sharp and the fundamental and its overtones are not harmonically related. The

---

* See Donkin's *Acoustics*.

† For meaning of Young's modulus see Wagstaff's *Properties of Matter*, Art. 109.

effect on the fundamental is the same as if the wire were perfectly flexible and its stretching force were increased by $\frac{\pi^3 r^4 Y}{4l^2}$ units of force.

Thus, taking the case of the fine steel wire in the Example in Art. 177, when the length was 42·1 cms. and the stretching force 14 lbs.,

$$Y \text{ or steel} = 2 \times 10^{12} \text{ dynes per sq. cm.};$$

$\therefore$ for this wire

$$\frac{\pi^3 r^4 Y}{4l^2} = \frac{\pi^3 \times (\cdot 024)^4 \times 2 \times 10^{12}}{4 \times (42)^2} = 146 \text{ dynes}$$

$$= \tfrac{1}{3000} \text{ lb.-wt.}$$

i.e. the note given is the same as would be given by a perfectly flexible wire whose stretching force was $\frac{1}{3000}$ lb. more than the actual stretching force. From this result the student will see that the result is practically negligible in the strings employed in music under the stretching forces usually exerted.

187. Use of Strings in Musical Instruments.—The production of musical notes by the vibration of strings finds many important applications in musical instruments. The pianoforte, all stringed instruments of the type of the violin, the harp, the guitar, and other similar instruments are examples of instruments on which musical notes are produced by the vibration of strings set in vibration by different methods.

A slight examination of any one of these instruments will show that the laws of vibration of strings are consistently applied in its construction.

In the pianoforte the "strings" are thin steel wire set in vibration by the stroke of soft leather-covered hammers actuated by the keys. The wires for the high notes are short and thin, and stretched very tightly. Those for the low notes are longer and thicker, but as the stretching force cannot be made very small if the wire is to be kept taut, the mass for unit length has to be further increased for the lowest notes by a covering of fine wire wound spirally round the stretched wire. This gives the necessary increase of the mass of the string without unduly decreasing the flexibility or requiring the stretching force to be too small for satisfactory action. The hammers too are adjusted to strike the wires at points which render the production of certain undesirable harmonics impossible.

For example, Helmholtz has shown that the most pleasing effect is got from a pianoforte wire by striking at a point from one-seventh to one-ninth of its length from the end. Points within this limit are not required as nodes by any of the lower overtones which accord well with the fundamental. On the other hand, the agitation of this part of the wire excludes the first dissonant overtones, viz. the 6th and the 8th.

Also the higher partials of a wire are more easily produced when the wire is struck with a hard body than with a soft body, hence the reason of covering the hammers of a piano with leather.

In the violin the strings are of the same length, but the mass per unit length differs with the pitch of the string, and in tuning the adjustment is made by adjusting the stretching force. Also, in playing the instrument, the note given by any string is varied by adjusting its length, and, when necessary, harmonics are produced by bowing and damping at the right points as described above.

The action of the bow, which is resined, is the same as that of the resined cloth in Art. 154; though it produces resonant vibration, its impulses depend on the vibration already existing, and the vibration it produces has the frequency of the free vibration of the string.

A beautiful effect is produced on the sonometer by bowing steadily near one end and drawing a feather lightly along the wire. The sweetness and variety of tones reminds one of the music of the *Aeolian harp*. This instrument consists of a number of different strings stretched upon a sound-box. It is placed on the sill of a window which is slightly opened so that a current of air flows over the strings and causes them to oscillate sometimes as a whole, sometimes in segments, but oftener in both ways at the same time.

188. Transverse Vibration of Rods.—A wave of transverse displacement may be sent along a rod of any elastic material in the same way as along a stretched cord, but the rod need not be stretched, and it is the elasticity of the material, not the tension, which restores the successive portions of the rod to their original relative positions. The velocity of such a transverse wave, if harmonic, is

independent of the thickness b of the rod in the direction perpendicular to the displacement; proportional directly to the thickness t in the direction of the displacement, and to the square root of Young's modulus Y for the substance; proportional inversely to the square root of the density D and to the length of the wave λ.

i.e. $$V \propto \frac{t}{\lambda}\sqrt{\frac{Y}{D}}.$$

A non-harmonic wave has no definite velocity, but the movement is the sum of the movements which would be due to the harmonic components, each travelling with the velocity proper to its wave-length. This dependence of velocity on wave-length makes the laws of transverse vibration of elastic rods much more complex than those of strings.

In elastic rods stationary transverse undulation is produced, as in strings, by the interference of wave-systems, which are travelling in opposite directions, and which have been reflected from the ends of the rod. The modes of stationary transverse undulation possible in a bar depend on what points, if any, are fixed, and whether the bar at these points is fixed both in direction and in position, as when a rod is held in a vice, or fixed only in position, as when it merely rests on fixed supports, as in the toy called the *harmonicon*. In any of these cases there may be nodes and antinodes, and an end which is fixed in position but not in direction is always a node. An end fixed in direction is not a true node, since it is not a point of maximum change of slope (Art. 165); nor is a free end a true antinode, since it is not a place where the rod moves parallel to itself. The point of no transverse motion nearest to a free end is also not a true node, since the change of slope there is not as great as at the free end itself.

All the true nodes are equidistant. The simpler modes of harmonic vibration for a bar entirely free or supported at two nodes—called a **free-free** bar—are shown in Fig. 141, and those for a bar fixed in direction at one end and free at the other—called a **clamped-free** bar—are shown in Fig. 142.

The frequency of the vibration, being the reciprocal of the time required for two travelling waves to pass each other whose wave-length λ is equal to twice the distance d between two true nodes, is proportional to $\frac{V}{d}$,

i.e. to $$\frac{t}{d^2}\sqrt{\frac{Y}{D}}.$$

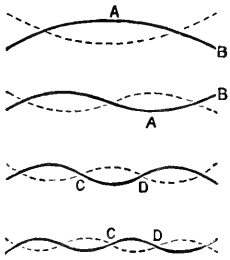


Fig. 141.

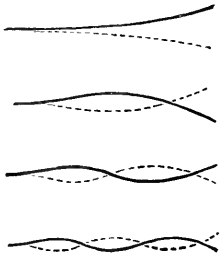


Fig. 142.

From this it follows that—

(1) If two rods vibrate in the same mode, d is the same fraction of the length l of the rod in each case, therefore their frequencies are proportional to $\frac{t}{l^2}\sqrt{\frac{Y}{D}}$.

(2) If the same rod vibrates in two different modes, their frequencies are approximately inversely proportional to the squares of the length of pieces which vibrate in the same way; for instance, the pieces AB or the pieces CD in Fig. 141.

189. The theory of the lateral vibrations of bars is much beyond the scope of the present book. The following results, mainly due to Lord Rayleigh, are worth noting.*

(*a*) *Free-free bar.* The frequency of vibration of such a bar is given by the formula

$$n = \frac{Uk}{2\pi l^2} \cdot m^2,$$

* A student desiring further knowledge of this part of the subject should consult Barton's *Text-book of Sound* (Macmillan), or Rayleigh's *Theory of Sound* (Macmillan).

where n = frequency of vibration,

$U = \sqrt{\frac{Y}{D}}$ = the velocity of a longitudinal wave in the rod,

l = length of the rod,

k = radius of gyration of the cross section of the rod about an axis perpendicular to the plane of bending,*

m = an abstract number, being $3{\cdot}01\,\frac{\pi}{2}$ for the fundamental tone

and $\frac{5\pi}{2}$, $\frac{7\pi}{2}$, $\frac{9\pi}{2}$, ... for the successive overtones.

The frequencies of the fundamental and successive overtones are therefore as $(3{\cdot}011)^2$, $(5)^2$, $(7)^2$, $(9)^2$, etc.

For two rods of the same material vibrating in the same mode

$$n \propto \frac{k}{l^2}.$$

For a rod of *rectangular* section

$$k^2 = \frac{l^2}{12}, \quad \therefore\ k = \frac{t}{2\sqrt{3}};$$

therefore for the fundamental note

$$n = \sqrt{\frac{Y}{D}} \cdot \frac{t}{2\sqrt{3}} \cdot \frac{(3{\cdot}01)\frac{\pi}{2}}{2\pi l^2}.$$

If the material is steel this reduces to

$$n = 538000\,\frac{t}{l^2}.$$

For a rod of *circular* section

$$k^2 = \frac{r^2}{2}; \quad \therefore\ k = \frac{r}{\sqrt{2}}.$$

$\therefore$ the frequency of the fundamental note

$$= \sqrt{\frac{Y}{D}} \cdot \frac{r}{\sqrt{2}} \cdot \frac{3{\cdot}01\,\frac{\pi}{2}}{2\pi l^2}.$$

---

* See Wagstaff's *Properties of Matter* (Art. 76) for values of k for different cross sections.

The particulars of the first few tones—calculated by Seebeck and Donkin—are given in the following table. The term node here includes both true and false nodes.

Number of Tone.	Number of Nodes.	Distances of Nodes from One End expressed in Fractions of the Length of the Bar.	Relative Frequencies.	Frequencies relative to Fundamental.
1	2	·224, ·776	$3{\cdot}01^2$	1
2	3	·132, ·5, ·868	5^2	2·76
3	4	·094, ·356, ·644, ·906	7^2	5·40
4	5	·073, ·277, ·5, ·723, ·927	9^2	8·93

(*b*) *Clamped-free bar.* Here

$$n = \frac{Uk}{2\pi l^2} \cdot m^2,$$

where the successive values of m are

$$1{\cdot}194\,\frac{\pi}{2},\ 2{\cdot}99\,\frac{\pi}{2},\ 5\,\frac{\pi}{2},\ 7\,\frac{\pi}{2}, \ldots\ldots$$

The frequency the fundamental tone of a steel rod of rectangular cross section reduces to

$$n = 84{,}600\,\frac{t}{l^2},$$

while for a steel rod of circular cross section it reduces to

$$n = 146{,}600\,\frac{r}{l^2}.$$

The particulars of the first few tones—calculated by Seebeck and Donkin—are given in the following table. The term node here includes both true and false nodes (but excludes the clamped end).

Number of Tone.	Number of Nodes.	Distances of Nodes from Free End expressed as Fractions of the Length of the Bar.	Relative Frequencies.	Frequencies relative to Fundamental.
1	0		$1{\cdot}19^2$	1
2	1	·226	$2{\cdot}99^2$	6·2
3	2	·132, ·500	5^2	17·6
4	3	·094, ·356, ·644	7^2	34·4

190. By making the assumptions that the distance from an end fixed in direction to the nearest true node, or from a free end to the nearest true node, is $1\frac{1}{4}$ times the distance between two true nodes, and that the distance from a free end to the nearest point of no transverse motion is a $\frac{1}{4}$ of the distance between two nodes, we can calculate approximately the relative frequencies of transverse vibration of rods.

Thus, taking the clamped-free bar, Fig. 142, let m be the total number of nodes (this excludes the fixed end, thus $m = 3$ in the lowest figure). If x be the average distance between the nodes, $\frac{5x}{4}$ = distance from the clamp to first node, and $\frac{x}{4}$ = distance from last node to free end.

$$\therefore\ l = \frac{5x}{4} + (m-1)\,x + \frac{x}{4} = (m + \tfrac{1}{2})\,x.$$

Omitting the first figure of Fig. 142, $m = 1, 2, 3$, respectively in the others, therefore $x = \frac{2l}{3}, \frac{2l}{5}, \frac{2l}{7}$, and therefore the frequencies are as $3^2 : 5^2 : 7^2$.

In the free-free bar, Fig. 141,

$$l = \frac{x}{4} + (m-1)\,x + \frac{x}{4} = (m - \tfrac{1}{2})\,x.$$

For the modes of vibration given in the figure $m = 2, 3, 4, 5$, hence $x = \frac{2l}{3}, \frac{2l}{5}, \frac{2l}{7}, \frac{2l}{9}$, and therefore the frequencies are $3^2 : 5^2 : 7^2 : 9^2$.

Exp. 91.—Make a set of rods to give the notes of the musical scale.

(1) Rectangular rods. Let the rods be of same material, length, and width, then since the frequency is proportional to the thickness, the thickness must be in the ratios of the numbers 24, 27, 30, 32, 36, 40, 45, 48. The rods may be cut from common deal. Suitable dimensions are: length 21 cms., width 3 cms., thicknesses ·70, ·79, ·88, ·93, 1·05, 1·17, 1·31, 1·40 cms.

(2) Cylindrical rods. Here it will be more convenient to take rods of same material and cross section and of different lengths. Since the frequency is inversely proportional to the square of the length, the squares of the lengths must be in the ratios of the numbers

$$\tfrac{1}{24},\ \tfrac{1}{27},\ \tfrac{1}{30},\ \tfrac{1}{32},\ \tfrac{1}{36},\ \tfrac{1}{40},\ \tfrac{1}{45},\ \tfrac{1}{48}.$$

Glass rod may be used. Suitable dimensions are: diameter ·7 cm., lengths 25·0, 23·6, 22·4, 21·7, 20·4, 19·4, 18·2, 17·7 cms.

Take either of these sets of rods and drop them in succession on a stone or hard wood block. The notes of the scale are plainly given. Now try a simple tune.

Exp. 92.—Clamp a steel knitting-needle in a vice. Make it sound its fundamental and adjust the length of the vibrating portion until the note given is the same as that of a fork of known pitch. Measure the diameter with a screw gauge and test the relation

$$n = 147{,}000 \frac{r}{l^2}.$$

191. The Tuning-fork.—If a rod is bent into a U-shape and made to vibrate with an even number of nodes, two of the nodes are close together at the bend, and at the antinode between them there is not only no change of direction, but very little transverse motion, as shown in Fig. 143. If the vibrating rod is held by this point, it shakes the holder very slightly, and so loses very little energy, except by communicating it to the ear; it thus vibrates a very long time.

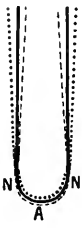


Fig. 143.

The *tuning-fork* is an application of this principle. It may be made to vibrate by striking one prong on an inelastic substance such as lead, or by drawing a violin bow across one prong, or (if the prongs are nearer together at the points than at the bend) by drawing between the points of the prongs a rod of wood just too large to pass without bending them. The relative frequencies of the different modes of vibration should in principle be calculated from those of free-free bars, but owing to the great thickening at *A* due to the attachment of the shank the frequencies of the different modes do not differ much from those of a straight bar fixed at one end (Fig. 142). The slowest mode (that shown in Fig. 143) lasts much longer than the others, so that the vibrations of a tuning-fork which has been vibrating for some time are always in this mode, and are (practically) perfectly harmonic.

Exp. 93.—*Obtain the value of the Young's modulus of steel from a tuning-fork.*

The formula above for the frequency of the fundamental note of a clamped-free bar is

$$n = \frac{Uk}{2\pi l^2} \times \left(1{\cdot}19 \frac{\pi}{2}\right)^2.$$

Let t = thickness of prong in the direction of vibration, then

$$k = \sqrt{\frac{t^2}{12}}, \text{ also } U = \sqrt{\frac{Y}{D}},$$

$$\therefore n = \sqrt{\frac{Y}{D}} \cdot \sqrt{\frac{t^2}{12}} \cdot \frac{\pi^2}{8l^2} \times (1{\cdot}19)^2,$$

$$\therefore Y = 39\,\frac{n^2 l^4 D}{t^2}.$$

EXAMPLE.—The prong of a g′ tuning-fork is 7·7 cms. long and ·32 cm. thick. Find the value of Young's modulus of steel. (The density of steel is 7·8 gms. per c.c.)

Then
$$Y = 39 \times \frac{(391)^2 \times (7{\cdot}7)^4 \times 7{\cdot}8}{(\cdot 32)^2}$$
$$= 1{\cdot}6 \times 10^{12} \text{ dynes per sq cm.}$$
(This is rather low.)

192. It will be noticed that the frequencies of the different modes of free harmonic vibration of a bar or tuning-fork are not exact multiples of the slowest, as they are in the case of perfectly flexible strings and (nearly) in perfectly cylindrical or conical tubes. The numbers are given in the tables in Art. 189, but the values for tuning-forks vary somewhat.

The following simple laws apply to most cases where the more rapid modes are not exact multiples of the slowest, *e.g.* to uniform rods, tuning-forks, plates, bells, columns of air of other forms than cylindrical or conical, such as resonators (Art. 97).

(1) The free vibration of such a body, if not harmonic, is the sum of two or more of its harmonic modes occurring together. The slowest of these is called the fundamental, and the others *partials*, *upper partials*, or *overtones*. These terms can be applied to any components of higher frequency than the fundamental, whether exact multiples of the fundamental or not. The term harmonics is restricted to exact multiples.

(2) When such a body is set in forced vibration by a periodic force, all the components of its motion are exact multiples of the frequency of the force, so that all the upper partials are harmonics of the fundamental. The components which have any considerable amplitude correspond

to components of the periodic force which nearly agree in frequency with possible free vibrations of the body.

It must be remembered that, even when a bar or fork vibrates quite harmonically, the air-waves from it are not exactly harmonic (Art. 25), and therefore they have harmonic components which are exact multiples of the frequency of the bar. So that, when a bar is in free non-harmonic vibration, the waves from it have two distinct sets of harmonic components: one set which correspond in frequencies to different modes in which the bar can vibrate, but are not exact multiples of the fundamental frequency; the other set having frequencies which are exact multiples of the fundamental frequency, but do not correspond to anything in the movement of the bar. The former set are called the *non-harmonic overtones*, the latter the *harmonic overtones*. The latter are still present when the motion of a fork has become quite harmonic.

If a tuning-fork vibrates in front of a spherical resonator whose slowest vibration frequency is the fundamental frequency of the fork, none of the overtones of the fork correspond with free-vibration frequencies of the resonator, and therefore the forced vibration of the resonator has no intense overtones, but is nearly harmonic. The waves so produced are the most nearly harmonic waves which can be made.

Exp. 94.—*Obtain the first overtone of a fork.* Carefully bow a tuning-fork near the middle point. The first overtone may be obtained (the vibration being performed is in the second figure of Fig. 142). The overtone is best obtained by bowing quickly, but

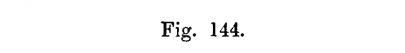


Fig. 144.

lightly taking the bow off the fork each time. If the fork be held horizontal and fine sand be sprinkled on the prongs the sand will collect at the node, which is about one-quarter the way down the prong (Fig. 144).

The frequency of the first overtone of a fork may be readily found by the dropping plate method.

193. Transversal Vibrations of Plates and Membranes.—Plates of glass, metal, or any elastic solid, clamped at one point and free everywhere else, can be made to vibrate transversely, as already explained in Art. 55. In the stationary undulation produced there are certain lines on the surface which have no movement in space, but where the surface slopes first one way, then the other. These lines are called nodal lines, and the parts of the surface separated by a nodal line are moving, at any one moment, in opposite directions. The antinodes, or places where the surface moves parallel to itself, are also lines. The nodal lines can be found by scattering sand on the surface and drawing a violin bow across the edge of the plate, which for this purpose must be fixed horizontally; the sand collects along the nodal lines, as it is thrown into the air from every other point. The figures so produced are called *Chladni's figures.*

In the case of a circular disc which is fixed at its centre the nodal lines are radii, and divide the disc into an even number of equal sectors. There are four such

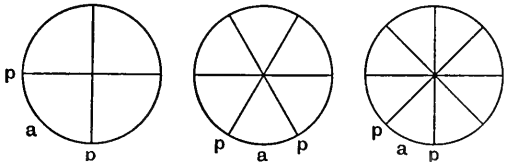


Fig. 145.

sectors if the edge of the disc is touched only by the bow, but if, while the plate is bowed, we touch with our fingers two points on the edge whose distance apart is contained an even number of times in the circumference, there will be that number of radial nodal lines, and two of them will run from the centre to the points touched by the fingers. Fig. 145 shows the arrangement for the first three tones of a circular plate fixed at the centre: points of bowing are marked *a*, and points of damping *p*. There may also be circular nodal lines concentric with the plate.

The frequencies of the same plate vibrating with different numbers of radial nodes are proportional to the squares of these numbers. The frequencies of plates of the same material vibrating with the same number of radial nodes are inversely as the squares of the radii and directly as the thicknesses of the plates.

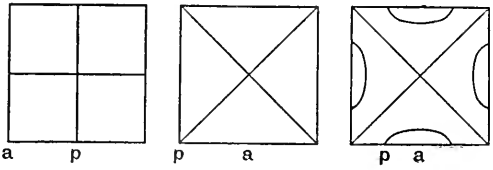


Fig. 146.

Circular stretched membranes, *e.g.* drumskins, vibrate in modes which are very similar, though the vibrations depend on the force with which they are stretched, not on elasticity, and the relative frequencies are not those of plates.

Plates of other forms can also be made to vibrate, their nodal lines being often very complex. Fig. 146 shows the

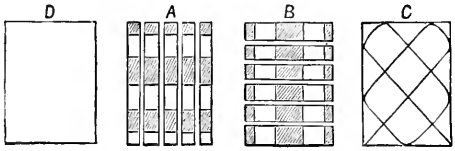


Fig. 147.

arrangement for the three tones of a square plate fixed at the centre: points of bowing are marked *a*, and points of damping *p*. In the case of square and rectangular plates they have a general resemblance to Lissajous' figures. The following device, due to Wheatstone, enables us in many cases to predict their forms:—A rectangular plate *D*, Fig. 147, might be supposed divided into a series of equal rods by parallel cuts in either of two directions. We will call these the *A* and *B* series respectively. Among the

modes of transverse vibration possible for these imaginary rods there is sure to be one mode for those of the *A* series, and one for those of the *B* series, of about equal frequency.

Suppose the rods made to vibrate in these modes, but with exactly equal frequencies and amplitudes, all the rods of the same series executing simultaneously the same movement, and the rods of both series reaching their stationary instants simultaneously. In the figure, the shaded parts of the rods are parts which might be moving all in one direction at the same time, while the unshaded parts were moving in the other direction. The actual movement of any part of the plate is the sum of the movements of the corresponding portions of the two fictitious rod-systems vibrating as above, and the nodal line of the actual vibration is one like that in *C*, at every point of which the movements of the fictitious rods would either both be zero or equal and opposite.

194. Bells.—A *bell*, which is simply a concave circular plate, bears to a flat plate the same relation as a tuning-fork to a straight bar, the advantage gained by the bent form being the same as explained above in the case of the fork. The modes of vibration of a bell are the same as those of a circular plate, the edge being divided by nodal points into any even number, not less than 4, of oppositely moving segments. This may be tested by holding a suspended pith ball against different parts of the edge, or by inverting the bell, filling it with water, and bowing the edge; ripples will be seen to proceed from all points except the nodes.

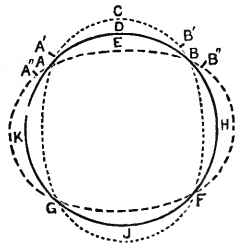


Fig. 148.

The curved form makes a difference which in the case of a bell is of some importance. The edge of a bell vibrating in four segments keeps changing its form from that of the dotted line in Fig. 148 to that of the broken one and back again, and its edge always passes through the points

ABFG, which are therefore considered nodes. These nodes, however, are not points where the substance of the edge is not displaced, as they are in a flat plate. The arc *ACB* is longer than the arc *ADB*, so that particles of the rim which were at *A*, *B* before the vibration began are at *A'*, *B'* when the rim has the dotted form and at *A''*, *B''* when it has the broken one. (The distances *AA'*, etc., are exaggerated in the figure.) There is therefore a tangential vibration of the material of the bell at *A*, *B*, *F*, *G*, as well as a radial vibration at *D*, *H*, *J*, *K*, and a vibration making an oblique angle with the edge at intermediate points.

As one of these movements cannot take place without the other (unless the edge alters in length), the whole movement is produced if we either make *D* vibrate radially or make *A* vibrate tangentially, so that striking the edge of the bell at *D*, or drawing a violin bow across that part of the edge, produces exactly the same vibration as applying a resined finger at *A* and drawing it along the edge. So that, when we run a wet finger round the edge of a tumbler, it vibrates just as if we struck it at a point 45° from the finger, and as the finger moves round, the mode of vibration moves round also, the finger being always at a point of purely tangential vibration, or node.

195. Torsional Vibrations of Wires and Rods.—If one end of a rod is twisted, the next portion twists, and a wave of twist or torsion travels along the rod. The velocity of such torsional waves depends on the rigidity of the substance, and their chief interest is as a means of determining this. They are reflected and form nodes and antinodes, exactly like longitudinal vibrations of rods.

For a rod of circular section the velocity of a torsional wave is

$$\sqrt{\frac{\text{dynamical measure of modulus of rigidity}}{\text{density}}}$$

$$= \sqrt{\frac{n}{D}}, \text{ say.}$$

If length of wire $= l$ cms.

then the wave-length $= 2l$ cms.

and the frequency $= \frac{1}{2l}\sqrt{\frac{n}{D}}$.

196. Law of Linear Dimensions.—A very important law, which applies to every kind of vibration of solids or fluids, is due to Bernouilli and is called the Law of Linear Dimensions. It states that bodies of geometrically similar form, and the same material, differing only in dimensions, when they vibrate in the same manner, have periods proportional to their linear dimensions.

EXAMPLES XII.

Elementary.

1. Describe experiments which illustrate the laws of transverse vibration of strings.

2. Two equally stretched strings of the same thickness, one of steel, the other of catgut, give the same note when struck. Which of them is the longer? Give reasons for your answer.

3. What variety of notes can you get out of a stretched string without altering its stretching force or length? What will be the effect of halving its length by a fixed bridge?

4. Explain the nature of the vibration of a stretched wire. What effect is produced by touching it at one-third its length from end to end?

5. State in what way the rate of transverse vibration of a stretched string depends upon the stretching force. How would you determine the rate of vibration of the string?

6. Explain the use of the monochord. How may it be employed to exhibit the relations which exist between the best known musical intervals?

7. Explain how the sonometer may be used for the determination of frequency.

8. Two similar strings on a sonometer are tuned to unison. One is 36 inches long and stretched by 100 lbs. Find the weight on the other one, which is 45 inches long.

9. Two similar strings are in unison. One is 36 inches long and stretched by 100 lbs. The other is stretched by 220 lbs. Find its length.

10. One wire is 100 inches long and bears a weight of 200 lbs. Another similar wire yielding the second higher octave of the first bears a weight of 130 lbs. Find the length of this wire.

11. A steel wire 30 inches long and $\frac{1}{50}$ in. diameter yields a note with a frequency of 200 vibrations per second. A second steel wire bears the same weight as the first, but is 20 inches long and $\frac{1}{40}$ in. diameter. Find the frequency of its fundamental note.

12. A steel wire one yard long, and stretched by a weight of 5 lbs., vibrates 200 times per second when plucked. If I wish to make two yards of wire vibrate twice as fast, with what weight must I stretch it?

13. Two forks when sounded together give 4 beats per second. One is in unison with a length of 96 cms. of a monochord wire under constant tension, and the other with 97 cms. of the same wire. Find the frequencies of the forks.

14. Explain how sounds of different quality but same frequency can be got from a sonometer wire by lightly touching it when bowed, or by bowing it at different points.

15. What is meant by the harmonics of a note of given frequency? How would you determine whether any of the notes given out by a bell were harmonics of its fundamental note?

16. A string is stretched by the weight of 60 lbs. and it is found that a hump (or small transverse wave) produced by striking it travels along it at 30 feet per second. By what weight must it be stretched to make the hump travel 90 feet per second?

17. A string stretched by the weight of 10 lbs. gives a certain note. By what weight must a second string of the same material, but of twice the length and twice the diameter, be stretched to make it give a note an octave higher than that of the first string?

18. Four violin strings, all of the same length and material, but of diameters in the ratios 4 : 3 : 2 : 1, are to be stretched so that each gives a note a fifth above the preceding. Compare the forces necessary.

ADVANCED.

19. A string whose mass is 500 grams hangs vertically, and a mass of 400 grams is fastened to its lower end. A transverse wave an inch in length is produced by shaking the lower end, and travels to the top of the string. What is the length of the wave at the middle and at the top?

20. A rope weighing half a pound to the foot is stretched horizontally by a force equal to the weight of 100 lbs. If the rope is shaken at one end, with what velocity do the waves run along it.

21. A wire a metre long, weighing $2\frac{1}{2}$ grams, is stretched on a sonometer by a weight of 18 kilograms, and when its length is adjusted to 80 centimetres and a vibrating fork applied, the string throws off a rider placed at any point except two, besides the ends. Find the frequency of the fork.

22. A bar 2 feet long and $\frac{1}{10}$ inch square, fixed at one end in a vice, makes 20 vibrations per second. How many vibrations will a bar of the same material, 1 foot long, $\frac{1}{4}$ inch wide, and $\frac{1}{8}$ inch thick, make per second if made to vibrate transversely in the direction of its thickness?

23. In performing Melde's experiment it was found that the string vibrated in 5 loops when 10 grams was placed in the scale-pan. What mass must be placed in the scale-pan to make the string vibrate in 7 loops? [Neglect the weight of the scale-pan.]

24. One end of a string is attached to a prong of a tuning-fork; the string passes over a small pulley and carries a weight at the other end. With a weight of 40 grams the string divides into four segments when the fork vibrates; what weight must be suspended to cause the string to vibrate in five and six segments? If the string is originally in the plane of vibration of the fork, what effect will be produced by turning the fork so that the plane of vibration of the fork is at right angles to the string?

25. Find the pitch of the fundamental note of a very light string of length l loaded in the middle with a mass m and stretched with a force f.

26. A uniform rope of length l hangs from a fixed point. It is set in transverse vibration. Find the velocity of the waves at any point and show that the time of passage of any wave is $\sqrt{\frac{4l}{g}}$ secs.

27. A sonometer wire fixed at the ends A and B is plucked aside at its middle point C so that AC, BC represent the two sides of an isosceles triangle. If it is released from rest, show by a set of figures how it will oscillate.

28. Describe the transverse vibration of a rod fixed at one end. Where is the node and where the antinode? What are the properties of the node and antinode?

29. Why does a steel knitting-needle fixed in a clamp at one end vibrate with greater frequency than a wooden rod of the same size fixed in the same way?

30. Describe an experiment to show that the frequencies of transverse vibration of rods increase as the lengths of the rods diminish.

31. Explain how the frequency of transverse vibration of a rod depends on its length, breadth, thickness, elasticity, and density. If a bar 2 feet long and $\frac{1}{10}$ inch square, fixed at one end, makes 5 transverse vibrations per second, how many vibrations per second would a bar of the same material, 1 foot long, $\frac{1}{4}$ inch wide, and $\frac{1}{5}$ inch thick make, if vibrating in the same manner in the direction of its thickness?

32. A steel knitting-needle exactly 2 mm. in diameter is fixed in a vice so that 6·23 cms. are free to vibrate. Find the frequency of the fundamental note of the transversal vibration.

EXAMINATION QUESTIONS IV.

1. In what way is the frequency of transverse vibration of a stretched wire affected (*a*) by halving the length of the wire, (*b*) by doubling the stretching force?
Describe any experiment which you would make in order to verify either of your statements.

2. How is it possible to cause a stretched string to emit a note having three times the frequency of its fundamental? Explain by the aid of a diagram the mode of vibration in this case.

3. According to what laws does the frequency of vibration of a stretched wire depend upon the length of the wire, the material of which it is composed, and the stretching force?
Describe experiments with the sonometer to verify these laws.

4. What do you understand by the harmonic modes of vibration of a stretched string?
When a violin string is bowed in the ordinary manner, several harmonic modes of vibration are induced in addition to the fundamental. How would you prove by experiment that this is the case?

5. How would you show that, when a stretched string is bowed, the octave above is generally sounded together with the fundamental note?
How should a stretched string be bowed to avoid sounding the octave?

6. Describe, giving possible numerical results, how you would prove that the frequency of vibration of a string vibrating transversely is proportional to the square root of the stretching force.
On increasing the weight stretching a given string by 2·5 kilograms, the frequency is altered in the ratio 3 : 2. Find the original stretching weight.

7. A string 50 cms. long, stretched by a weight of 10 kilograms, makes 256 transverse vibrations per second. How could the frequency of the note emitted be raised to 384 (1) by altering the length of the string, (2) by altering the stretching weight?
Could the string be made to emit a harmonic note of frequency 384 without altering either the length or the stretching weight? Explain fully.

8. The upper end of a given copper wire is fixed to a peg, and it supports a weight at its lower end. When bowed it emits a certain note. Show that, if the wire were drawn out to four times its original length, it would, when supporting the same weight and bowed, emit a note an octave lower than the first.

9. How would you find by means of the sonometer whether two wires of different diameters were made of the same material?

10. A wire stretched on a sonometer is touched with a needle at a point whose distance is one-fourth of the length of the wire from one end and the shorter section is then lightly bowed. Describe and explain the state of vibration of the longer section, and show how to test your statement experimentally.
What happens if the needle is moved gradually to either side of the point?

11. What conditions determine the velocity of transmission of a transverse wave in a stretched string?

12. Describe how experiments may be carried out to determine the law connecting the velocity at which a disturbance travels along a stretched wire with the tension.

13. A string stretched with a weight of 25 lbs., when made to vibrate transversely, gives a certain note. What force must be applied to a string of the same material, but of twice the length and twice the thickness, to make it give the octave above that note?

14. A string is stretched by such a weight that a hump raised upon it runs along it at the rate of 64 feet a second. Two points on this string, four feet apart, are firmly clamped to a sound-board without altering the tension of the string; if this part of the string be tweaked, what is the frequency of the note it will emit?

15. Four exactly similar and equal strings stretched with the same force are vibrating side by side; how will the note emitted be affected if they be fastened together so as to form one string by winding round them an extremely thin piece of silk?

16. Two strings of the same length and thickness, one (a) made of a material whose density is 1·3, and the other (b) of a density of 20·8, are stretched by forces equal respectively to the weights of

(*a*) 31 pounds and (*b*) 124 pounds. Calculate the ratio of their frequencies.

17. Compare the frequencies of vibration of two strings stretched with weights of 10 kilograms and 1 kilogram respectively. They are one metre long each, and they are of the same diameter, but their specific gravities are 7·8 and 1 respectively.

18. How may tones of different *quality* be obtained from the same stretched string? Explain the reason of the difference in quality.

19. A string is fastened to the end of one of the prongs of a tuning-fork, which is kept in vibration, and the tension of the string is gradually increased. Describe and explain how the string vibrates.

20. If four strings are all of the same length and material, but of diameters in the ratios of 1 : 2 : 3 : 4, and are all stretched to half their breaking stress, compare their vibration frequencies.

21. How would you illustrate resonance on a piano?

22. Point out the difference between a wave of sound, a wave of light, and a wave traversing a stretched string.

23. Show that if a train of waves moving with constant velocity along a string is totally reflected at a certain point, the direct and reflected trains will form a stationary system. Hence deduce the modes of transverse vibrations of a stretched string.

A string is under such tension that its length is increased by $1/n$ of the original length. Show that the frequency of the fundamental longitudinal vibration is $\sqrt{n} \times$ the frequency of the fundamental transverse vibration.

24. Discuss the modes of transverse vibration of a stretched string. A cycle wheel has 44 spokes, each 40 cms. long, and 0·2 cm. diameter, and their density is 7·8 grams per c.c. If they sound, when plucked, a note of frequency 256, calculate the stretching force of each, and find an approximate value for the thrust in the rim of the wheel.

25. Obtain an expression for the velocity of propagation of transverse waves along a stretched flexible cord.

A circle of flexible cord of radius r is rotating in its own plane with angular velocity ω. Show that if a small transverse disturbance is given to it, one wave of half the amplitude will move round the circle with velocity $2\omega r$ and another will remain at rest.

26. If two notes forming a major third, a little out of tune, are sounded together on a pianoforte or on the principal stop of an organ (rather narrow open pipes), show why the imperfection is more easily perceived than if the same two notes are sounded on two tuning-forks or on wide-stopped organ pipes.

ANSWERS.

EXAMPLES I. (*Page* 34.)

2. 4,000 $\frac{\text{cm.}}{\text{sec.}^2}$. 40,000 dynes.

4. Period $= 2\pi\sqrt{\frac{l}{g}}$, where l = length of string, and g = acceleration due to gravity. 2 secs.

8. $\frac{1}{200}$ sec.

9. Average velocity $= \frac{4 \times \text{amplitude}}{\text{period}}$; $\therefore$ ratio of average velocities $= \frac{20}{10} \times \frac{20}{10} = 4$.

11. The curve is Fig. 12 shifted through a right angle.

12. $\frac{1}{4}$ or $\frac{3}{4}$. **13.** (1) $\frac{2}{12}$, $\frac{4}{12}$, $\frac{8}{12}$, $\frac{10}{12}$; (2) $\frac{1}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{11}{12}$.

17. (1) A straight line inclined at 45° to the N.S. or E.W. line; (2) a circle; (3) a straight line perpendicular to (1); (4) a circle described in the opposite direction to (2).

18. The path is an ellipse, the longer axis being parallel to the direction of motion of the point with the greater amplitude.

EXAMPLES II. (*Page* 64.)

7. 135 cms. per second.

9. ·5 cm. $\frac{\text{Intensity at 100 metres}}{\text{Intensity at 200 metres}} = 4$.

12. 200π $\frac{\text{cms.}}{\text{sec.}}$, Four million ergs.

EXAMPLES III. (*Page* 98.)

1. 1,010,000 dynes per sq. cm. **5.** 19·65° C. **6.** $\frac{20}{21}$ sec.

7. 1,144 ft. per sec. **8.** 1,130 ft./sec., 1,120 ft./sec., 1,150 ft./sec.

9. − 45° C., 15° C., 55° C., − 55° C.

10. 555 ft., 4,440 ft., 2,815 ft. **11.** 819° C. **12.** 36·3.

13. $4\frac{3}{8}$ ft., $4\frac{27}{64}$ ft. 14. $4\frac{3}{8}$ ft. 15. $16\frac{17}{64}$ ft. 16. 278.
17. 900 ft./sec. 18. 10° C. 19. − 25° C. 20. 900° C.
21. 34,500 cms. per sec. 22. (i) 1,140; (ii) 1,140.
23. 44,300 cms. per sec. 24. 130,600 cms. per sec.
25. 1,410,000 $\frac{\text{dynes}}{\text{cm.}^2}$. 26. 29,700 cms. per sec. 27. −193° C.

Examples IV. (*Page* 116.)

5. 540.

Examples V. (*Page* 139.)

2. 1,860. 3. 256; 4·4 ft. 5. 12·8. 6. 406·7.

Examination Questions I. (*Page* 139.)

2. 5 cms., 30 cms. per sec., $\frac{1}{6}$ sec. 5. 1,680 ft.
8. 275 vibrations per sec.; frequency (275) unchanged, velocity 4,400 ft./sec., wave-length 16 ft.
11. 560. 16. 56° C. 18. 220.

Examples VI. (*Page* 153.)

7. If x = displacement at any instant the accelerations due to the two forces are $\frac{4\pi^2}{T_1^2}x$ and $\frac{4\pi^2}{T_2^2}x$, ∴ acceleration due to both $= 4\pi^2\left\{\frac{1}{T_1^2} + \frac{1}{T_2^2}\right\}$, and this $= \frac{4\pi^2}{T^2}$; ∴ $T = \frac{T_1T_2}{\sqrt{T_1^2 + T_2^2}}$.
Let T_1 = gravity period, T_2 = elasticity period. Period in hanging position $= \frac{T_1T_2}{\sqrt{T_1^2 + T_2^2}}$;
period in upright position $= \frac{T_1T_2}{\sqrt{T_2^2 - T_1^2}}$.

Examples VII. (*Page* 171.)

2. Yes. One is able to foresee what resonance the wave system is likely to produce.
3. One of the harmonic components must have a period equal to that of the free vibration of the body.

Examples VIII. (*Page* 215.)

8. $\frac{3}{2}$, or a fifth. 10. $\frac{2}{1}$. $\frac{9}{8} \times \frac{10}{9} \times \frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} \times \frac{9}{8} \times \frac{16}{15} = 2$.
11. $\frac{5}{4}$, $\frac{4}{3}$, 1·059 (= $\frac{18}{15}$ nearly), $\frac{81}{80}$ (called a *comma*).

12. The four notes whose frequencies are in the ratio 4 : 5 : 6 : 8. Doh-Me-Soh-Doh'.

13. 384, 768, 192. **14.** $213\frac{1}{3}$ (or $426\frac{2}{3}$ for the higher C).

18. $2^{\frac{4}{12}} \times 2^{\frac{4}{12}} \times 2^{\frac{4}{12}} = 2$. **20.** 2.

23. (*a*) Infinite; (*b*) 100; (*c*) 400; (*d*) 0. **24.** $\frac{21}{19}$.

Examples IX. (*Page* 248.)

7. 4. **13.** 5 secs. **14.** 672 ft.

Examples X. (*Page* 264.)

5. 4·25 cms. and every odd multiple of 4·25 cms. from the wall.

6. 5,000.

Examination Questions II. (*Page* 264.)

9. 600.

Examples XI. (*Page* 305.)

5. The open pipe gives a note an octave above the other.

8. 1,808 ft./sec. **9.** 41·5; 124·5, 207·5 cms.; 83, 166, 249 cms.

10. (*a*) 50, 100, 150, 200; (*b*) 25, 75, 125, 175; (*c*) 50, 100, 150, 200.

11. 360. **12.** 42·7. **13.** 60.

14. 52·9 cms. **15.** 15·4, 48·6, 81·8 cms. from top.

16. 8 ins. **17.** At the middle. **18.** (*a*) 19,000. (*b*) 236.

20. 1,310 ft./sec. **21.** $1{\cdot}39 \times 10^8$ poundals per sq. ft.

22. 715.

Examination Questions III. (*Page* 307.)

1. 266.

2. Maximum resonance when air column is 16 cms., $49\frac{1}{3}$ cms., and $82\frac{2}{3}$ cms. long.

3. Frequency, 366 (nearly); wave length, 136·7 cms. (Same as in air).

4. $n \propto \frac{\sqrt{\text{absolute temperature}}}{\text{length}}$. A wider pipe is effectively a little longer than a narrow one of the same length because it has a greater end correction.

5. 1 ft. **6.** (*a*) Yes, greater. (*b*) Yes, less.

7. In the middle of a closed tube the air is always at rest, in the middle of an open tube the air is always in motion except at two instants during one complete vibration.

13. Odd multiples of 256.
15. An antinode occurs at the hole.
20. 25.
21. Tube is 3 wave-lengths long.
24. Pitch rises a tone.

Examples XII. (*Page* 365.)

2. The catgut.
3. Doubling the frequency.
4. Frequency trebled.
8. 156 lbs.
9. 53·4 ins.
10. $20\frac{1}{7}$ ins.
11. 240.
12. 80 lbs.
13. 388, 384.
16. 540 pds.-wt.
17. 640 pds.-wt.
18. 1024 : 1296 : 1296 : 729.
19. $\sqrt{\frac{3}{8}}$ in., $1\frac{1}{2}$ ins.
20. 80 ft. per sec.
21. 498.
22. 100.
23. 5·1 gms.
24. 25·6 gms., 17·8 gms., doubled.
25. $\frac{1}{l}\sqrt{\frac{f}{m}}$.
31. 40.
32. 380.

Examination Questions IV. (*Page* 368.)

1. (*a*) Doubled. (*b*) Increased to $\sqrt{2}$ times its original value.
5. Bow at the middle.
6. 2 kilos.
7. (1) Alter length to $33\frac{1}{3}$ cms. (2) Alter stretching weight to $22\frac{1}{4}$ kilos.
13. 1,600 lbs.
14. 8.
15. Not at all.
16. 2 : 1.
17. 1·13 : 1.
19. 3 : 2.
20. Equal.
24. $1{\cdot}03 \times 10^8$ dynes, $7{\cdot}2 \times 10^8$ dynes.

INDEX.

ACOUSTIC CLOUDS, 248
Acoustic pendulum, 2
Action of resin, 292
Adiabatic elasticity, 69, 72
Aeolian harp, 352
Aerial echoes, 248
Air blast, 283
Air columns, 267
" " , Resonant vibrations of, 269
Alteration of pitch by reflection, 242
Amplitude, 14, 60
" of forced vibrations, 145
Antinodes, 254, 255, 320
Appunn and *Preyer*, 180
Appunn's tonometer, 137
Audibility, Limit in pitch for, 180

BAR, CLAMPED-FREE, 353
" , Free-free, 353
Beats, 113, 136, 149, 150, 189, 203, 325
Beat-tone, 207
Bells, 363
Bird-call, 225, 260
Blackburn's pendulum, 30
Bravais and *Martin*, 90
Bulk (or volume) elasticity, 67
" " " modulus, 68

CHANGE OF SIGN, 218
Characteristics of musical sound, 176
Characteristics of stationary undulation, 255
Cheshire's disc, 258
Chladni's figures, 361
Chord, 192
" , Common, 193
" , Major, 193
Chronograph, 124
Circular motion, 12
Clamped-free bar, 353
Clarinet, 289
Closed tube, 268, 275
Cochlea, 172, 174
Colladon and *Sturm*, 92
Combination tones, 203, 205
" or resultant tones, 152
Comparison of frequency, 127, 332
Components, 155
" , Change of, 166
" , Harmonic, 159
Composition of vibrations, 21
Compression modulus, 79
Concert pitch, 202
Concertina, 291
Concord, 191, 192
Condensation, 37
" in a wave system, 43
Condition of air in resounding tube, 273
Configurations of a vibrating string, 345
Conical pipes, 283
Constitution of a wave system, 167
Crova's disc, 50

DENSITY MOMENT, UNIform, 252, 256, 275
Derham, 88
Determination of frequency, 119
Diatonic scale, 194
Difference tone, 153, 205
Disc, *Crova's*, 50
Discord, 189
Displacement of curve, 8
,, in a wave system, 43
Dominant, 194
Donkin, 356
Doppler's principle, 210
Dropping plate, 124
Drum-skin, 172, 208
Duhamel's vibroscope, 123
Dust tube experiment, *Kundt's*, 301

EAR, STRUCTURE OF THE, 172
Ear trumpets, 238
Echoes, 90, 224, 234
,, , Aerial, 248
Edison, 131
Effect of wind, 243
Effects of temperature, 245
Elasticity, 67
Electrical maintenance of fork, 119
Embouchure, 287
End-correction, 280
Energy of progressive undulation, 53
,, of stationary undulation, 256
,, of vibrating particle, 15
,, of waves, 111
Equally tempered scale, 200

FIXED PITCH THEORY, 290
Flute, 285
Forced vibration, 142, 145, 182
Fourier's theorem, 157
Free reed, 288
,, vibration, 142
French horn, 289
Frequency, 8
,, by resonance, 282
,, , Comparison of, 127, 332
,, , Determination of, 119
,, , Determination of, by indirect methods, 138
,, , high, Determination of, 262
,, , Measurement of, 331
Fundamental, 161, 194, 280

GALTON'S WHISTLE, 181
Gas-flame, Reflection from, 225
Gas-lens, Refraction through, 241
Greely, 90

HARMONIC ANALYSIS, 163
Harmonic components, 134, 156, 162
,, curve, 17
,, overtones, 360
,, waves, 48
Harmonics, 161, 280
,, of a string, 333
Harmonium, 291
Harmonograph, *Tisley's*, 31
Heat production, 60
Helmholtz, 133, 180, 203, 347, 352
Helmholtz's experiment on quality, 184
,, notation, 202
,, resonators, 169
Hollow sound, 267
Hopkin's forked tube, 110
Huyghen's construction for reflected waves, 229
,, construction for refracted rays, 239

INTENSITY OF SOUND, 59
Interference, 101, 114
Interferometer, 206
Interval, 196
Inverse squares, Law of, 59
Isochronous, 16
Isothermal elasticity, 69, 71

KALEIDOPHONE, *WHEATSTONE'S*, 28
Koenig, 203
Koenig's manometric capsules, 137
„ manometric flames, 296
„ wave siren, 187
Kundt, 298
„ and *Warburg*, 301
Kundt's dust tube experiment, 301

LAPLACE'S CORRECTION, 86
Law of inverse squares, 59
„ „ linear dimensions, 365
Laws of plates, 362
„ „ rods, 354
„ „ strings, 326
Limits of hearing, 180
Liquids, Waves in, 312
Lissajous' figures, 21, 128, 129
Longitudinal vibrations, 267
Loudness, 176, 181

MAJOR DIATONIC, 194
„ tone, 195
Manometric capsule, 169
„ flames, *Koenig's*, 296
Measurement of frequency, 331
Melde's experiment, 342
Membrana basilaris, 174
Mersenne's laws, 326
Metronome, 236
Minimum amplitude, 62
Minor diatonic scale, 195
Minor tone, 194
Modulus of elasticity, 67
Moisture and velocity, 85
Molecular structure, 93
„ velocities, 93
Monochord, 322
Musical instruments, 351
„ notes by reflection, 236
„ scales, 192
„ sound, 117
„ „ , Characteristics of, 176

NEWTON, 90
Newton's velocity of sound, 86
Nodes, 254, 255, 319
Noise, 187
Noises, 290
Non-harmonic overtones, 360
„ vibration, 285
Note, 186

OBSTACLES, EFFECT OF, 229
Octave, 191
Open pipes, 287
„ tube, 268, 275
Organ, 288
„ pipes, 286
Oscillograph, 131
Overtones, 280, 360

PANDEAN PIPES, 285
Pendulum, 12
Pendulum, Acoustic, 2
„ , *Blackburn's*, 30
Pendulums, Experiments with, 144
Perception of sound direction, 214
Period, 8
„ of forced vibrations, 145
Persistence of sound sensations, 234

Phase, 20
„ difference, 20
Phonautograph, 130, 168
Phonograph, 131, 168
„ records, Manufacture of, 132
Physiological difference tones, 208
Piano scale, 201
Pitch, 176, 177
„ by reflection, Alteration of, 242
„ with motion, Variation of, 210
Pitch-fork, 136
Plates, Transverse vibrations of, 361
Polysyllabic echoes, 235
Pressure and velocity, 83
Progressive undulation, 42
„ „ , Transverse, 312
Pulse, 43
„ along a fluid, 74
„ „ a rod, Velocity of a, 72
Pythagorean Scale, 197

QUALITY, 132, 176, 183
„ of note of air column, 286
„ of note of rod, 293
„ of sound from fork, 359
„ of sound from string, 338
Quincke's branched tube, 107

RAREFACTION, 37
Rayleigh, 60, 62, 114, 180, 183, 204, 214, 243, 354
Rayleigh and *Ramsey*, 305
Rayleigh's experiment, 261
Rays, Sound, 246
Reed pipes, 288
Reflected waves, *Huyghen's* construction for, 229
Reflection, 223
„ of spherical waves, 231
„ of transverse waves, 317
„ with change of sign, 217, 317
„ without change of sign, 220, 317
Refracted rays, *Huyghen's* construction for, 239
Refraction of a spherical wave, 240
Regnault, 91
Relative pitch theory, 290
Resonance, 120, 147, 150, 175, 325
„ box, 136
„ , True and false, 150
„ with a bottle, 271
Resonant vibration, 143, 161
„ „ of air columns, 268
Resonators, 169
Resultant tones, 205
„ „ , Combination or, 152
Rigidity correction for a string, 350
Ripples, 312
Rods, 267
„ of *Corti*, 174
„ , Transverse vibration of, 352
Rucker and *Edser's* experiment, 205

SAVART, 295
Savart's toothed wheel, 118, 133, 178
Scale, Major, 194
Schiebler's tonometer, 137
Seebeck, 356
Seebeck's siren, 119, 133
Semitone, 194

Sensitive flame, 225
Shadow, Sound 229
Shape elasticity, 67
Simple harmonic motion, 14
Sine curve, 17
Siren, 133
„ , Double, 135, 202
„ , *Koenig's* wave, 187
Sonometer, 322
Sound, Cause of, 1
„ , Determination of velocity by indirect methods, 298
„ direction, 214
„ , Hollow, 267
„ in air, Velocity of, 81, 87
„ in water, „ „ 91, 92
„ , Intensity of, 3, 59
„ , Musical, 117
„ rays, 246
„ shadow, 229
„ , Speed of, 6
„ through apertures, Transmission of, 225
„ , Velocity of, 97, 298
Sounding board, 182
Speaking tube, 237, 263
Specific heats of a gas, Ratio of, 81, 305
Speed of sound, 6
Spherical waves, Reflection of, 231
„ „ , Refraction of, 240
Spiral wire, 51
Standards of pitch, 202
Stationary undulation, 251, 254
Stationary undulation caused by reflection, 259, 263
Stationary undulation, Characteristics of, 255
Stationary undulation, Energy of, 256
Stationary undulation, Transverse, 313
Stethoscope, 6
Stokes, 87
Stone, 89
Stopped pipes, 287
Striking reed, 288
String, Compound vibration of, 336
„ , Harmonics of a, 331
„ , Specification of, 313
Strings, Resonant vibrations of, 342
Stroboscope, 121
Structure of the ear, 172
Subdominant, 194
Summation tone, 153, 205
Superposition, 227
„ , Principle of, 101
Synthesis of sound, 184

TELEPHONE, TOY, 5
Telephonic theory of audition, 175
Temperature and velocity, 84
„ , Effects of, 245
Tempered scale, 200
Timbre, 177, 183
Tisley's harmonograph, 31
Tone, 186
Tonic, 194
„ sol-fa, 195
Tonometer, 137
Torsional vibrations of wires, 364
Transmission of sound through apertures, 225
Transversal vibrations of plates, 361
Transverse progressive undulation, 312
„ stationary undulation, 318
„ undulation, 311
„ vibration of rod, 352
„ vibration of string, 321
„ wave, 311
„ waves along strings, Velocity of, 313
„ waves, Reflection of, 317
Tube, Speaking, 237, 263

Tuning-fork, 136, 358
Tunnel, Sound effect in, 245
Tympanum, 172
Tyndall, 248
Tyndall's sensitive flame, 261

UNDULATION, 43
Undulation caused by reflection, Stationary, 259, 263
Undulation, Characteristics of stationary, 255
,, , Energy of stationary, 256
,, , Progressive, 42
,, , Stationary, 251, 254
,, , Torsional, 364
,, , Transverse, 311
,, , Transverse progressive, 312
,, , Transverse stationary, 318
Uniform density moment, 252, 256, 275
Unison, 145

VELOCITY CURVE, 10
Velocity in air, 79
Velocity of a pulse along a fluid, 74
,, of a pulse along a rod, 72
,, of sound, 97, 298
,, ,, by resonance, 281
,, ,, in air, 81, 87
,, ,, in water, 91, 92
,, of transverse waves along strings, 313
Vibration, 8, 43
,, , Forced, 142, 145, 182
,, , Free, 142
,, , Longitudinal, 267
,, microscope, 129, 340, 347
Vibration, Non-harmonic, 285
,, of air columns, Resonant, 268
,, of liquid columns, 291
,, of plates, Transverse, 361
,, of rods, 352
,, of solid rods, 291
,, of strings, Transverse, 321
,, of wires, Torsional, 364
,, , Resonant, 143
,, , Simple harmonic, 14
Vibrations, Composition of, 21
Vibroscope, *Duhamel's*, 123
Vocal chords, 290
,, sounds, 290
Volume or bulk elasticity, 67
,, modulus, Bulk or, 68

WAVE-FORM, 45, 132, 168, 186
Wave-form, Gradual change of, 97
Wave-front, 58
Wave-length, 47, 59
,, ,, by resonance, 280
Wave-machine, 222
,, ,, , *Weinhold's*, 51
Waves, Energy of, 111
,, , Harmonic, 48
Weinhold's wave-machine, 51
Wheatstone, 362
Wheatstone's kaleidophone, 28
Whispering gallery, 238
Whistle, Double, 209
,, , Tin, 285, 291
Wind, Effect of, 243
Wires, Torsional vibrations of, 364

YOUNG'S modulus, 67

PRINTED AT THE BURLINGTON PRESS, FOXTON, NEAR CAMBRIDGE, ENGLAND

A complete Catalogue of Text-Books published by the University Tutorial Press, and separate Sectional Catalogues in English Language and Literature, French, Mathematics, and Science, may be had on application to the Publisher.

SELECTED TEXT-BOOKS IN SCIENCE

PUBLISHED BY THE

University Tutorial Press Ld.

25 High St., New Oxford St., W.C. 2.

Chemistry.

The Tutorial Chemistry. By G. H. Bailey, D.Sc., Ph.D. Edited by Wm. Briggs, LL.D., M.A., B.Sc., F.C.S.

Part I. Non-Metals. *Fourth Edition.* **5s.**

Part II. Metals and Physical Chemistry. *Fifth Edition.* **7s. 6d.**

Qualitative Analysis. By Wm. Briggs, LL.D., M.A., B.Sc., F.C.S., and R. W. Stewart, D.Sc. **4s.**

Elementary Quantitative Analysis. By Wm. Briggs, LL.D., M.A., B.Sc., F.C.S., and H. W. Bausor, M.A., F.C.S., F.I.C. **3s.**

Senior Chemistry. By G. H. Bailey, D.Sc., Ph.D., and H. W. Bausor, M.A., F.C.S., F.I.C. *Second Edition.* **6s. 6d.**

Senior Practical Chemistry. By H. W. Bausor, M.A., F.C.S., F.I.C. **4s.**

Senior Volumetric Analysis. By H. W. Bausor, M.A., F.C.S., F.I.C. **2s.**

Junior Chemistry. By R. H. Adie, M.A., B.Sc. *Second Edition.* **3s. 6d.**

Preliminary Chemistry. By H. W. Bausor, M.A., F.C.S., F.I.C. **2s.**

Bailey's Elementary Chemistry. By Dr. G. H. Bailey. Edited by Dr. Wm. Briggs. *Fourth Edition.* **6s. 6d.**

Chemistry for Matriculation. By G. H. Bailey, D.Sc., Ph.D., and H. W. Bausor, M.A., F.C.S., F.I.C. *Second Edition.* **8s. 6d.**

An Introductory Course of Chemistry. By H. W. Bausor, M.A., F.C.S., F.I.C. **3s. 6d.**

Biology—*continued.*

Plant Biology. By Dr. CAVERS. 6s.

Life Histories of Common Plants. By Dr. CAVERS 4s. 6d.

Plant and Animal Biology, A First Course in. By W. S. FURNEAUX. 3s. 6d.

Zoology, Text-book of. By H. G. WELLS, B.Sc., and A. M. DAVIES, D.Sc. Revised by J. T. CUNNINGHAM, M.A *Sixth Edition.* 8s. 6d.

Applied Sciences, etc.

Electrical Engineering, Continuous Current. By W. T. MACCALL, M.Sc., M.I.E.E. *Second Edition.* 15s.

Electrical Engineering, Alternating Current. By W. T. MACCALL, M.Sc., M.I.E.E. 15s.

Technical Electricity. By H. T. DAVIDGE, B.Sc., M.I.E.E., and R. W. HUTCHINSON, M.Sc., A.M.I.E.E. *Fourth Edition.* 12s. 6d.

Junior Technical Electricity. *Second Edition.* By R. W. HUTCHINSON, M.Sc., A.M.I.E.E. 4s. 6d.

Wireless Telegraphy and Telephony, A Text-book of. By W. GREENWOOD, B.Sc., A.M.I.E.E., A.C.G.I. 5s. 6d.

Wireless, A First Course in. By R. W. HUTCHINSON, M.Sc., A.M.I.E.E. 3s. 6d.

Engineering Science, A First Course in. By P. J. HALER, M.B.E., M.Sc., A.M.I.Mech.E., A.I.E.E., and A. H. STUART, Ph.D., B.Sc., F.R.A.S. *Third Edition.* 3s. 6d.

Engineering Science, A Second Course in. By P. J. HALER and A. H. STUART. 5s.

Heat Engines, A First Course in. By J. W. HAYWARD, M.Sc., A.M.I.Mech.E. *Second Edition.* 4s.

Building Construction, A First Course in. By BRYSSON CUNNINGHAM, D.Sc., B.E., F.R.S.E., M.Inst.C.E. *Third Edition.* 4s.

Hygiene, A First Course in. By R. A. LYSTER, M.D., Ch.B., B.Sc., D.P.H. *Seventh Edition, Revised.* 4s.

Hygiene, A Second Course in. By A. E. IKIN, LL.D., B.Sc., and R. A. LYSTER, M.D., D.P.H. *Third Edition.* 5s.

QC
225
C35
1913
cop. 3

Catchpool, Edmund
Text-book of sound
5th ed.

Physical &
Applied Sc.

PLEASE DO NOT REMOVE
CARDS OR SLIPS FROM THIS POCKET

UNIVERSITY OF TORONTO LIBRARY